Part 4
Realization and comparison of reference frames

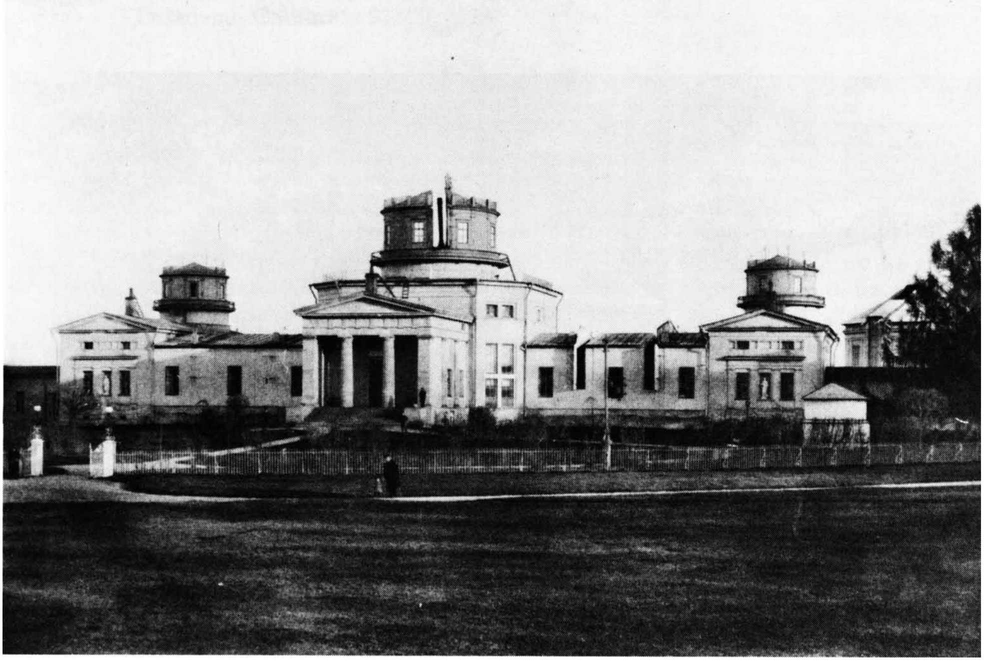

Plate VIII. The original Pulkovo Observatory viewed from the northwest. The 15-inch refractor can be seen in the open center turret. Nineteenth century photograph courtesy of W.R. Dick.

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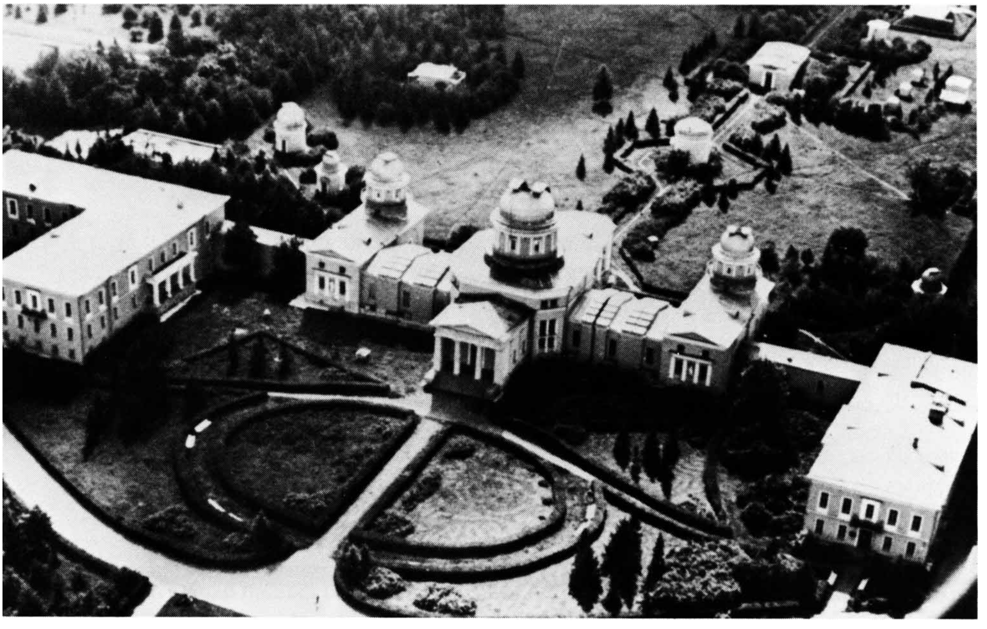

Plate IX. The reconstructed Pulkovo Observatory viewed from the northwest in an aerial view. Photograph provided by Fulkovo Observatory.

