Inside the Stars, IAU Colloquium 137
ASP Conference Series, Vol. 40, 1993
Werner W. Weiss and Annie Baglin (eds.)

COULOMB CORRECTIONS AND GLOBULAR CLUSTER STELLAR EVOLUTION

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ABSTRACT The effects of Coulomb corrections on the evolution of globular clusters stars are discussed. Coulomb corrections alter the equation of state by about 1% in most of the stellar interior, and for stars of fixed initial parameters, this results in an 8% increase in the ZAMS luminosity and an 8% decrease in the age at the main sequence turnoff. Ages for globular clusters measured by comparing to the turnoff luminosity of theoretical isochrones are lowered by $\approx 4\%$ when Coulomb effects are included.

Keywords: stellar interiors; globular clusters; equation of state

COULOMB CORRECTIONS AND THE EQUATION OF STATE

We assume the expression given by Graboske, Harwood and Rogers (1969) for the Coulomb correction term to the free energy. Corrections to other thermodynamic quantities can be derived from this expression, and explicit formulae for many of these are given by Mihalas et al. (1988), and Däppen et al. (1988). We are most interested in the pressure correction:

$$\delta P_C = -\frac{\partial F_C}{\partial V} = -\frac{2\pi^{1/2}e^3}{3(kT)^{1/2}}\left(\sum_i n_i Z_i^2\theta_i\right)^{3/2}\tau(x)$$
$$\times\left[\frac{1}{2}+\frac{3}{2}\frac{n_e\theta_e'/\theta}{\sum_i n_i Z_i^2\theta_i}+x\frac{\tau'}{\tau}\left(\frac{3}{2}-\frac{3F_{1/2}(\eta)}{2\theta_e F_{3/2}(\eta)}+\frac{n_e\theta_e'/\theta}{2\sum_i n_i Z_i^2\theta_i}\right)\right]. \quad (1)$$

The summations are over all charged particles, with $\theta_i = 1$ for ions, while for electrons $\theta_e = F_{-1/2}(\eta)/2F_{1/2}(\eta)$. F_k is the Fermi-Dirac integral of order k, and η is the degeneracy parameter. When nondegenerate, $\theta_e \to 1$, while as the electrons become degenerate $\theta_e \to 0$. The factor $\tau(x)$ is a correction for the finite size of the ions (usually near 1). Also $\theta_e' = d\theta_e/d\eta$, and $\tau' = d\tau/dx$.

For a fully ionized mixture of H and He the Coulomb correction can be written in terms of the electron density using

$$\sum_i n_i Z_i^2\theta_i \approx n_e\left(\frac{2}{1+X}+\theta_e\right). \quad (2)$$

This formulation is especially convenient when adding Coulomb corrections to the Eggleton et al. (1973) (EFF) eos (which uses T and a function of the degeneracy parameter as the basic thermodynamic variables), as n_e can

https://doi.org/10.1017/S0252921100018224 Published online by Cambridge University Press

be directly written in terms of the Fermi-Dirac integrals. Thermodynamic quantities then are easily calculated. When $X = 1$, equation (2) is correct even for partially ionized plasmas. When $X = 0.765$, equation (2) overestimates the Coulomb correction by 10% when all the electrons come from singly ionized particles. Thus, when there is no He^{++} we apply 110% of the correction implied by equation (1), but this affects only a small part of the total stellar mass.

In Däppen et al. (1990), results calculated with the Mihalas et al. (MHD) and Livermore eos were compared for $Z = 0$ with X, Y, ρ and T similar to the solar center. For these parameters we match the MHD P to better than 0.1%, while the Livermore eos finds P to be about 0.15% smaller than MHD.

Our results do *not* agree with the eos results of Straniero (1988). We find consistently smaller pressures at a given ρ and T, (apparently significantly larger Coulomb corrections). Chieffi and Straniero (1989) found that Coulomb corrections had negligible effects on the time scale for main sequence evolution, but we find a substantially larger effect.

