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DIV-CURL TYPE THEOREMS ON LIPSCHITZ DOMAINS

ZENGJIAN LOU

For Lipschitz domains of $\mathbb{R}^n$ we prove div-curl type theorems, which are extensions to domains of the Div-Curl Theorem on $\mathbb{R}^n$ by Coifman, Lions, Meyer and Semmes. Applying the div-curl type theorems we give decompositions of Hardy spaces on domains.

1. INTRODUCTION

In [4] two Hardy spaces are defined on domains Ω of $\mathbb{R}^n$, one which is reasonably speaking the largest, and the other which in a sense is the smallest. The largest, $\mathcal{H}^1_r(\Omega)$, arises by restricting to Ω arbitrary elements of $\mathcal{H}^1(\mathbb{R}^n)$. The other, $\mathcal{H}^1_z(\Omega)$, arises by restricting to Ω elements of $\mathcal{H}^1(\mathbb{R}^n)$ which are zero outside $\overline{\Omega}$. Norms on these spaces are defined as following

$$\|f\|_{\mathcal{H}^1_r(\Omega)} = \inf \|F\|_{\mathcal{H}^1(\mathbb{R}^n)},$$

the infimum being taken over all functions $F \in \mathcal{H}^1(\mathbb{R}^n)$ such that $F|_\Omega = f$,

$$\|f\|_{\mathcal{H}^1_z(\Omega)} = \|F\|_{\mathcal{H}^1(\mathbb{R}^n)},$$

where F is the zero extension of f to $\mathbb{R}^n$.

From [2], the dual of $\mathcal{H}^1_z(\Omega)$ is $\mathrm{BMO}_r(\Omega)$, a space of locally integrable functions with

$$\|f\|_{\mathrm{BMO}_r(\Omega)} = \sup_{Q\subset\Omega}\left(\frac{1}{|Q|}\int_Q |f(x) - f_Q|^2\,dx\right)^{1/2} < \infty,$$

where $f_Q = 1/|Q|\int_Q f(x)\,dx$, and the supremum is taken over all cubes Q in the domain Ω. The dual of $\mathcal{H}^1_r(\Omega)$ is $\mathrm{BMO}_z(\Omega)$, the space of all functions in $\mathrm{BMO}(\mathbb{R}^n)$ supported in $\overline{\Omega}$, equipped with the norm $\|f\|_{\mathrm{BMO}_z(\Omega)} = \|f\|_{\mathrm{BMO}(\mathbb{R}^n)}$.

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Let Ω denote a Lipschitz domain - an assumption which is enough to ensure the existence of a bounded extension map from $\mathrm{BMO}_r(\Omega)$ to $\mathrm{BMO}(\mathbb{R}^n)$ ([6]). We use $H(\Omega)^n := H(\Omega, \mathbb{R}^n)$ to denote a space of functions $f : \Omega \to \mathbb{R}^n$ (when $n = 1$, write $H(\Omega)^1$ as $H(\Omega)$). For simplicity we introduce the following spaces

$$L^2_{\mathrm{div}}(\Omega)^n = \{f \in L^2(\Omega)^n : \mathrm{div}\, f = 0,\ \nu \cdot f|_{\partial\Omega} = 0,\ \|f\|_{L^2(\Omega)^n} \leqslant 1\};$$
$$L^2_{\mathrm{curl}}(\Omega)^n = \{f \in L^2(\Omega)^n : \mathrm{curl}\ f = 0,\ \nu \times f|_{\partial\Omega} = 0,\ \|f\|_{L^2(\Omega)^n} \leqslant 1\},$$

where ν denotes the outward unit normal vector. When $\Omega = \mathbb{R}^n$

$$L^2_{\mathrm{div}}(\mathbb{R}^n)^n = \{f \in L^2(\mathbb{R}^n)^n : \mathrm{div}\ f = 0,\ \|f\|_{L^2(\mathbb{R}^n)^n} \leqslant 1\};$$
$$L^2_{\mathrm{curl}}(\mathbb{R}^n)^n = \{f \in L^2(\mathbb{R}^n)^n : \mathrm{curl}\, f = 0,\ \|f\|_{L^2(\mathbb{R}^n)^n} \leqslant 1\}.$$

In [5, Theorems II.1 and III.2], among other results, Coifman, Lions, Meyer and Semmes established the following theorems.