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THE DEFINITION AND STABILITY OF LOCAL INERTIAL REFERENCE FRAMES

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ABSTRACT. Inertial reference frames spanning approximately 10^o-30^o square on the sky and capable of locating objects to few-hundred microarcsecond accuracies are useful for a broad class of astrometric measurements. Deep space tracking and general relativistic angular deflection experiments are examples of astrometric measurements which can profitably reference the positions and/or motions of objects to a field of radio sources in a local frame. A method for defining local inertial reference frames has been developed based on Very Long Baseline Interferometry (VLBI) measurements of extragalactic radio sources. By observing the radio emission from the object to be located in the frame, as well as that from about five radio sources which define the frame, dominant systematic astrometric errors can be minimized through parameter estimation. The entire reference frame measurement is of the order of 30 minutes including all the sources in a frame. The limiting error for single-epoch position determination in a local frame is the unknown structure of both target and reference objects. Structure can cause systematic milliarcsecond-level errors. The limiting error for epoch-to-epoch differential position measurements is tropospheric fluctuations, assuming that the radio source structures do not change from one epoch to the next. Preliminary results of an epoch-to-epoch measurement of relativistic gravitational deflection by Jupiter, in which the total deflection was about 600 microarcseconds, suggest that the local reference frame is stable at the 240-microarcsecond level over twelve days. Data have been taken at longer time intervals to determine the annual stability of the frames. At the time of preparation of these proceedings, those data have not yet been analyzed.

1. Defining a Local Inertial Reference Frame

A local reference frame in this report can extend over 10^o to 30^o square on the sky. It is inertial in the sense that positions and motions of an object referenced to the frame are attributable to forces *on* the object, as opposed to measurement errors associated with the object or the frame. The local inertial reference frame technique can be seen as an intermediary between the full sky radio frame and the few arcminute frame defined by a radio antenna beamwidth. In addition to being intermediate in spatial extent, its $\approx$200-microarcsecond (μas) precision is, on the average, better than global reference frame results, which are based on much larger data sets taken over years [Sovers, 1989, and Ma, 1989]; but the accuracy is poorer than the 5-10 μas achieved in the antenna beamwidth frame [Marcaide and Shapiro, 1983]. The accuracy attained in a given frame is a measure of how "inertial" the frame is. That is, it is a measure of the level at which positions and

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motions measured in that frame can be attributed to forces acting on the object being measured.

Covariance studies have shown that dual frequency (for charged particle calibration) Very Long Baseline Interferometry (VLBI) measurements of five reference radio sources, with one source measured twice, define a frame which enables approximately 200-μas position measurements [Treuhaft, 1988]. This accuracy results by assuming perfectly known reference source positions, which implies that the structure of the reference and target sources is also perfectly known. In the presence of milliarcsecond structure [Charlot, 1989], a single local reference frame measurement can only produce few-milliarcsecond results. Unless all sources in the frame are either mapped, or their structures parameterized to much better than 1 milliarcsecond, the full potential of the local reference frame approach can only be realized in epoch-to-epoch differential measurements. If the epochs of the individual reference frame determinations are sufficiently close that the reference source structure effects do not vary at more than the 100-μas level, then differential accuracies of the order of 300-μas should be attainable on the differential measurement.* Epochs separated by less than a few months should satisfy that criterion [Porcas, 1987].

As described in detail elsewhere [Treuhaft, 1988], the seven observations in a minimal local reference frame observation set determine seven parameters. One parameter is the projection of the position of the target object onto the baseline vector at the time of the measurement. Two parameters are associated with clock behavior: epoch and rate. Two parameters are associated with orthogonal earth rotations, and two with static zenith tropospheric delays, one for each station. Assuming 20 picoseconds system noise on each VLBI group delay measurement implies 120-μas accuracy for the single-epoch measurement. Tropospheric and source structure/position uncertainty effects, when treated with a consider analysis [Bierman, 1977], yield the errors quoted above.