EFFECT OF COULOMB CORRECTIONS ON STELLAR MODELS

We find, for models of typical Pop. II main sequence stars, the Coulomb correction to the equation of state to be about -1% throughout most of the interior. It is significantly larger in a thin region near the surface.

The pressure at any point in a hydrostatic stellar interior balances the weight of the overlying material. When Coulomb corrections are included a higher density and/or temperature is needed to give the same pressure. To provide this extra pressure the central regions need to be slightly more compact. As a result, for models with given initial parameters, including the Coulomb correction results in slightly *increased* interior temperatures and densities, as well as increased nuclear reaction rates and luminosity. Near the ZAMS, a $0.754 M_{\odot}$ Pop. II model with Coulomb corrections is 8% more luminous, the central T is increased by 1%, the central ρ by 0.4%, and T_{eff} by 78 K. *The turnoff point of the track occurs 8% earlier in time at essentially the same luminosity.* The giant branch tracks are almost exactly coincident in the HR diagram.

Different results are found for solar models. To have the same luminosity at 4.5 Gyr, models with the Coulomb correction need an initial He abundance lower by about 0.01. The central density and temperature are then *lower* than in models without the Coulomb correction (see also Guenther et al. 1992).

EFFECTS ON ISOCHRONES

We will compare to the non-diffusion isochrones of Proffitt and VandenBerg (1991) (PV), which assume a scaled solar T-τ relation with $\alpha = 1.9$. If we require that the isochrones computed here be coincident with those of PV on the lower main sequence near 5100 K (effectively normalizing the T_{eff} scale using the local subdwarf calibration), then $\alpha = 1.6$ instead of 1.9 is needed.

Figure 1 compares 13 and 14 Gyr isochrones computed with Coulomb corrections and $\alpha = 1.6$ to isochrones from PV. *The age for a given turnoff luminosity is found to be ≈ 0.6 Gyr smaller when isochrones including Coulomb corrections are used.* Observationally, it is difficult to define the luminosity of the turnoff to better than 0.1 magnitudes. VandenBerg et al. (1990), suggest choosing the luminosity of a point on the main sequence below the turnoff and 0.05 mag redder in $B-V$ (≈ -0.02 in $\Delta \log T$). This approach gives ages $\approx$ 0.6–0.8 Gyr younger when Coulomb corrections are included.

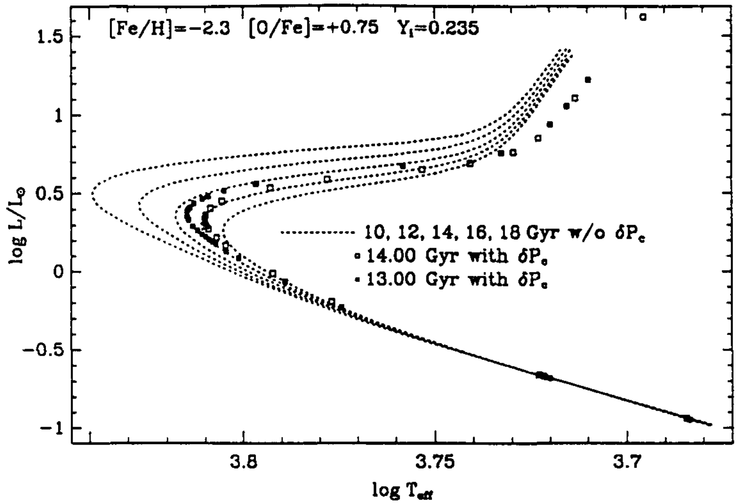

Fig. 1: Isochrones calculated with and without Coulomb corrections.

SUMMARY

- Coulomb corrections have an $\approx 1\%$ effect on the equation of state in most of the interior mass of typical main sequence Pop. II stars.
- Including Coulomb corrections decreases the evolutionary time scale for stars of fixed mass and initial composition by $\approx 8\%$.
- The ages for globular clusters estimated by comparing to the turnoff luminosity of isochrones will be decreased by $\approx 4\%$.
- The effects on isochrone shapes are large enough that Coulomb corrections should be included in all models of low mass stars.

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