Theorem CLMS1. *Let $1 < p,\ q < \infty$, $1/p + 1/q = 1$, $E \in L^p(\mathbb{R}^n)^n$, $\mathrm{div}\, E = 0$, $F \in L^q(\mathbb{R}^n)^n$, curl $F = 0$. Then $E \cdot F \in \mathcal{H}^1(\mathbb{R}^n)$ and*

$$\|E \cdot F\|_{\mathcal{H}^1(\mathbb{R}^n)} \leqslant C\|E\|_{L^p(\mathbb{R}^n)^n}\|F\|_{L^q(\mathbb{R}^n)^n} \tag{1.1}$$

for a constant C depending only on the dimension n.

Theorem CLMS2. *For $b \in \mathrm{BMO}(\mathbb{R}^n)$*

$$\|b\|_{\mathrm{BMO}(\mathbb{R}^n)} \approx \sup_{E,F} \int_{\mathbb{R}^n} b\, E \cdot F\, dx, \tag{1.2}$$

where the supremum is taken over all $E \in L^2(\mathbb{R}^n)^n$, $F \in L^2(\mathbb{R}^n)^n$ with $\mathrm{div}\, E = 0$, $\mathrm{curl}\, F = 0$ and $\|E\|_{L^2(\mathbb{R}^n)^n} \leqslant 1$, $\|F\|_{L^2(\mathbb{R}^n)^n} \leqslant 1$, and the implicit constants in (1.2) depend only on n.

A natural question to ask is: under what conditions on domains Ω does the equivalence (1.2) hold on Ω? As a main theorem of this paper, we solve this problem for Lipschitz domains in $\mathbb{R}^n$.

Theorem 1.1. *Let Ω be a Lipschitz domain of $\mathbb{R}^n$.*

(1) *If $b \in \mathrm{BMO}_r(\Omega)$, then*

$$\|b\|_{\mathrm{BMO}_r(\Omega)} \approx \sup_{e,f} \int_{\Omega} b\, e \cdot f\, dx, \tag{1.3}$$

the supremum being taken over all $e \in L^2_{\mathrm{div}}(\Omega)^n$, $f \in L^2_{\mathrm{curl}}(\Omega)^n$.

(2) *If $b \in \mathrm{BMO}_z(\Omega)$, then*

$$\|b\|_{\mathrm{BMO}_z(\Omega)} \approx \sup_{e,f} \int_\Omega b\, e \cdot f \, dx, \tag{1.4}$$

the supremum being taken over all $e = E|_\Omega$, $f = F|_\Omega$, $E \in L^2_{\mathrm{div}}(\mathbb{R}^n)^n$, $F \in L^2_{\mathrm{curl}}(\mathbb{R}^n)^n$. *The implicit constants in* (1.3) *and* (1.4) *depend only on the domain* Ω *and the dimension* n.

REMARK. Results for other BMO-type spaces, such as dual of divergence-free Hardy spaces, can be found in [**8**] and [**9**].

COROLLARY 1.2.

(1) *A function* $b \in \mathrm{BMO}_r(\Omega)$ *if and only if there exists a constant* C *such that* $\int_\Omega b\, e \cdot f \, dx \leqslant C$ *for all* $e \in L^2_{\mathrm{div}}(\Omega)^n$ *and* $f \in L^2_{\mathrm{curl}}(\Omega)^n$.

(2) *A function* $b \in \mathrm{BMO}_z(\Omega)$ *if and only if there exists a constant* C *such that* $\int_\Omega b\, e \cdot f \, dx \leqslant C$ *for all* $e = E|_\Omega$ *and* $f = F|_\Omega$ *with* $E \in L^2_{\mathrm{div}}(\mathbb{R}^n)^n$, $F \in L^2_{\mathrm{curl}}(\mathbb{R}^n)^n$.

Here and afterwards, unless otherwise specified, C denotes a constant depending only on the domain Ω and the dimension n. Such C may differ at different occurrences.

