Volume/area scaling of glaciers and ice caps and their longitudinal profiles

VALERIO FARAONI

Physics Department and Environmental Science Programme, Bishop's University, 2600 College Street, Sherbrooke, Québec J1M 1Z7, Canada
Correspondence: Valerio Faraoni <*vfaraoni@ubishops.ca*>

ABSTRACT. Empirical data on the volume/area scaling of glaciers and ice caps are compared with theoretical models of longitudinal glacier profiles. Ice caps are described well by theoretical solutions of the differential equations modeling longitudinal profiles, but glaciers for which the surface slope is approximated by the ice thickness divided by the length are not. A power-law profile recently derived analytically, reproduces exactly the thickness-length scaling, first predicted by scaling analysis for ice caps.

KEYWORDS: ice thickness, longitudinal profiles of glaciers and ice caps, volume/area scaling

1. INTRODUCTION

A volume/area scaling relation for glaciers and ice caps is widely used to estimate the volume of glaciers and the loss of ice in relation with global climate change and sea-level rise (e.g., Raper and Braithwaite, 2006; Meier and others, 2007; Bahr and others, 2009; Leclercq and others, 2011; Radić and Hock, 2011; Mernild and others, 2013). Other scaling relations (e.g. basal stress/area or response timescaling are used successfully in glaciology (e.g. Jóhannesson and others, 1989; Pfeffer and others, 1998; Harrison and others, 2001, 2003; Lüthi, 2009; Raper and Braithwaite, 2009; Clarke and others, 2013). A volume/area scaling of the form

$$V = c S^\gamma, \tag{1}$$

where V and S are the volume and surface area of glaciers or ice caps (km^3 and km^2, respectively), has been shown to fit empirical data. From the theoretical point of view, this relation Eqn (1) is derived using scaling analysis (Bahr and others, 1997, 2015) according to the Buckingham Pi theorem (Buckingham, 1914). Note the recent review (Bahr and others, 2015), which dispels common misconceptions, and references therein for a detailed exposition. While it is easy to determine the area S using, for example, aerial surveys, it is much more difficult to assess directly the volume, V of the ice. The exponent γ in Eqn (1) is believed to be a universal constant, as is theoretically sound in scaling analysis, while the value of c is characteristic of the specific glacier or ice cap considered (Bahr and others, 2015). Different values of γ are predicted for ice caps and for valley glaciers, but the exponent is believed to be universal within each of these two classes (Bahr and others, 2015). It is conceivable, at least in principle, that more than two classes exist, and other authors report a range of values for different types of glaciers (Adhikari and Marshall, 2012). However, the transition from one value of γ to another appears to be abrupt, not gradual, as described by percolation theory (Bahr and Pfeffer, 2016).

The law Eqn (1) is used in a statistical sense and it can also be used for individual glaciers and ice caps if one contents oneself with obtaining an order of magnitude estimate for the volume V.

The universal exponent γ is predicted using scaling analysis and assuming certain closure relations that determine how various quantities scale with length (Bahr and others, 2015). Fundamental quantities involved in the scaling relations for glaciers and ice caps are the glacier longitudinal dimension l, width w and thickness h. For valley glaciers (but not for ice sheets and ice caps), a shape factor F is also introduced, which describes how the valley walls partially support the ice (Hooke, 2005; Greve and Blatter, 2009; Cuffey and Paterson, 2010). While the shape factor is not necessary in every derivation of the volume/area scaling law (in fact it is used only in one of the three derivations of Bahr and others (Bahr and others, 2015), it is assumed (Bahr and others, 1997) that $S \sim [l][w]$ and

$$V \sim [l][w]F[h] \tag{2}$$

with

$$[w] \sim [l]^q, \tag{3}$$

$$F \sim [l]^f, \tag{4}$$

$$[h] \sim [l]^s, \tag{5}$$

where, a posteriori, $f = 0$ is found to be appropriate (Bahr and others, 2015). A further assumption used in scaling analysis is that the mass-balance rate $\dot{b}$ of the ice is described by a power law,

$$\dot{b} = c_m l^m. \tag{6}$$

Numerous authors have analyzed empirical data for glaciers and ice caps determining values for the parameters q, m and γ (Bahr and others, 2015). Scaling analysis does not need to assume a priori a scaling relation $h \sim [l]^s$. However, it is clear that the extension in the third-spatial dimension is relevant for the volume of the ice and the volume/area relation Eqn (1) clearly has implications for theoretical models of the local ice thickness $h(x)$ (where x is a longitudinal coordinate along the ice bed). Longitudinal glacier profiles have been the domain of a separate body of literature aiming at solving analytically ordinary differential equations derived

from specific assumptions on the physical behaviour of ice under stress, which a priori has nothing to do with the theory of scaling. It is, however, interesting to relate these two separate areas of glaciology.

