Ronald C. Kirkpatrick<br>Los Alamos Scientific Laboratory

ABSTRACT: Several theories seek to explain the peculiar shapes of planetary nebulae. Those of Louise, Kirkpatrick, and Phillips and Reay rely on progenitor rotation. The velocity-radius relation for the shells of well observed planetaries do not extrapolate back through the origin, but rather fall short, suggesting that the shell acquires its velocity over a significant period of time. Kirkpatrick's theory relies heavily on long term acceleration of the nebular shell, and other theoretical studies support the idea of acceleration of the nebular shell up to the time it becomes optically thin to the ionizing radiation from the central star.

## INTRODUCTION

Planetary nebulae show marked axial symmetry. Some, such as NGC 7027, that appear irregular in the visible have quite axi-symmetric radio images (Atherton et. al., 1979) and it is known that internal dust is responsible for their irregular visible image. Also, many planetary nebulae have double shell structure (Weedman, 1968) or show spectral evidence of double shell structure (Kirkpatrick, 1972). Giant halo's around several planetaries have been discussed by Capriotti (1978), and astrophysical mass determinations such as that by Vauclair (1968) indicate that the mass range for the nebular shells is considerable. Inhomogeneities are evident for the better resolved objects and are best seen in the low excitation lines (Capriotti, et. a1., 1971)

The ejection theories may be divided into four groups which have been discussed by Miller (1974) and by Roxburg (1978). They are 1) enhanced nuelear burning (e.g., flashes), 2)ionization instability, 3) radiation pressure, and 4) a wind/wind model. Kwok and Purton (1978) proposed the wind/wind model and more recently Giuliani (1980) has studied various wind and radiation front interactions. The basis for this model is a high velocity wind interpretation of the observations of Wallerstein (1978) and of Fitzgerald (1973), an interpretation of questionable correctness.

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## EVIDENCE FOR PROLONGED ACCELERATION

Using the distances to planetary nebulae derived by Cahn and Kaler (1971) and radial velocities obtained by Wilson (1950), Bohuski and Smith (1974) derived a velocity-radius relationship for twenty five planetaries. To these objects they added several based on their expansion velocity measurements. The relationship for the first 25 objects is approximately linear. If

$$
\begin{equation*}
\mathrm{R}=\mathrm{a}+\mathrm{bV} \tag{1}
\end{equation*}
$$

then

$$
\begin{equation*}
\mathrm{dV} / \mathrm{dt}=\mathrm{bV} \mathrm{o}_{\mathrm{o}} \exp [\mathrm{~b}(\mathrm{t}-\mathrm{to})] \tag{2}
\end{equation*}
$$

where $R$ is radius, $V$ is velocity, $t$ is time, $a$ and $b$ are constants, so that a linear velocity relationship implies a positive continually increasing outward acceleration of the nebular shell. Since the detection of nebular shells depends their surface brightness ( $\sim M_{n}^{2} / R^{5}$, where $M_{n}$ is the nebular mass), for a mass independent driving force one would expect a family of trajectories in the velocity-radius plane, with the lower mass (higher limiting velocity) objects achieving a smaller detectable radius. One would therefore interpret the 25 objects using Cahn and Kaler distances to all have similar masses. In fact Cahn and Kaler derived a mean mass of $0.18 \mathrm{M}_{9}$ which they then used as the basis for their individual distances.

There is no theoretical basis for assuming all planetaries to have similar mass, and the "old" planetary data points of Bohuski and Smith would suggest a range of nebular masses. Also, the astrophysical masses of Vauclair (1968) indicate a large range of nebular masses. However, there are other complications to a straight forward interpretation of the velocity-radius relations. The data, of Liller, Welther and Liller (1960) constrain NGC 7293 to $4.5 \times 10^{-1}$ $\mathrm{kms}^{-1} \mathrm{~cm}^{-1}$, and Caranza et. al. (1968) measure higher expansion velocities for NGC 7293 than Bohuski and Smith use. Also, since the study by Cahn and Kaler, internal extinction in many planetaries has been observed. Therefore, whereas the linear relation discussed by Bohuski and Smith has interesting implications, as do their additional data points, the whole basis for their velocity-radius relation is questionable and needs further study.

## NEBULAR SHAPES

Weedman (1968) used the expansion velocity of various ions together with the observed ionization stratification for specific objects to deduce a linear velocity-radius relationship for each. These were combined with several line profiles from high dispersion
slit spectra for each object to derive nebular shapes. He provided outer shell shapes for a few objects. Generally, most objects were prolate spheroidal, with the outer shell being more nearly spherical. Atherton et. al., (1979) used a different method to obtain their velocity radius relationship for NGC 7027. They obtained the mean nebular velocites as a function of position along the major axis. This relation was then applied to their many line profiles across the face of the nebula, and they were able to deduce prolate spheroidal shape for that highly internally reddened object. Their derived form reasonably reproduced the radio profiles.