Applying Theorem 1.1 we have the following theorem which gives decompositions of $\mathcal{H}^1_z(\Omega)$ and $\mathcal{H}^1_r(\Omega)$ into quantities of forms "$e \cdot f$".

THEOREM 1.3.

(1) *Any function* $u \in \mathcal{H}^1_z(\Omega)$ *can be written as*

$$u = \sum_{k=1}^{\infty} \lambda_k \, e_k \cdot f_k,$$

where $e_k \in L^2_{\mathrm{div}}(\Omega)^n$, $f_k \in L^2_{\mathrm{curl}}(\Omega)^n$ *and* $\sum_{k=1}^{\infty} |\lambda_k| < \infty$.

(2) *Any function* $u \in \mathcal{H}^1_r(\Omega)$ *can be written as*

$$u = \sum_{k=1}^{\infty} \lambda_k \, e_k \cdot f_k,$$

where $e_k = E_k|_\Omega$, $f_k = F_k|_\Omega$, $E_k \in L^2_{\mathrm{div}}(\mathbb{R}^n)^n$, $F_k \in L^2_{\mathrm{curl}}(\mathbb{R}^n)^n$ *and* $\sum_{k=1}^{\infty} |\lambda_k| < \infty$.

2. Proof of Theorem 1.1

To prove Theorem 1.1, we need the following lemmas.

Lemma 2.1. ([**6**, Theorem 1].) *Let $b \in \mathrm{BMO}_r(\Omega)$. Then there exists $B \in \mathrm{BMO}(\mathbb{R}^n)$ such that*

$$b = B|_\Omega$$

and

$$\|B\|_{\mathrm{BMO}(\mathbb{R}^n)} \leqslant C\|b\|_{\mathrm{BMO}_r(\Omega)}. \tag{2.1}$$

Lemma 2.2. ([**7**, Theorem 3.1].) *Let b be a locally integrable function on Ω. Then*

$$\|b\|_{\mathrm{BMO}_r(\Omega)} \approx \|b\|_{\mathrm{BMO}^H(\Omega)}, \tag{2.2}$$

where

$$\|b\|_{\mathrm{BMO}^H(\Omega)} = \sup_Q \left(\frac{1}{|Q|} \int_Q |b - b_Q|^2 \, dx \right)^{1/2},$$

the supremum being taken over all cubes Q with $2Q \subset \Omega$, the implicit constants in (2.2) depend only on Ω and n.

Lemma 2.3. *For $b \in L^2_{loc}(\Omega)$*

$$\|b\|_{\mathrm{BMO}^H(\Omega)} \leqslant C \sup_{e,f} \int_\Omega b\, e \cdot f \, dx, \tag{2.3}$$

the supremum being taken over all $e \in L^2_{\mathrm{div}}(\Omega)^n$ and $f \in L^2_{\mathrm{curl}}(\Omega)^n$.

The proof of Lemma 2.3 is given in the last section.

Proof of Theorem 1.1: (1) Let $B \in \mathrm{BMO}(\mathbb{R}^n)$ be an extension of $b \in \mathrm{BMO}_r(\Omega)$ such that $b = B|_\Omega$ and (2.1) holds. For $e \in L^2_{\mathrm{div}}(\Omega)^n$, $f \in L^2_{\mathrm{curl}}(\Omega)^n$, define

$$E = \begin{cases} e & \text{in } \Omega; \\ 0 & \text{in } \mathbb{R}^n \setminus \Omega, \end{cases}$$

$$F = \begin{cases} f & \text{in } \Omega; \\ 0 & \text{in } \mathbb{R}^n \setminus \Omega. \end{cases}$$