We examine the implications of the empirical values of the exponent γ and of the parameters (q, m, n) for analytical theoretical solutions of the differential equations that model the longitudinal profile $h(x)$ of the ice. These theoretical models are based on assumptions about the microphysics of ice (Hooke, 2005; Greve and Blatter, 2009; Cuffey and Paterson, 2010). The simplest assumption used in the early days of theoretical modeling is that of perfectly plastic ice (Nye, 1952), which is however unrealistic. A better model to describe the microscopic behaviour of ice is Glen's law (Glen, 1955), which relates the strain rates $\dot{\varepsilon}_{ij}$ and the deviatoric stresses s_{ij} in the ice,

$$\dot{\varepsilon}_{ij} = \mathcal{A}(\sigma_{\text{eff}})^{n-1} s_{ij}, \tag{7}$$

where $\mathcal{A}$ is a (temperature-dependent) constant, σ_{eff} is the effective stress, and $n = 3$ for ice flow, while $n \to \infty$ for perfectly plastic ice (Hooke, 2005; Greve and Blatter, 2009; Cuffey and Paterson, 2010). A few analytical solutions $h(x)$ are known for the differential equations modeling the local ice thickness (Hooke, 2005; Greve and Blatter, 2009; Cuffey and Paterson, 2010) and power-law relations $h \sim [l]^s$ emerge from these solutions, although they are not usually noted in the literature. These relations are compared below with the empirical values of the exponent γ and the parameters (q,m,n) relevant for the scaling analyses leading to Eqn (1).

2. USING DATA ON VOLUME/AREA SCALING TO TEST LONGITUDINAL PROFILE MODELS

Let us consider now how the characteristic thickness h of a glacier or ice cap scales with length, $h \sim [l]^s$. Since the ice volume is $V \sim [l][w][h]$ and $S \sim [l][w]$, a scaling relation Eqn (1) is related to the scaling of the thickness. Theoretical models of longitudinal glacier profiles $h(x)$ solve nonlinear ordinary differential equations and make precise predictions for the value of the scaling exponent s, which can be compared with the result of scaling analysis and with data.

The scaling $h \sim [l]^s$ with exponent

$$s = \frac{m+n+1}{2(n+1)} \tag{8}$$

is predicted for ice caps by Bahr and others (2015). Since $V \sim [l][w][h] \sim l^{q+s+1}$ and $S \sim [l][w] \sim [l]^{q+1}$, it is $V \sim S^{\frac{q+s+1}{q+1}}$ or

$$\gamma = 1 + \frac{s}{q+1} = 1 + \frac{m+n+1}{2(n+1)(q+1)} \tag{9}$$

for ice caps (Eqn (132) of Bahr and others, 2015). For glaciers, instead, it is predicted that

$$s = \frac{m+1}{n+2} \tag{10}$$

and (Eqn (122) of Bahr and others, 2015)

$$\gamma = 1 + \frac{s}{q+1} = 1 + \frac{m+1}{(n+2)(q+1)}. \tag{11}$$

2.1. Ice caps

For ice caps, it is believed that $m \simeq 0$, corresponding to a mass-balance rate independent of the length. For a horizontal bed and thick ice, as is commonly assumed for ice caps (Paterson, 1994; Hooke, 2005; Greve and Blatter, 2009; Cuffey and Paterson, 2010), an order of magnitude estimate provides $h \sim [l]^{1/2}$ (Faraoni and Vokey, 2015). This argument assumes a surface slope $\theta \sim h/l$, which is typical of ice caps and large glaciers. Setting $m = 0$ in Eqn (8) gives $s = 1/2$ and then $\gamma = 1 + (1/2(q + 1))$ irrespective of the value of n (although $n = 3$ is certainly the appropriate value for ice flow). Because of the approximate cylindrical symmetry of ice caps burying the underlying topography, $S \sim [w][l] \sim [l]^{q+1} \sim \pi l^2$ and $q = 1$, giving $\gamma = 5/4 = 1.25$, which matches the data (Bahr and others, 2015). Since, for the appropriate value $m = 0$ of the mass-balance rate parameter it can only be $s = 1/2$, theoretical models for longitudinal ice-cap profiles $h(x)$ do not have wiggle room. Let us review them.

