A QUANTITATIVE STUDY OF MAGNETIC FLUX TRANSPORT ON THE SUN

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Abstract: A computational model, based on diffusion, differential rotation, and meridional circulation, has been developed to simulate the transport of magnetic flux on the Sun. Using Kitt Peak magnetograms as input, we have determined a best-fit diffusion constant by comparing the computed and observed fields at later times. Our value of 730 ± 250 km^2/s is consistent with Leighton's (1964) estimate of 770-1540 km^2/s and is significantly larger than Mosher's (1977) estimate of 200-400 km^2/s. This suggests that diffusion may be fast enough to account for the observed polar magnetic field reversal without requiring a significant assist from meridional currents.

This paper presents the initial results of a project to simulate the transport of solar magnetic flux using diffusion, differential rotation, and meridional circulation. The study concerns the evolution of large-scale fields on a time scale of weeks to years, and ignores the rapid changes that accompany the emergence of new magnetic regions and the day-to-day changes of the supergranular network itelf.

Our initial objective was to determine the value of an effective diffusion constant that would provide the best fit between the computed and observed fields. To our knowledge, no fully quantitative determination has been attempted using modern computational techniques and high-quality observations. A resolution of the discrepancy between Leighton's (1964) model-dependent estimate of (770-1540 km^2/sec) and Mosher's (1977) semi-quantitative estimate of (200-400

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km^2/sec) would help to determine the relative importance of diffusion and meridional circulation in transporting flux to the Sun's poles. With a value as low as Mosher's, diffusion would require a substantial assist from meridional flow to produce a timely reversal of the polar magnetic field, whereas with a value as high as Leighton's, diffusion could accomplish the reversal alone.

We assume that the photospheric field's radial component satisfies a continuity equation containing the transport effects of Leighton diffusion, differential rotation, and meridional circulation. In spherical coordinates this equation is:

$$\frac{\partial B}{\partial t} = \frac{1}{\tau_1}\left[\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial B}{\partial\theta}\right) + \frac{1}{\sin^2\theta}\frac{\partial^2 B}{\partial\varphi^2}\right] + \frac{1}{\tau_2}\left[\cos^2\theta\frac{\partial B}{\partial\varphi}\right] + \frac{1}{\tau_3}\left[\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(B\sin\theta\sin 2\theta\right)\right],$$

where the time constants are $\tau_1 = R^2/K$ (K is the diffusion constant), $\tau_2 = 1/\omega_o$ (ω_o is the differential rotation rate), and $\tau_3 = R/V_o$ (V_o is the amplitude of the meridional flow speed). Typical values are τ_1 = 20 years (for K = 800 km^2/sec), τ_2 = 21 days (for a Newton and Nunn rate of 2.77 deg/day), and τ_3 = 1 year (for a meridonal circulation amplitude of 20 m/sec (Duvall, 1977)).

Our study has progressed through several increasingly difficult and realistic phases. First, to test our modeling procedures, we used photographic prints of Kitt Peak magnetograms to estimate the location, pole separation and strength of new bipolar magnetic regions during 1976-1981 as the present sunspot cycle evolved. Using these doublet sources, and an assumed initial global dipole field of 1 Gauss, we calculated the evolution of the large-scale fields and displayed the fields in Carrington synoptic charts which we updated at 27-day intervals to include the contributions of flux from newly emerging regions.

Second, to estimate the diffusion constant, we selected four relatively isolated magnetic regions which were observed on several consecutive solar rotations. For each of these regions, our objective was to deposit the Kitt Peak digital magnetic measurements into the simulator during the initial solar rotation, and to compute the evolving large-scale magnetic fields at elapsed times of 1, 2, and 3 rotation periods for a range of values of the diffusion time τ_1. In practice, we also considered other combinations such as depositing the flux during its second disk appearance and comparing the computed and observed fields on the third rotation. This procedure not only provided additional tests of our measurement consistency but also helped to avoid possible magnetic field calibration errors associated with the presence of sunspots during the initial rotation. In this second phase of our study, we deposited digital data for only a limited area including the source region. The surrounding area contained the typical, but unrealistic, background field computed from the doublet sources of phase one. To minimize the unknown errors associated with this unrealistic background field we limited our comparisons to the immediate area of the expanding bipolar region.