Since $\operatorname{div} e = 0$ on Ω and $e \cdot \nu|_{\partial\Omega} = 0$, it is easy to show that $\operatorname{div} E = 0$ on $\mathbb{R}^n$. So $E \in L^2_{\mathrm{div}}(\mathbb{R}^n)^n$. Similarly, $\operatorname{curl} f = 0$ on Ω and $f \times \nu|_{\partial\Omega} = 0$ imply that $\operatorname{curl} F = 0$

on $\mathbb{R}^n$. Therefore $F \in L^2_{\text{curl}}(\mathbb{R}^n)^n$. By duality $\mathcal{H}^1(\mathbb{R}^n)^* = \text{BMO}(\mathbb{R}^n)$, Lemma 2.1 and (1.1), we have

$$\begin{aligned}\int_\Omega b\, e \cdot f\, dx = \int_{\mathbb{R}^n} B\, E \cdot F\, dx &\leqslant \|B\|_{\text{BMO}(\mathbb{R}^n)} \|E \cdot F\|_{\mathcal{H}^1(\mathbb{R}^n)} \\ &\leqslant C\|b\|_{\text{BMO}_r(\Omega)} \|E\|_{L^2(\mathbb{R}^n)^n} \|F\|_{L^2(\mathbb{R}^n)^n} \\ &= C\|b\|_{\text{BMO}_r(\Omega)} \|e\|_{L^2(\Omega)^n} \|f\|_{L^2(\Omega)^n} \leqslant C\|b\|_{\text{BMO}_r(\Omega)}.\end{aligned}$$

The proof of the reversed inequality in (1.3) follows from (2.2) and (2.3).

(2) Let $b \in \text{BMO}_z(\Omega)$ and B be its zero extension to $\mathbb{R}^n$. Then $B \in \text{BMO}(\mathbb{R}^n)$ and $\|B\|_{\text{BMO}(\mathbb{R}^n)} = \|b\|_{\text{BMO}_z(\Omega)}$. Using (1.1) again,

$$\begin{aligned}\int_\Omega b\, e \cdot f\, dx = \int_{\mathbb{R}^n} B\, E \cdot F\, dx &\leqslant \|B\|_{\text{BMO}(\mathbb{R}^n)} \|E \cdot F\|_{\mathcal{H}^1(\mathbb{R}^n)} \\ &\leqslant C\|b\|_{\text{BMO}_z(\Omega)} \|E\|_{L^2(\mathbb{R}^n)^n} \|F\|_{L^2(\mathbb{R}^n)^n} \\ &\leqslant C\|b\|_{\text{BMO}_z(\Omega)}\end{aligned}$$

for all $e = E|_\Omega$, $f = F|_\Omega$, $E \in L^2_{\text{div}}(\mathbb{R}^n)^n$, $F \in L^2_{\text{curl}}(\mathbb{R}^n)^n$.

For the converse, let $b \in \text{BMO}_z(\Omega)$ and define B as above. Applying (1.2) yields

$$\begin{aligned}\|b\|_{\text{BMO}_z(\Omega)} = \|B\|_{\text{BMO}(\mathbb{R}^n)} &\leqslant C \sup_{E \in L^2_{\text{div}}, F \in L^2_{\text{curl}}} \int_{\mathbb{R}^n} B\, E \cdot F\, dx \\ &= C \sup_{e = E|_\Omega, f = F|_\Omega, E \in L^2_{\text{div}}, F \in L^2_{\text{curl}}} \int_\Omega b\, e \cdot f\, dx.\end{aligned}$$

Theorem 1.1 is proved. □

3. Proof of Theorem 1.3

The proof of Theorem 1.3 relies on Theorem 1.1 and the following facts from functional analysis which can be found in [**5**, Lemmas III.1, III.2].

Lemma 3.1. *Let V be a bounded subset of a normed vector space X. We assume that $\overline{V}$ (closure of V for the norm of X) contains the unit ball (centred at 0) of X. Then, any x in that ball can be written as*

$$x = \sum_{j=0}^{\infty} \frac{1}{2^j} y_j,$$

where $y_j \in V$ for all $j \geqslant 0$.