- *Nye's parabolic profile*: for perfectly plastic ice is (Nye, 1951a, b; Cuffey and Paterson, 2010)

$$h(x) = H\sqrt{1 - \frac{x}{L}}, \tag{12}$$

$$H = \sqrt{\frac{2\tau_{\text{b}}}{\rho g}}\sqrt{L}, \tag{13}$$

where τ_{b} is the basal stress. The maximum ice thickness H corresponds to $x = 0$ and the glacier terminus is located at $x = L$. Clearly, this model fits the data but the assumption of perfectly plastic ice is unphysical (Paterson, 1994; Hooke, 2005; Cuffey and Paterson, 2010).

- *Vialov profile*: The Vialov longitudinal profile for ice satisfying Glen's law (Vialov, 1958; Paterson, 1994; Hooke, 2005; Greve and Blatter, 2009; Cuffey and Paterson, 2010) is

$$h(x) = H\left[1 - \left(\frac{x}{L}\right)^{n+1/n}\right]^{n/2(n+1)}, \tag{14}$$

$$H = \left(\frac{2}{\rho g}\right)^{n/2(n+1)}\left[\frac{(n+2)C}{2A}\right]^{1/2(n+1)}\sqrt{L}. \tag{15}$$

It is a solution of the nonlinear Vialov differential equation for the longitudinal profile $h(x)$ (Vialov, 1958) when $\dot{b} =$ const. (corresponding to $m = 0$). Also for the Vialov exact profile, the maximum thickness H scales with $\sqrt{L}$.

- *New power-law solutions of the Vialov equation*: A new analytical formal solution of the Vialov differential equation for longitudinal glacier profiles corresponding to mass-balance rate $\dot{b} = c_m x^m$ is the power-law profile (Faraoni and Vokey, 2016),

$$h(x) = H\left(\frac{x}{L}\right)^s, \quad s = \frac{n+m+1}{2(n+1)}, \tag{16}$$

$$H = \left[\left(\frac{n+2}{2A}\right)\frac{c_m L^{n+m+1}}{(\beta \rho g)^n}\right]^{1/2(n+1)} \equiv h_0 L^s, \tag{17}$$

where now, $x = 0$ is the glacier terminus and the maximum thickness H of the ice is attained at $x = L$. The value

Eqn (16) of s reproduces exactly, for all values of the parameters (n, m, q), that of Eqn (8) first predicted by Bahr and others (2015) by scaling analysis.

For $\dot{b} = \text{const.} \equiv c_0$ (corresponding to $m = 0$) one obtains (Faraoni and Vokey, 2016)

$$h(x) = H\sqrt{\frac{x}{L}}, \qquad (18)$$

$$H = \left(\frac{2}{\rho g}\right)^{n/2(n+1)} \left[\frac{(n+2)c_0}{2A}\right]^{1/2(n+1)} \sqrt{L} \equiv h_0\sqrt{L}. \quad (19)$$

Apart from the value of the constant H (which is, however, still proportional to $\sqrt{L}$), this new profile reproduces the parabolic profile originally derived by Nye for plastic ice. Although this assumption is almost universally regarded as unphysical, a parabolic profile is nevertheless widely used in studies of both ice caps and glaciers (e.g. Benn and Hulton, 2010; Ng and others, 2010). The solution Eqn (18) of the Vialov equation assuming Glen's law with $n = 3$ justifies this widespread use (Faraoni and Vokey, 2016) and agrees with the empirical value $1/2$ of the exponent s for ice caps. It is important to be aware of the fact that this value of s is not tied to the unphysical assumption of perfectly plastic ice, as is instead widely believed.

- *Böðvarsson's profile*: This analytical longitudinal profile is derived from a model in which driving stress is balanced by sliding resistence. The profile is (Böðvarsson, 1955)

$$h(x) = H\left[1 - \left(\frac{x}{L}\right)^2\right] \qquad (20)$$

with

$$h(0) = H = \frac{ak}{6}L^2, \qquad (21)$$

where a and k are constants related to the mass-balance rate (through $M = a(H - H_0)$) and basal stress (through $\tau_b = -k\rho g h u$) (Böðvarsson, 1955). Note that the ice thickness scales as $H \propto L^2$. In this model, Eqn (8) with $(m, n, q) = (0, 3, 1)$ gives $s = 1/4 = 0.25$. We conclude that the Bövarsson model is not a realistic description of ice caps longitudinal profiles.

