descriptive set theory, computable analysis, game characterizations, function classes, generalized computable theory.

Abstract

Game characterizations of classes of functions in descriptive set theory have their origins in the seminal work of Wadge, with further developments by Van Wesep, Andretta, Duparc, Motto Ros, and Semmes, among others. In this thesis we study such characterizations from several perspectives.

We define modifications of the game characterization of the Borel functions by Semmes [3], obtaining game characterizations of the Baire class α functions for each fixed $\alpha < \omega_1$. Some of our results were independently proved by Louveau and Semmes in unpublished work. We also define a construction of games which transforms a game characterizing a class Λ of functions into a game characterizing the class of functions which are piecewise Λ on a countable partition of their domains by $\mathbf{\Pi}^0_\alpha$ sets, for each $0 < \alpha < \omega_1$.

We then define a framework of parametrized Wadge games by using tools from computable analysis, and show how the choice of parameters can be used to fine-tune what class of functions is characterized by the resulting game. As an application, we recast our games characterizing the Baire classes into this framework.

Furthermore, we generalize our game characterizations of function classes to generalized Baire spaces, i.e., the spaces of functions from an uncountable cardinal to itself. We also show how the notion of computability on Baire space can be generalized to the setting of generalized Baire spaces, and show that this is indeed appropriate for developing a generalized version of computable analysis by defining a representation of Galeotti's generalized real line (cf. [1]) and analyzing the Weihrauch degree of the intermediate value theorem for that space (cf. also [2]).

In the final part of the thesis, we show how the game characterizations of function classes discussed lead in a natural way to a stratification of each class into a hierarchy, intuitively measuring the complexity of functions in that class. This idea and the results presented open new paths for further research.

REFERENCES

[1] L. Galeotti, *A candidate for the generalised real line*, ***Pursuit of the Universal, 12th Conference on Computability in Europe, CiE 2016, Paris, France, June 27–July 1, 2016, Proceedings*** (A. Beckmann, L. Bienvenu, and N. Jonoska, editors), Lecture Notes in Computer Science, vol. 9709, Springer, Cham, Switzerland, 2016, pp. 271–281.

[2] L. Galeotti and H. Nobrega, *Towards computable analysis on the generalised real line*, ***Unveiling Dynamics and Complexity, 13th Conference on Computability in Europe, CiE 2017, Turku, Finland, June 12–16, 2017, Proceedings*** (J. Kari, F. Manea, and I. Petre, editors), Lecture Notes in Computer Science, vol. 10307, Springer, Cham, Switzerland, 2017, pp. 246–257.

[3] B. Semmes, ***A game for the Borel functions***, Ph.D. thesis, Universiteit van Amsterdam, 2009.

Abstract prepared by Benedikt Löwe and Hugo Nobrega.
E-mail: hugonobrega@dcc.ufrj.br
URL: https://www.hugonobrega.com/thesis

Rose Weisshaar, ***Some Results in Computability Theory***, University of Notre Dame, USA, 2019. Supervised by Julia Knight. MSC: 03D99. Keywords: mathematical logic, computability theory.

Abstract

We consider the question of *universality* among computable ω-branching trees. In particular, we construct a computable ω-branching tree T_{KP} whose paths compute the complete

diagrams of the countable ω-models of Kripke–Platek set theory (KP). We show that, given a path f through T_{KP}, representing a model $\mathcal{M}$ of KP, and another computable ill-founded ω-branching tree T, if f fails to compute a path through T, then $\mathcal{M}$ assigns to T a nonstandard ordinal tree rank. Furthermore, we indicate some circumstances in which, given computable ω-branching trees T_0 and T_1, a fixed path through T_{KP} helps the paths through T_0 compute paths through T_1.

In a different line of work, we consider *effective forcing notions*. In particular, we define a class of effective forcing notions that are similar to versions of Mathias forcing and Cohen forcing defined in the literature, and prove some results about how these notions relate. As a consequence, we see that the generics for an effective version of Mathias forcing compute generics for an effective version of Hechler forcing, and vice-versa. Later, we focus on a notion of Mathias forcing over a countable Turing ideal, defined by Cholak, Dzhafarov, and Soskova. We show that there are nested Turing ideals for which the Mathias generics for the larger ideal do not all compute Mathias generics for the smaller ideal.

Abstract prepared by Rose Weisshaar.
E-mail: roseweis@math.upenn.edu

Purbita Jana, ***A Study of the Interrelation between Fuzzy Topological Systems and Logics***, University of Calcutta, India, 2016. Supervised by Mihir K. Chakraborty. MSC: 03B52, 03B60, 97H50. Keywords: geometric logic, fuzzy topological system, graded frame.

Abstract

The ultimate objective of the thesis is to develop fuzzy geometric logic and fuzzy geometric logic with graded consequence. The motivation mostly comes from the main topic of Vickers' book "Topology via Logic," where he introduced the notion of topological system and indicated its connection with geometric logic. A topological system is a triple $(X, \models, A)$, where X is a nonempty set, A is a frame, and $\models$ is a binary relation between X and A. A frame is a lattice which is closed under arbitrary join and finite meet together with the property that binary meet distributes over arbitrary join. The relationships among topological space, topological system, frame and geometric logic play an important role in the study of topology through logic (geometric logic). Generalizations of topological space to fuzzy topological space were introduced by C. L. Chang and R. Löwen, and these concepts have been studied extensively and intensively. Naturally the question "from which logic can fuzzy topology be studied?" comes to mind. If such a logic is obtained what could be its significance?

To answer these questions, as basic steps, we first introduce some notions of fuzzy topological systems and establish the interrelation with appropriate topological spaces and algebraic structures. These relationships are studied in a categorical framework. As a matter of fact, study of duality takes place as one of the important parts of the thesis.

Geometric logic has been discussed in various works by P. T. Johnstone, S. Mac Lane & I. Moerdijk, S. J. Vickers. However for our purpose the reference point is Vickers' books and articles. The formulae of geometric logic are based on two propositional connectives viz. $\wedge$, the binary conjunction and $\bigvee$, the arbitrary disjunction over arbitrary set of formulae including null set. As a special case the binary disjunction $\vee$ is obtained. In addition, the logic has an existential quantifier $\exists$. It is noteworthy that geometric logic does not have negation, implication or universal quantifier. Also in this logic sequents of the form $\alpha \vdash \beta$ are derived from a set (possibly null) of sequents. These special sequents have exactly one formula on either side of the symbol $\vdash$ (turnstile), the intention of the symbol being, as usual, that β follows from α.

Some motivations behind generalizing the geometric logic to multivaluedness are the following:

- Geometric logic is endowed with an informal observational semantics: whether what has been observed does satisfy (match) an assertion or not. Now, observations of facts and assertions about them may corroborate with each other partially. It is a fact of reality

https://doi.org/10.1017/bsl.2019.50 Published online by Cambridge University Press