Lemma 3.2. *Let V be a bounded symmetric ($x \in V \Rightarrow -x \in V$) subset of a normed vector space X. Then, the closed convex hull $\widetilde{V}$ of V (in X) contains a ball centred at 0 if and only if, for any $l \in X^*$,*

$$\|l\|_{X^*} \approx \sup_{x \in V} \langle l, x \rangle.$$

Proof of Theorem 1.3: (1) Let $X = \mathcal{H}_z^1(\Omega)$ and

$$V = \{e \cdot f : e \in L^2_{\text{div}}(\Omega)^n, f \in L^2_{\text{curl}}(\Omega)^n\}.$$

It is easy to check that V is a bounded subset of X. In fact, for $e \in L^2_{\text{div}}(\Omega)^n$, $f \in L^2_{\text{curl}}(\Omega)^n$, let E and F be their zero extensions to $\mathbb{R}^n$ respectively. Then $E \in L^2_{\text{div}}(\mathbb{R}^n)^n$, $F \in L^2_{\text{curl}}(\mathbb{R}^n)^n$. From Theorem CLMS1, $E \cdot F \in \mathcal{H}^1(\mathbb{R}^n)$ and

$$\|E \cdot F\|_{\mathcal{H}^1(\mathbb{R}^n)} \leqslant C\|E\|_{L^2(\mathbb{R}^n)^n}\|F\|_{L^2(\mathbb{R}^n)^n} \leqslant C.$$

Therefore $e \cdot f \in \mathcal{H}_z^1(\Omega)$ with $\|e \cdot f\|_{\mathcal{H}_z^1(\Omega)} \leqslant C$. Applying Theorem 1.1 (1) and Lemmas 3.1 and 3.2, we have the decomposition of Theorem 1.3 (1).

(2) Let $X = \mathcal{H}_r^1(\Omega)$ and

$$V = \{e \cdot f : e = E|_\Omega, f = F|_\Omega, E \in L^2_{\text{div}}(\mathbb{R}^n)^n, F \in L^2_{\text{curl}}(\mathbb{R}^n)^n\}.$$

Similar to the case (1), we have $e \cdot f \in \mathcal{H}_r^1(\Omega)$ with

$$\|e \cdot f\|_{\mathcal{H}_r^1(\Omega)} = \inf_{e \cdot f = G|_\Omega, G \in \mathcal{H}^1(\mathbb{R}^n)} \|G\|_{\mathcal{H}^1(\mathbb{R}^n)} \leqslant \|E \cdot F\|_{\mathcal{H}^1(\mathbb{R}^n)} \leqslant C$$

for $e \cdot f \in V$. Using Theorem 1.1 (2) and those two lemmas again we finish the proof of Theorem 1.3. ∎

4. Proof of Lemma 2.3

To prove Lemma 2.3 we need the following result due to Nečas (see [**10**, Lemma 7.1, Chapter 3]). In Lemma 4.1, $W_0^{1,2}(\Omega)^n$ denotes the closure of $C_0^\infty(\Omega)^n$ in the Sobolev space $W^{1,2}(\Omega)^n$ and $\nabla\varphi = \left((\partial\varphi_i)/(\partial x_j)\right)_{n\times n}$ a $n \times n$ matrix (see [**1**] for Sobolev spaces).

Lemma 4.1. *Let Ω be a Lipschitz domain in $\mathbb{R}^n$. If $f \in L^2(\Omega)$ has zero integral, then there exists $\varphi \in W_0^{1,2}(\Omega)^n$ such that*

$$f = \text{div } \varphi$$

and

$$\|\nabla\varphi\|_{L^2(\Omega)^{n\times n}} \leqslant C\|f\|_{L^2(\Omega)}.$$

Corollary 4.2. *Let Q be a cube in $\mathbb{R}^n$. If $f \in L^2(Q)$ has zero integral, then there exists $\varphi \in W_0^{1,2}(Q)^n$ such that $f = \text{div } \varphi$ and*

$$\|\nabla\varphi\|_{L^2(Q)^{n\times n}} \leqslant C_0\|f\|_{L^2(Q)}$$

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for a constant C_0 independent of Q.