One dimensional modeling of the nebular hydrodynamies such as that by Ferch and Salpeter (1975) provide confidence that the approximately linear velocity gradient based on ionization stratification observations is reasonably correct. However, it should be realized that the ion velocities and the ion radii refer to different parts of the nebula, and no amount of logic can devise a proof that a linear gradrient actually exists. Furthermore, the velocity gradient is inappropriately applied when it is used to transform expansion velocities into radii. The approach taken by Atherton, et, al. is more nearly correct, but the difficulty of this problem is well illustrated by the oblate conclusion reached for NGC 7027 in a previous paper (Hieks, et. al, 1976). For a given object only two slopes in the velocity-radius plane will provide axial symmetry for a nebular axis skew relative to the plane of the sky. Other slopes give triaxial nebulae, a distinct possibility, but thus far unobserved. Munch (1968) and Carranza et. al (1968) preferred to stop short of assigning a shape, but rather presented their data as position-velocity relation ships which strongly suggested helical shapes for NGC 6543 and NGC 7293.

## NEBULAR SHAPING MECHANISMS.

Gurzadyan (1962) and more recently Hieligman (1980) have studied the effect of magnetic fields on planetary nebulae. Menzel (1968) presented strong arguments against stellar and nebular fields, and Hieligman finds that only under extreme circumstances will interstellar fields significantly shape planetaries. Isaacman (1979) reaches the same conclusion about the interstellar medium. Louise (1972) proposed that progenitor rotation and gravitational retardation could account for the shapes of planetaries. Kirkpatriek (1976) showed that prolonged accelerations of an initially slightly oblate shell thin at the poles would transform that shell into a substantially prolate shape by the time it reached typical planetary dimensions. More recently Phillips and Reay (1977) have combined the gravitational retardation and the prolonged acceleration effects.

Currently it would appear that progenitor rotation definitely plays a role in the shaping of planetary nebulae. However, no two dimensional modeling of nebular dynamics has been attempted. Rather, trajectories have been calculated which are suggestive of the results to be expected from hydrodynamic studies, and these simple calculations side-step such issues as Rayliegh-Taylor instabilities and angular redistribution due to internal pressure gradients. A detailed treatment of this problem would include the hydrodynamics, the radiation front, the detail balance, etc., and this seems formadable indeed for a 2-D calculation. Probably some benefit could still be derived from modeling which incorporates various simplifying assumptions before the complete 2-D problem is attempted.

Qualitative arguments can be made that shed some light on the nebular dynamics and nebular shaping. If the nebular shell is accelerated by the pressure of a fixed mass of hot HII inside, then Rayliegh Taylor instabilities should develop and the shell should break up according to the linear theory for plane geometry. However, in spherical geomertry, the expansion should provide some stability (Plessett, 1954). Still the internal pressure of a fixed mass of HII falls rapidly as the nebula expands and the acceleration continually decreases. This is contrary to the linear velocity-radius relationship discussed by Bohusky and Smith, and also ignores the material being added to the HII as the radiation front drives into the neutral shell whieh tends to a) keep the pressure up and b) absorb UV photons. If the shell were aceelerated only by the reaction when the UV photons ablate the neutral shell, then it is possible to show that there is an approximately linear velocity-radius relationship for the early expansion phase when $R$ is small:

$$
\begin{equation*}
R=\left(R_{0}-\frac{M_{0}}{M_{t}} v_{o}\right)+\frac{M}{M_{t}} v \tag{3}
\end{equation*}
$$

where $R_{0}, V_{0}$, and $M_{0}$ are the initial radius, velocity, and mass, and $M_{t}$ is the (constant) rate at which the UV photons ablate material from the neutral shell. The diffuse radiation contribution to the UV flux at the radiation front tends to stabilize the front against Rayliegh Taylor instabilities. This is because there is a net fraction of the UV lost for the photoionization-recombination cycle so that the crest of a Rayliegh Taylor instability receives more incident UV flux than the trough. Of course the ablated material adds to the mass of ionized HII and tends to reduce the UV flux reaching the neutral shell while adding to the internal pressure of the HII. Thus, a regulatory process is established whereby the ablation and pressure compete with each other in the acceleration of the neutral shell.

CONCLUSIONS
It is presently unclear whether or not a linear velocity-radius relationship exists for planetary nebulae. If it does, then it has
some interesting implications concerning the development of the nebular shell and the character of the progenitor. Shapes have been deduced for several planetaries. Although the basis for transforming from a position-velocity relation as observed to a position-radius relation which provides the shape is open to criticism, the most careful work done thus far indicates that those planetaries with complete shells are prolate spheroidal in shape. Other studies suggest that at least two nebulae have helical components.