- *Bueler profile*: In this longitudinal profile (Bueler, 2003; Bueler and others, 2005) the basal stress $\tau_b = -\rho g h \, dh/dx$ is finite at the ice terminus, contrary to the Vialov and other profiles. It is given in (Bueler, 2003; Bueler and others, 2005; Greve and Blatter, 2009):

$$h(x) = H\left[(n+1)\frac{x}{L} - n\left(\frac{x}{L}\right)^{n+1/n} + n\left(1 - \frac{x}{L}\right)^{n+1/n} - 1\right]^{n/2(n+2)}, \qquad (22)$$

$$H = \left\{\frac{2}{n}\left[\frac{(n+2)\alpha}{2A}\right]^{1/n}\frac{1}{\rho g}\right\}^{n/2(n+1)} L^{n/2(n+1)}, \qquad (23)$$

where α is a parameter appearing in the mass-balance rate (Bueler, 2003; Bueler and others, 2005)

$$\dot{b}(x) = \alpha\left[\left(\frac{x}{L}\right)^{1/n} + \left(1 - \frac{x}{L}\right)^{1/n} - 1\right]^{n-1}. \qquad (24)$$

In this profile the maximum ice thickness scales as

$$H \propto L^{n/2(n+1)} = L^{3/8} \simeq L^{0.375} \qquad (25)$$

for $n = 3$. The mass-balance rate Eqn (24) is not of the form $c_m x^m$ and one cannot discuss the parameter m here. However, Eqn (8) can only agree with $s = n/2(n + 1)$ if $m = -1$, which is unphysical. Therefore, the Bueler profile is not a reliable match of ice caps longitudinal profiles.

2.2. Glaciers

For glaciers, the parameter values $(m, n, q) = (2, 3, 0.6)$ are believed to be appropriate (Bahr and others, 2015). Equations (10) and (11) then give $s = 3/5 = 0.60$ and $\gamma = 1.375$, while the observed value is $\gamma = 1.36$ (Bahr and others, 2015). The theoretical models for longitudinal glacier profiles make the following predictions.

- *Nye's parabolic profile*: The longitudinal profile (Eqns (12) and (13)) for perfectly plastic ice predicts $H \propto \sqrt{L}$ and $s = 0.5$; hence this model seems inadequate to describe glaciers.
- *Vialov profile*: For ice satisfying an $n = 3$ Glen law and for constant mass-balance rate, the Vialov profile, Eqns (14) and (15), again predicts $s = 0.5$. This model also seems inappropriate for glaciers.
- *Power-law profiles*: For mass-balance rate of ice described by Eqn (6), the new power-law profile obtained as an analytical solution of the Vialov equation by Faraoni and Vokey (2016) is

$$h(x) = H\left(\frac{x}{L}\right)^s, \quad s = \frac{n+m+1}{2(n+1)}, \qquad (26)$$

$$H = \left[\left(\frac{n+2}{2A}\right)\frac{c_m L^{n+m+1}}{(\beta\rho g)^n}\right]^{1/2(n+1)} \equiv h_0 L^s. \qquad (27)$$

These equations apply to large glaciers where the slope $\theta = h/l$. Assuming $(m, n, q) = (2, 3, 0.6)$ gives $s = 3/4 = 0.75$ and $\gamma = 1.47$. This value of γ does not coincide with the observed value $\gamma = 1.36$ but is close to the number 1.46 reported by Adhikari and Marshall (2012) for a population of glaciers in steady state, in contrast with the value $\gamma = 1.36$ attributed by these authors to a state of glacier retreat causing greater reduction in V than in S. However, letting γ vary continuously while keeping c in Eqn (1) fixed, as done by Adhikari and Marshall (2012), is inconsistent with the scaling theory of Bahr and others (2015), as pointed out in this reference. Whether the numerical coincidence above is significant could be established by further analysis of the data.

- *Böðvarsson's profile*: Eqns (20) and (21) predict $s = 2$ which, together with $n = 3$ yields $m = 12$, ruling out this model for the description of glaciers.
- *Bueler's profile*: Eqns (22) and (23) has $s = (n/2(n + 1)) = 0.375$ for $n = 3$, which is reproduced only if $m = -1$. We conclude that the Bueler profile is also ruled out as a model of glaciers.