PROOF OF LEMMA 2.3: Suppose $b \in L^2_{loc}(\Omega)$. We shall show that for all cubes Q with $2Q \subset \Omega$ there exists $e \in L^2_{\text{div}}(\Omega)^n$ and $f \in L^2_{\text{curl}}(\Omega)^n$ such that

$$\left(\frac{1}{|Q|}\int_Q |b-b_Q|^2\,dx\right)^{1/2} \leqslant C\left|\int_\Omega b\, e\cdot f\,dx\right|. \tag{4.1}$$

Let $h = b - b_Q$, then $h \in L^2(Q)$ with $\displaystyle\int_Q h\,dx = 0$. From Corollary 4.2, there exists $\varphi := (\varphi_1, \dots, \varphi_n) \in W_0^{1,2}(Q)^n$ such that $h = \operatorname{div}\varphi$ and

$$\|\nabla\varphi\|_{L^2(Q)^{n\times n}} \leqslant C_0\|h\|_{L^2(Q)}, \tag{4.2}$$

where C_0 is independent of Q. So

$$\begin{aligned}\|h\|^2_{L^2(Q)} &= \int_Q h\sum_{i=1}^n \frac{\partial\varphi_i}{\partial x_i}\,dx \leqslant n\max_{1\leqslant i\leqslant n}\left|\int_Q h\frac{\partial\varphi_i}{\partial x_i}\,dx\right| \\ &= n\left|\int_Q h\frac{\partial\varphi_{i_0}}{\partial x_{i_0}}\,dx\right|\end{aligned} \tag{4.3}$$

for some choice of i_0 $(i_0 = 1, \dots, n)$. Assuming without loss of generality that $i_0 = 1$ in (4.3). To prove (4.1), it is sufficient to show that

$$\left|\int_Q h\|h\|^{-1}_{L^2(Q)}\frac{\partial\varphi_1}{\partial x_1}\,dx\right| \leqslant C|Q|^{1/2}\left|\int_Q h\, e\cdot f\,dx\right|. \tag{4.4}$$

We next construct e and f. Define

$$f = \left(-\frac{\partial\varphi_1}{\partial x_i}, 0, \dots, 0, \frac{\partial\varphi_1}{\partial x_1}, 0, \dots, 0\right)C_0^{-1}\|h\|^{-1}_{L^2(Q)},$$

where $(\partial\varphi_1)/(\partial x_1)$ is the i-th component of f. Then $f \in L^2(Q)^n$ with $\operatorname{div} f = 0$ and $\|f\|_{L^2(Q)^n} \leqslant 1$ by (4.2).

Let $\psi_0 \in C_0^\infty(\mathbb{R}^n)$ such that

$$\psi_0 = \begin{cases} 1 & \text{on } [-1,1]^n; \\ 0 & \text{outside } [-2,2]^n. \end{cases}$$

Define

$$e = \gamma C_0|Q|^{-1/2}\nabla\big((x_i - x_i^0)\psi_Q(x)\big), \quad 1 \leqslant i \leqslant n,$$

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where $\psi_Q(x) = \psi_0\Big(\big(x - x^0\big)/\big(l(Q)/2\big)\Big)$, $x^0 = \big(x_1^0, \dots, x_n^0\big)$ and $l(Q)$ denote the centre and the side-length of the cube Q, $\gamma > 0$ is a normalisation constant (independent of x^0 and $l(Q)$) so that $\|e\|_{L^2(\Omega)^n} \leqslant 1$. It is obvious that $e \in C_0^\infty(2Q)$ and $e = \gamma C_0 |Q|^{-1/2}\varepsilon_i$ on Q, where $\varepsilon_i = (0, \dots, 0, 1, 0, \dots, 0)$, 1 is the i-th component of ε_i. From the construction of e and f, we get

$$e \cdot f = \gamma |Q|^{-1/2} \|h\|_{L^2(Q)}^{-1} \frac{\partial \varphi_1}{\partial x_1} \quad \text{on} \quad Q$$

and (4.4) is proved. ∎

NOTE. It should be added that at the time the paper was finished, the author was unfortunately unaware of a similar but unpublished work [**3**] (with different proof). Thanks go to Galia Dafni (Department of Mathematics & Statistics, Concordia University, Canada) for informing us her paper with Chang and Sadosky.

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Department of Mathematics
Shantou University
Shantou
Guangdong 515063
Peoples Repbulic of China
e-mail: zjlou@stu.edu.cn