For each region, the best fit was obtained numerically by minimizing the sum of the squared errors between the computed and observed fields and by allowing for a possible bias in the zero level of the Kitt Peak measurements. A relatively broad minimum was always obtained probably due to the unknown influence of the weak background fields. However, in half of these cases the minimum solution also provided close agreement between the observed and computed values of the centroids and the total surviving flux of the magnetic region.

The results for the case of V_o=0 are shown in the following table:

TABLE 1 - DIFFUSION CONSTANT MEASUREMENTS

Region	A	B	B	C	C	C	D	D
Deposition Rotation	1	0	2	0	0	1	0	2
Comparison Rotation	2	1	3	1	2	2	1	3
Diffusion Constant	470*	680*	700*	1160	1140	1220	1690	1070*

Summary: K = 1050 $\pm$ 400 km^2/s (all 8 measurements)
K = 730 $\pm$ 250 km^2/s (4 best measurements)

* most reliable determinations, based on considerations of the location and total flux of the evolving region.

The eight determinations in Table 1 yield an average diffusion constant of 1050 $\pm$ 400 km^2/sec, but the four most reliable measurements give the smaller value of 730 $\pm$ 250 km^2/sec. Both are significantly larger than Mosher's (1977) 200-400 km^2/sec, and are consistent with Leighton's (1964) 770-1540 km^2/sec. In future phases, we plan to study the effects of including the observed background field as well as a non-zero value of meridional circulation speed.

Figure 1 illustrates some of our procedures graphically. The upper left frame shows a Kitt Peak magnetogram on December 30, 1976. A large bipolar magnetic region (BMR) is visible in the southern hemisphere on this otherwise quiet day near sunspot minimum. The upper middle frame shows these fields after suppressing the short wavelength components of their Fourier representation. Obviously on this day the Kitt Peak zero level was biased in the negative (blue) direction and the positive-polarity north polar field is not visible. The upper right frame, displaying the normal component rather than the line-of-sight component of field, shows the same BMR deposited within the artificial background field. Relatively few BMRs have contributed to this background field at this early stage of the new sunspot cycle.

The lower frames in Figure 1 show the situation 28 days later on January 27, 1977. The lower left and center frames show the raw and smoothed Kitt Peak measurements, respectively. The lower right frame shows the BMR and its surroundings as they would have evolved with a diffusion constant of 680 km^2/sec. At this time, a newly emerged BMR, visible in the magnetogram's northern hemisphere, has not yet been deposited in the simulator. A recently deposited doublet is visible west of the large BMR in the southern hemisphere. Our measurements are based on a comparison of the flux distributions only in the immediate vicinity of the evolving BMR and not in the surrounding areas where disagreement is both obvious and expected.