2. The Short Term Stability of Local Reference Frames

The stability of a reference frame will be defined as the rms scatter of repeated position determinations in that frame. A test of the short term stability of local reference frames was performed by measuring the relativistic gravitational deflection of the radiation from P 0201+113, which passed within 200 arcseconds of Jupiter on 21 March 1988. Two VLBI experiments were performed, one on 21 March 1988 and one on 2 April 1988. The Deep Space Network stations involved were DSS 13 and DSS 15, the 26-m and 34-m antennas at Goldstone, California, and DSS 43, the 70-m antenna near Canberra, Australia. Data were taken at 2.3 GHz and 8.5 GHz with spanned bandwidths of approximately 40 and 100 MHz for the low and high frequency band respectively. A total of 28 channels, each sampling at 4 Mbits/second were used. According to general relativity, the maximum difference in apparent position of the source, projected onto the baseline vector, between the two observation epochs was about 600 μas; this difference in apparent position is due to deflection by Jupiter's gravitational field on 21 March. The effect of Jovian gravitation on 2 April was down to 25 μas. Ten local reference frame measurements were obtained for each day. The frames used are described below.

In figure 1, preliminary results of the measured angular deflection of P 0201+113 are shown versus observation time. Only the DSS 13 to DSS 43 baseline is shown. The times given are the universal times of the observations of the target source on 2 April. The

* The differential accuracy is $\sqrt{2}$ times the "structure-free" single-epoch accuracy, because the measurement consists of a difference between two effectively independent measurements.

TABLE 1. Radio Sources Used in Local Reference Frame Stability Test

Source Name	Right Ascension hr min sec	Declination degrees min sec
P 0201+113	02 03 46.65734	11 34 45.3629
P 0019+058	00 22 32.44128	06 08 04.2715
P 0106+01	01 08 38.77113	01 35 00.3200
GC 0119+04	01 21 56.86177	04 22 24.7377
P 0202+14	02 04 50.41399	15 14 11.0450
CTD 20	02 37 52.40576	28 48 08.9919
GC 0235+16	02 38 38.93021	16 36 59.2753
OD 166	02 42 29.17103	11 01 00.7285
3C 454.3	22 53 57.74796	16 08 53.5614

Details of the data shown follow. The data in figure 1 are for DSS 13 to DSS 43 only. The solid curved line shows the expected gravitational deflection by Jupiter versus time, and the horizontal line shows the result if the experiment were totally insensitive to the gravitational deflection difference between the two epochs. The rms scatter of the differential results about the expected Jovian deflection is 336 μas, which implies a single-epoch stability of 240 μas. The reduced χ^2 values in the figure are calculated for the expected Jovian deflection between epochs, χ^2_J, no detectable change between epochs, χ^2_0, and a constant bias between epochs, χ^2_C. The probability that Jovian deflection would produce a χ^2_J of the value quoted or greater is approximately 0.2. The probability that zero actual deflection would produce the χ^2_0 value shown or greater is much less than 0.005, and the corresponding probability for a constant bias is 0.008. Thus this preliminary result is more consistent with planetary gravitational deflection than with either a null detection or a constant bias between epochs. However, the value of χ^2_J indicates that errors have been underestimated by about 20%, on the average. Actually, it is clear from inspection of the figure that underestimation of the error on only one or two points is a more likely explanation of the large χ^2. A possible explanation of the underestimate of the error on the first point is given in the next paragraph.

The results of figure 1 are preliminary in several respects. The reduced χ^2 values calculated do not include known correlations between measurements. These correlations can be substantial between small-net determinations and the large-net measurement from which earth rotation and troposphere parameters were derived. A precise treatment of these correlations is underway. The underestimation of errors is in part due to the failure to account for tropospheric fluctuation effects. Incorporating these effects in the formal error using a fluctuation model [Treuhaft and Lanyi, 1987] is also underway. For example, a calculation of the fluctuation effect on the first data point, which was taken at 10^o elevation in Australia for P 0201+113, yields an adjusted error bar of approximately 500 μas, as opposed to the 240 μas bar shown. It should also be noted that in the last large network (the ninth point from the left), one of the seven observations failed, and seven parameters could not be estimated. Earth rotation parameters were therefore used from the previous large network observed two hours beforehand (the fourth point from the left). The errors introduced by this approach should be well below the formal error of the result, but this must be verified. There may also be effects due to errors in the cross correlation of the VLBI data which are currently under investigation. These errors could be of the order of the error bars, changing the conclusion substantially. Given those cautions, the angular shifts shown are the difference in measured position between the two epochs at equal sidereal times. That is, the observation schedule of 21 March was exactly repeated on 2 April, except for one or two instrumental failures on reference frame sources. If a reference frame source failed at one epoch, the analogous scan was excluded from the analysis at the other epoch. Failure to do so–e.g. using four reference sources for a P 0201+113 position determination at one epoch and five at the other–resulted in errors as large as 1 milliarcsecond. These milliarcsecond errors are probably due to apriori source coordinate errors of that order.