Of the several mechanisms proposed for shaping planetary nebulae, those which on physical grounds seem most likely all rely on rotation of the progenitor. The current epoch velocity-radius relation for planetaries as discussed by Bohuski and Smith could provide a clue as to which mechanism actually operates, but only if the underlying data can be refined so that assumptions regarding planetargy mass, etc. don't bias the results. Although a complete numerical model is desirable for studing the shaping problem, attempts should be made to understand the basic problem using various simplifying assumptions until $2-$ D modeling becomes possible.

## REFERENCES

Atherton, P.D., Hicks, T.R., Reay, N.K., Robinson, G.J., and
Worswiek, S.P.: 1979, Astrophys. J. 232, 786.
Bohuski, T.J., and Smith, M.G.: 1974, Astrophys. J. 193, 197.
Cahn, J.H., and Kaler, J.B.: 1971, Astrophys. J. Suppl. 22, 319.
Capriotti, E.R.: 1978, in Y. Terzian (ed.), IAU Symp. No. 76:
Planetary Nebulae, Reidel Publ. Co., Dordrecht, Boston, p. 270.
Capriotti, E.R., Cromwell, R.H., and Williams, R.E.: 1971,
Astrophys. Letters 7, 241.
Carranza, G., Courtes, G., and Louise, R.: 1968, in D.E. Osterbrock
(ed.) IAU Symp. No 34: Planetary Nebulae, Reidel Publ. Co.,
Dordrecht, Boston, p. 249.
Ferch, R.L. and Salpeter, E.E.: 1975, Astrophys. J. 202, 195.
Fitzgerald, M.P.: 1973, Nature: Phys. Sci. 245, 58.
Giuliani, J.L.: 1980, "Ionization Fronts in the Colliding Wind Model of Planetary Nebulae", (preprint).
Gurzadian, G.A.: 1962, Vistas Astron. 5, 40.
Hicks, T.R., Phillips, J.P., and Reay, N.K.: 1976, Monthly Notices
Roy. Astron. Soc. 176, 409.
Hieligman, G.M.: 1980, Monthly Notices Roy. Astron. Soc. 191, 761.
Isaacman, R.: 1979, Astron. Astrophys. 77, 327.
Kirkpatrick, R.C.: 1973, Mem. Roy Soc. Liege 5, 343.
Kirkpatrick, R.C.: 1976, Astrophys. Letters 17, 7.
Kwok, S., and Purton, C.R.: 1978, Astrophys, J. Letters 219, 125
Liller, M.H., Welther, B.L., and Liller, W.: 1966, Astrophys. J. 144, 280.

Louise, R.: 1973, Mem. Roy. Soc. Liege 5, 465
Menzel, D.H.: 1968, in D.E. Osterbrock (ed.), IAU Symp. No. 34:
Planetary Nebulae, Reidel, Publ. Co., Dordrecht, Boston, p. 279. Miller, J.: 1974, Ann. Rev. Astron. Astrophys. 12, 352.
Munch, G.: 1968, in D.E. Osterbrock (ed.), IAU Symp. No. 34:
Planetary Nebulae, Reidel Publ. Co., Dordrecht, Boston, p. 259. Phillips, J.P., and Reay, N.K.: 1977, Astron. Astrophys. 59, 91. Plesset, M.S.: 1954, J. Appl. Phys. 25, 96.
Roxburg, I.W.: 1978, in Y. Terzian (ed.), IAU Symp. No. 76:
Planetary Nebulae, Reidel Publ. Co., Dordrecht, Boston, p. 295. Vauclair, G.: 1968, Ann. Astrophys. 31, 199. Wallerstein, G.: 1978, Publ. Astron. Soc. Pacific 90, 36. Weedman, D.W.: 1968, Astrophys. J. 153, 49. Wilson, O.C.: 1950, Astrophys. J. 111, 279.

## DISCUSSION

J. COX: Is this the translation of an oblatespheroid into a prolate spheroid?
KIRKPATRICK: Yes.
J. COX: Is there any simple way of understanding that?

KIRKPATRICK: Yes, there is a very simple way of understanding it. If you have some mechanism which acts uniformly at the base of the shell, which accelerates the shell, and if you have an oblate spheroidal shell which initially has the same properties of density and temperature, etc. along the equipotential lines, you will have more mass at the equator per unit solid angle than you will have at the poles. Therefore, the same accelerating mechanism, the same force per unit solid angle, will accelerate it more rapidly at the poles than at the equator. Therefore, although the poles start out behind, they very rapidly obtain higher velocity and eventually you go from an oblate to a prolate spheroid.


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