The latter is a certain kind of fixed-point: It comes with an "almost" monotone collapse $\vartheta : T[\mathrm{BH}(T)] \to \mathrm{BH}(T)$ (we cannot expect full monotonicity, since the order-type of $T[X]$ may always exceed the order-type of X). The Bachmann-Howard principle asserts that such a collapsing structure exists. In fact we define three variants of this principle: They are equivalent but differ in the sense in which the order $\mathrm{BH}(T)$ is "computed".

On a technical level, our investigation involves the following achievements: a detailed discussion of primitive recursive set theory as a basis for set-theoretic reverse mathematics; a formalization of dilators in weak set theories and second-order arithmetic; a functorial version of the constructible hierarchy; an approach to deduction chains (Schütte) and β-completeness (Girard) in a set-theoretic context; and a β-consistency proof for Kripke–Platek set theory.

Independently of the Bachmann-Howard principle, the thesis contains a series of results connected to slow consistency (introduced by S.-D. Friedman, Rathjen, and Weiermann): We present a slow reflection statement and investigate its consistency strength, as well as its computational properties. Exploiting the latter, we show that instances of the Paris–Harrington principle can only have extremely long proofs in certain fragments of arithmetic.

Abstract prepared by Anton Freund (coincides with thesis abstract)
URL: `http://etheses.whiterose.ac.uk/20929/ (PhD thesis) and https://arxiv.org/abs/1809.06759 (paper version)`

Gianluca Paolini, ***Independence in Model Theory and Team Semantics***, University of Helsinki, Finland, 2016. Supervised by Tapani Hyttinen and Jouko Väänänen. MSC: Primary 03C45, Secondary 03B48, 81P10. Keywords: model theory, classification theory, independence calculi, independence logic, team semantics, probability logic, quantum logic.

Abstract

The subject of this doctoral thesis is the mathematical theory of *independence*, and its various manifestations in logic, mathematics, and computer science. The topics covered in this study range from model theory and combinatorial geometry, to database theory, quantum logic and probability logic. The thesis consists of seven articles ([1–6], and [7]), grouped along two main themes:

(1) Independence calculi and combinatorial geometry ([1–3], and [4]);
(2) New perspectives in team semantics ([5–6], and [7]).

The first topic is a classical topic in model theory, which we approach from different directions (implication problems, abstract elementary classes, and unstable first-order theories). The second topic is a relatively new logical framework where to study nonclassical logical phenomena (dependence and independence, probabilistic reasoning, and quantum foundations). The fundamental thesis defended in this work is that these two themes are deeply intertwined, under the guiding thread of independence.

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Filippo Calderoni, ***A Descriptive View of the Bi-embeddability Relation***, Università degli Studi di Torino, Italy, 2018. Supervised by Luca Motto Ros. MSC: 03E15. Keywords: bi-embeddability, analytic equivalence relations, Borel reducibility, torsion-free abelian groups.

Abstract

In this thesis we use methods from the theory of Borel reducibility to analyze the bi-embeddability relation.

We continue the work of Camerlo, Marcone, and Motto Ros investigating the notion of invariant universality, which is a strengthening of the notion of completeness for analytic equivalence relations. We prove invariant universality for the following relations: bi-embeddability between countable groups, topological bi-embeddability between Polish groups, bi-embeddability between countable quandles, and bi-embeddability between countable fields of a fixed characteristic different from 2. Our work strengthens some results previously obtained by Jay Williams; Ferenczi, Louveau, and Rosendal; and Fried and Kollár.

Then, we analyze the bi-embeddability relation in the case of countable torsion-free abelian groups, and countable torsion abelian groups. We obtain that the bi-embeddability relation on torsion-free abelian groups is strictly more complicated than the bi-embeddability relation on torsion abelian groups. In fact, we prove that the former is a complete analytic equivalence relation, while the latter is incomparable up to Borel reducibility with the isomorphism relation on torsion groups. Furthermore, we argue that the bi-embeddability relation between countable torsion abelian groups is strictly below isomorphism up to $\mathbf{\Delta}^1_2$-reducibility.

In the end, we analyze the bi-embeddability relation on torsion-free abelian groups in the framework of generalized descriptive set theory. We use a categorical construction to prove that bi-embeddability on κ-sized graphs Borel reduces to bi-embeddability on torsion-free abelian groups of size κ, for every uncountable cardinal κ which satisfies $\kappa^{<\kappa} = \kappa$. It follows that the bi-embeddability relation on torsion-free abelian groups of size κ is as complicated as possible among analytic equivalence relations.

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Raphaël Carroy, ***Functions of the first Baire class***, Université Paris 7 - Denis Diderot & Université de Lausanne, France, 2013. Supervised by Olivier Finkel & Jacques Duparc. MSC: Primary 26A21, 54C05. Keywords: Continuous functions, infinite games, well-quasi-orders, continuous reducibility.

Abstract

We aim at starting an analysis of definable functions similar to the Wadge theory for definable sets, focusing more specifically on Baire class 1 functions between 0-dimensional Polish spaces. To parallel Wadge's analysis, we break this study in two parts. The first concerns subclasses of the first Baire class characterisable by infinite games, while the second looks at the quasi-order of continuous reducibility on continuous functions.

Here X, Y, X', and Y' are variables for Polish 0-dimensional (P0D for short) spaces, considered as closed subspaces of the Baire space of infinite sequences of natural numbers.

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