As mentioned above, each point in figure 1 corresponds to a local reference frame measurement consisting of a target observation of P 0201+113, and one or more reference sources. Only the first, fourth, and ninth measurements consisted of the seven nominal observations needed for a complete parameter set determination. All other points were derived from smaller networks, in which either two or three observations were used. In these smaller networks, a clock epoch (for only two observations) or a clock epoch and rate (for three observations) were estimated along with the projection of the source vector onto the baseline. All other parameters in smaller networks were fixed at the values derived from the previous 7-observation measurements. A complete list of the target (first entry) and reference source J2000 positions [Sovers, 1988] is given in table 1 below. Reference sources for the larger networks differed slightly, depending on mutual visibility. The reference source used in the smaller networks was the one closest to P 0201+113, P 0202+14.

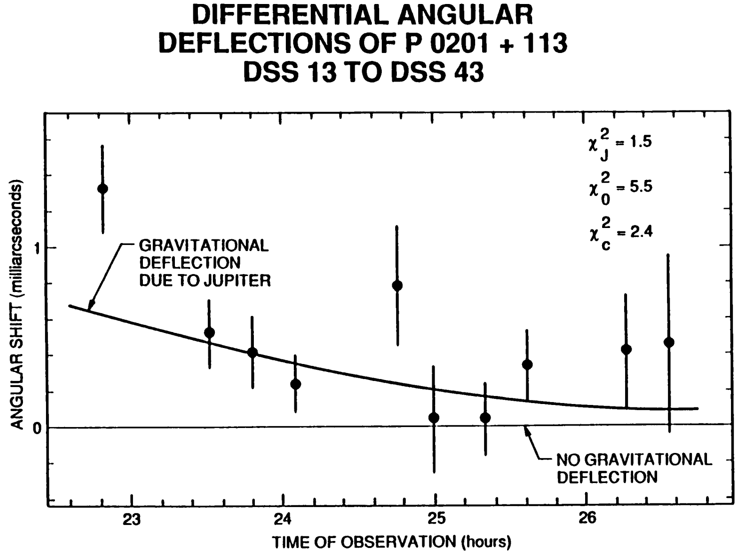

Figure 1. Differential angular deflections of P 0201+113, between observation epochs separated by 12 days, versus time of observation at second epoch. The deflections were measured over the DSS 13 to DSS 43 baseline.

data currently suggest that 10 repeated local reference frame differential measurements– i.e. 20 single-epoch measurements, of approximate accuracy 340 μas, were sensitive to the change in target position induced by planetary relativistic gravitational deflection over the course of twelve days. If this conclusion withstands further analysis and checks, this local reference frame measurement will be the first to detect planetary deflection; all other solar system deflections have been observed in the sun's gravitational field. It is worth noting that a single deflection measurement of the above accuracy 2° from the sun's center would enable a post-Newtonian γ parameter determination to 1 part in 1000, equaling the accuracy of the best determination [Reasenberg, et. al., 1979], if the phase fluctuations induced by the solar plasma could be calibrated. Repeated independent measurements could substantially improve the knowledge of γ.

3. Summary and Future Directions

A local inertial reference frame can be defined by six reference observations of five radio sources. Measuring a radio emitter's position within that frame requires one more observation of that object, totalling seven observations for a complete single-epoch measurement. The six reference source observations are used to solve for six parameters relating to the dominant errors associated with clock behavior, earth rotation, and static tropospheric delay. Single-epoch measurement accuracies are limited by radio source position uncertainties and source structure effects at the milliarcsecond level. They are also limited by tropospheric fluctuations. Differential measurements made between epochs over which radio source structures, although possibly unknown, do not change, are limited by tropospheric fluctuations. Differential measurements made over few-month time scales should therefore be insensitive to uncalibrated, but stationary, radio source structure or position uncertainty.

A test of the differential measurement technique, aimed at detecting relativistic gravitational deflection by Jupiter, currently suggests that over a 12-day interval, 240-μas single-epoch stabilities were observed. The differential measurement results imply that single-epoch measurements are much more stable than accurate, as expected. Assuming three different hypotheses, 1) the observed deflections arose from gravitational deflection, 2) they arose from zero actual deflection, and 3) they arose from a constant bias in the deflection measurements, yields probabilities of 0.2, much less than 0.005, and 0.008. These numbers are the probability of observing the quoted reduced χ^2 or greater for each hypothesis. The preliminary nature of these probabilities and the error analysis in the experiments was discussed.

