On 3-groups of second maximal class

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This thesis describes an algorithm and an implementation of this
algorithm for generating finite p-groups. It is essentially an extension
algorithm which given a group G calculates new groups $H_1, \ldots, H_m$ all
of which have G as a certain kind of quotient. The new groups
$H_1, \ldots, H_m$ can be used as input for the algorithm and more new groups are
calculated. The p-groups generated in this way can be used to form a
tree. These trees are frequently infinite. Parts of the trees are drawn
as tree diagrams. These tree diagrams are a succinct way of representing
some results about p-groups.

Detailed tree diagrams of 2-groups and 3-groups of maximal class and
2-groups of second maximal class are drawn using previously known results.
The implementation of the algorithm has been used to calculate all
2-generator 3-groups of second maximal nilpotency class up to order 3^8
and many up to order 3^{10}. This is approximately 2,500 groups. The
calculations were used to draw a tree diagram for some 2-generator
3-groups of second maximal class. Seventeen infinite branches of this tree
are exhibited. These are of several different types. For one type, one
infinite branch has been studied in detail and a complete description of
the tree associated with this infinite branch is given.

The presentations of the p-groups calculated by the algorithm are
thought of as being standard. The final chapter describes a process for

recognizing such a presentation or a presentation which gives a group
isomorphic to a group with a standard presentation.