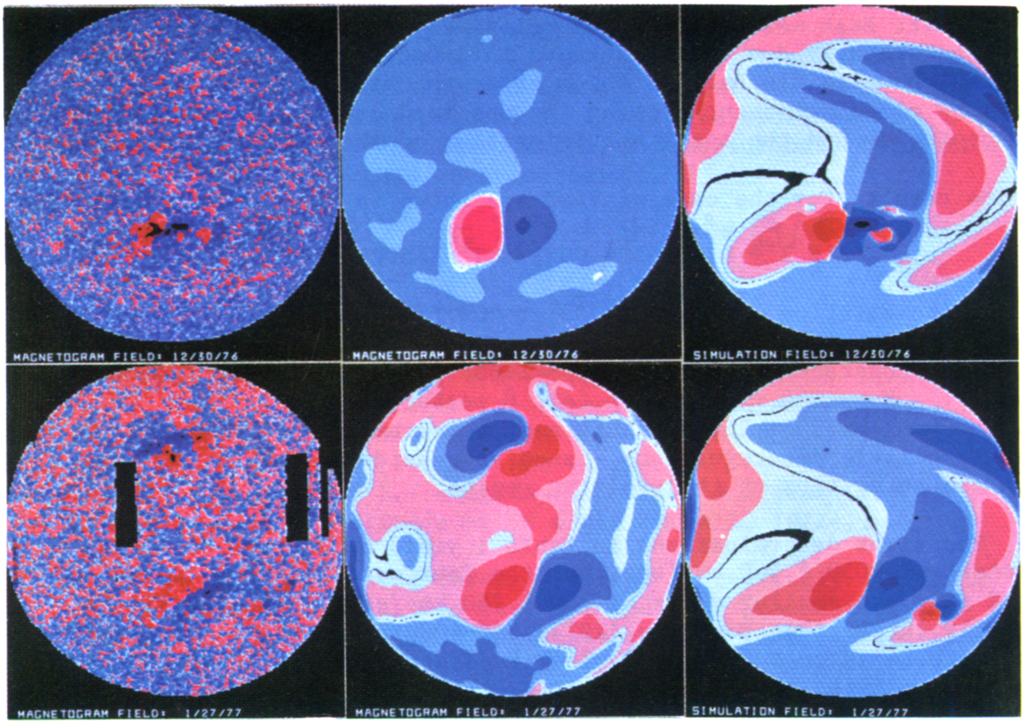

Fig. 1: Observed BMR (upper left and center) is deposited into simulated field (upper right), evolved (lower right), and compared with later observed field (lower left and center).

In conclusion, we should like to emphasize that our study is just reaching the point where most of the technical problems have been overcome. While our results to date are tentative, they are encouraging and in particular give a diffusion constant that is consistent with Leighton's original estimate.

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DISCUSSION

IONSON: Could you give us some idea of the physics hidden in your black box?

SHEELEY: We begin with a continuity equation for the variation of the normal component of the magnetic field. We assume that the magnetic flux is transported by the combined effects of random-walk diffusion, differential rotation, and meridional circulation. Then by continually entering new sources of flux as they occur, we calculate the expected evolution of the large-scale fields.

GIOVANELLI: This is a highly important investigation. I hope that it will be possible to study flux migrations on a shorter time scale than one rotation, particularly when dealing with the fate of flux originating near the equator, which I suspect is of vital importance for the cycle mechanism.

SHEELEY: Eventually we do plan to consider a short time scale, perhaps using daily magnetograms or magnetograms every few days. Meanwhile to establish a realistic background field for the future short-term study we will try to use 27-day synoptic data and run the program for several years into the present sunspot cycle.

GOKHALE: This is a very interesting and instructive way of studying the magnetic flux transport on the Sun. Won't it be useful to include source and sink terms in the equations to account for the emergence of new flux and for disappearance of flux either by dissipation or by fluxtubes going above the surface?

SHEELEY: In fact we do include the eruption of new sources in our study of the evolution of the large-scale field during the sunspot cycle. We provide a sink by adding the positive and negative fields algebraically so that they annihilate when they overlap.

KOUTCHMY: I would like to know if the method of "smearing" you used can reproduce the neutral lines we observe over the sun, in particular through tracers like filaments?

SHEELEY: Neutral lines are clearly visible in our synoptic plots of the large-scale field. In fact, they evolve dramatically as the sunspot cycle advances, the wavy equatorial neutral line being the most prominent example. Incidentally, we have also seen the evolution of saddle-point-like field configurations associated with coronal holes.

LOW: It seems that you have constructed your equation in analogy with the conservation law for mass flow, with diffusion added. We do have, already, the hydromagnetic equation for magnetic field transport. Have you tried to show from first principles that your constructed equation is derivable from the hydromagnetic equation? This is important, because a missing or additional term can overwhelm otherwise physically real effects.

NORDLUND: Concerning the possibility of deriving your equation from first principles: To the extent that the photospheric magnetic fields are vertical, I think your equation follows from the induction equation. The vertical component is transported by horizontal flows as a scalar quantity.

SHEELEY: Yes, our equation follows directly from the induction equation $\partial \mathbf{B}/\partial t = \nabla \times (\mathbf{v} \times \mathbf{B})$, provided that $\mathbf{B}$ is purely radial, and the velocity field is given by $\mathbf{v} = (-K \nabla \mathbf{B}/B) + \mathbf{v}_{differential\ rot.} + \mathbf{v}_{meridional\ flow}$.

GAIZAUSKAS: In the examples of bipolar regions selected for your simulation, how sure are you that there were not just one but multiple injections of flux?

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SHEELEY: Starting near sunspot minimum in 1976, we looked systematically for those regions whose evolution was both isolated and free from subsequent eruptions of flux. Only in one case did we find an anomalous field distribution, which subsequently was proved to be a secondary eruption.

VAN BALLEGOOIJEN: Can polar field reversals be explained with this model, or is it necessary to involve additional forces on the surface fluxtubes that are polarity dependent (poleward for following flux, equatorward for leading flux)?

SHEELEY: Polar field reversals can be explained with this model. Diffusion provides a meridional component of transport, and the meridional component of the doublet sources insures that the net poleward flow has the "following" polarity in each hemisphere.