In addition to further analysis of the data presented, analysis of the DSS 15 to DSS 43 data will also be completed. At the moment, the conclusions from that largely independent measurement are similar to those presented here. Data taken on the DSS 15 to DSS 45 (the 34-m antenna in Australia) in September of 1989 will have been processed by the time these proceedings are published. The same observing schedules were run at two week time intervals. Differential measurements of P 0201+113 between the two-week epochs will allow a further check of instrumentation and the differential technique. Because Jupiter was many tens of degrees away for both observations, the angular shifts should be consistent with a null effect over that time interval. Comparing the 1988 epochs to the 1989 epochs will allow an assessment of the level to which source structure fluctuations and uncalibrated instrumental changes corrupt the data on annual time scales.

Should experiment confirm that the single-epoch stability of local reference frames is at the 240-μas level, as suggested by the current data set and error analysis, reducing dominant error sources could then be addressed [Treuhaft, 1988]. Applying water vapor radiometer (WVR) calibrations to the data should reduce the tropospheric fluctuation error due to the wet component of the atmosphere. The level to which that component can be calibrated over 1 to 30 minute time scales is not determined as yet. If the 200-μas level of the wet fluctuation error is reduced, for example, by an order of magnitude, then system noise errors of the order of 100 μas will become dominant. Successful use of the VLBI phase delay observable, as opposed to the group delay observable on which the described technique has been based, would reduce the system noise component of the error to about 4 μas. Beyond using phase delays and WVRs, dry tropospheric fluctuations at about 50 μas might be calibrated with barometric arrays around each station of the baseline, in an attempt to increase the local reference frame stability to that of the antenna beamwidth technique. It is also conceivable that radio source structure fluctuations could be parameterized and estimated with additional reference frame observations, which would then allow single-epoch accuracies to be of the order of single-epoch stabilities. This approach has not yet been considered quantitatively.

4. Acknowledgment

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Discussion

YATSKIV: Some years ago, Dr Kovalevsky has proposed to use the terms "small field astrometry" and "global astrometry." You are using "local" and "inertial" frames. What is the reason for these changes?

TREUHAFT: I agree in principle with the terms "small field" and "global" astrometry. The only reason for the change is that "small field" usually applies to fields of the order of 1° x 1°, but in this report I am discussing fields of about 20° x 20°.

KHARIN: Is it possible to obtain radiointerferometric positions of the natural satellites and, if so, what is the accuracy of these observations?

TREUHAFT: Because the natural satellites are angularly broad, on the order of one arcsec, they can only be observed on short baselines. For example, observations of the Galilean satellites, with the Very Large Array in New Mexico (having a 30 km maximum baseline), have yielded approximate accuracies of 30–50 mas. This should be compared to the milliarcsecond or better accuracy attainable on compact extragalactic sources.

XU: How can you get enough reference sources in such a small field?

TREUHAFT: The density of current radio source catalogs (about 200 sources around the sky above –45° declination) is sufficient to guarantee approximately 5 reference sources in the 20° x 20° field used.

YE: What would be the approaches you would use in order to measure phase delay instead of group delay?

TREUHAFT: The phase delay is a higher accuracy data type than the group delay. However, much more accurate *apriori* information is required to employ this data type. We plan to use high precision group delays as the *apriori* information needed for employing the phase delays.

XU: Your results are from differential VLBI. Does that mean it is the internal precision?

TREUHAFT: The preliminary results shown in this paper are epoch-to-epoch differential radio source coordinate determinations. I have referred to the results in terms of "differential accuracy" but the term "precision" is also appropriate. Since two independent data sets were compared, however, I would not use the term "internal precision" which, to me, implies an internal consistency check on one data set.

ZHDANOV: What are the prospects for "measuring" the post-newtonian parameter γ?

TREUHAFT: If 200–300 microarcsecond precision data could be obtained on a raypath 2° from the Sun's center, then a γ measurement of about 2 parts in 1000 would result. This accuracy, which is comparable to the current knowledge of γ, could be obtained with a *single* pair of 30-minute measurements. Many such pairs of measurements could be made to statistically improve the result. The problem of solar plasma fluctuations is now being studied.

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