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Finite irreducible linear 2-groups of degree 4

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Conlon [1] gave, for each prime p, a list of the finite irreducible linear p-groups of degree p. Here, a linear group of degree n is a subgroup of $GL(n, \mathbb{C})$. Since the natural equivalence relation for linear groups is conjugacy in the relevant $GL(n, \mathbb{C})$, a "list" of linear groups means a list of conjugacy class representatives of the groups. Given Conlon's results, one might ask for lists of irreducible p-groups of degree p^2. In this thesis, attention is restricted to the irreducible 2-groups of degree 4.

Part of the motivation arises from a current aim in computational group theory to develop better "soluble quotient algorithms"; the thesis is a preparatory step towards extending the databases presently available for such algorithms. The next step will be to obtain lists of irreducible 2-subgroups of $GL(4, p)$, for each odd prime p.

A finite 2-subgroup of $GL(4, \mathbb{C})$ is conjugate to a group of monomial matrices. The group of all monomial matrices in $GL(4, \mathbb{C})$ is the semidirect product of the group of all diagonal matrices with the group of all permutation matrices. There are two important subgroups of each group G of monomial matrices: the intersection of G with the group of diagonal matrices, and the projection of G in the group of permutation matrices. When G is irreducible, this projection T is transitive. The transitive 2-groups of permutation matrices in $GL(4, \mathbb{C})$ fall into three conjugacy classes. Thus the problem splits up into consideration of three separate cases, according to whether T is cyclic or noncyclic of order 4 or dihedral of order 8.

The group B of all diagonal matrices of 2-power order is a module for each T. Furthermore, $B \cap G$ is a finite T-submodule of B, and G is an extension of $B \cap G$ by T. The first major task to be undertaken is the determination of all finite T-submodules of B. (This listing problem is the source of much of the complexity of the full listing problem.) It is useful to have information about the inclusion relations between these submodules, and so the poset of finite T-submodules of B is studied. When T is of order 4, and apart from a minor perturbation, this poset is the lattice of finite submodules of the direct sum of two modules. Therefore, we study in general

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the lattice of finite submodules of the direct sum of any two modules, considered as a directed graph (Hasse diagram) in the usual way. A description of the edge set of this Hasse diagram is given which complements a well-known description of the vertex set—see Remak [**3**].

Of course, $B \cap G$ and T do not in general determine G. It is necessary to solve the following "extension problem": given a transitive 2-group T of permutation matrices and a finite T-submodule U of B, determine all finite subgroups G of BT such that $B \cap G = U$ and $BG = BT$. Such a group G need not be irreducible, and therefore one must be able to identify the reducible G. The major part of the solution to this extension problem is the calculation of orders of certain second cohomology groups. The tool used is the Lyndon-Hochschild-Serre spectral sequence. Taking a paper of Huebschmann [**2**] as starting point, the actions of the relevant differentials are determined. These results may be used to construct a list of BT-conjugacy classes of extensions. However, non-conjugate subgroups of BT may still be conjugate in $GL(4, \mathbb{C})$, and these remaining conjugacies must then be determined to produce the final list.

The list constructed in this thesis, like Conlon's, is infinite. The conjugacy class representatives are sorted into a finite number of families. The members of any one family admit a common description in terms of integer parameters: each member of a family is labelled by a unique parameter string. This description amounts to giving a generating set of monomial matrices for each group.

The procedures outlined above are carried out in the thesis for the case that T is noncyclic of order 4. The results are developed in a general context which allows them to be applied routinely to the other cases. The calculations in those cases are significantly easier and are not included in the thesis; a full solution will be published at a later stage.

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