For glaciers, no known analytical model of longitudinal profile derived from theory fits well the volume/area scaling law. For ice caps, the theoretical model of Vialov for constant
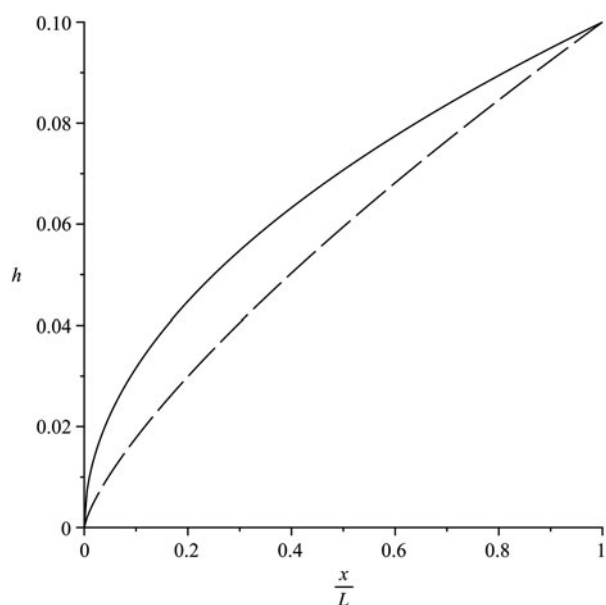
Fig. 1. The profiles Eqn (18) (solid) and Eqn (16) (dashed) for the parameter values $H = 0.1L$, $n = 3$, $m = 2$.

mass-balance rate and the new parabolic solution of Faraoni and Vokey (2016) corresponding to $m = 0$ fit well the empirical data for the volume/area scaling law. The power-law profile Eqn (9) originally predicted with scaling analysis by Bahr and others (2015) is reproduced only as an exact solution of the Vialov differential equation obtained by assuming the ice follows Glen's law in Faraoni and Vokey (2016). The profiles, Eqns (18) and (16) are compared in Figure 1.

3. CONCLUSIONS

The volume/area scaling widely used, especially in relation to climate change, ice loss and sea-level rise, does not need to assume explicit models for the local ice thickness, i.e. the longitudinal glacier profile. Nevertheless, it is interesting to relate it to these models, thus connecting two branches of glaciology, which were, thus far, separated. Since the volume of a glacier or ice cap depends on its third-spatial dimension (the thickness), the empirical values obtained for the volume/area scaling law have implications for theoretical models of longitudinal glacier profiles. Assuming the ice thickness scales with length as $[h] \sim [l]^s$, ice caps are well modelled by a parabolic profile, but this cannot come from the microphysical assumption of perfectly plastic ice, which is unrealistic. Instead, a parabolic profile can be obtained by the Vialov equation based on Glen's law for the behaviour of ice (Faraoni and Vokey, 2016). What is more, the power-law profile Eqn (16) obtained analytically by Faraoni and Vokey (2016) reproduces the scaling exponent first predicted by scaling analysis by Bahr and others (2015) for ice caps.

Contrary to ice caps, for valley glaciers no known theoretical solution of the ordinary differential equations describing longitudinal profiles fits well the empirical data on volume/area scaling. In any case, the volume/area scaling law Eqn (1) applies to *ensembles* of glaciers (i.e. to fictitious average glaciers and to other statistics of many glaciers) and it is not expected to apply to measurements of single, specific glaciers. The discrepancy noted here between theory and observations suggests that more accurate theoretical modeling of longitudinal profiles is needed for valley glaciers.

REFERENCES

Adhikari S and Marshall SJ (2012) Glacier volume–area relation for high-order mechanics and transient glacier states. *Geophys. Res. Lett.*, **39**, L16505 (doi: 10.1029/2012GL052712)

Bahr DB and Pfeffer WT (2016) Crossover scaling phenomena for glaciers and ice caps. *J. Glaciol.* (doi: 10.1017/jog.2016.6)

Bahr DB, Meier MF and Peckam SD (1997) The physical basis of glacier volume–area scaling. *J. Geophys. Res.*, **102**, 355–362 (doi: 10.1029/97JB01696)

Bahr DB, Dyurgerov M and Meier MF (2009) Sea-level rise from glaciers and ice caps: a lower bound. *Geophys. Res. Lett.*, **36**, L03501 (doi: 10.1029/2008GL036309)

Bahr DB, Pfeffer WT and Kaser G (2015) A review of volume–area scaling of glaciers. *Rev. Geophys.*, **10**, 95–140 (doi: 10.1002/2014RG000470)

Benn DI and Hulton NRJ (2010) An excel spreadsheet program for reconstructing the surface profile of former mountain glaciers and ice caps. *Comput. Geosci.*, **36**, 605–610 (doi: 10.1016/j.cageo.2009.09.016)

Böðvarsson G (1955) On the flow of ice-sheets and glaciers. *Jökull*, **5**, 1–8

Buckingham E (1914) On physically similar systems: illustrations of the use of dimensional equations. *Phys. Rev.*, **4**, 345–376

Bueler E (2003) Construction of steady state solutions for isothermal shallow ice sheets. *Report Tech. Rep. 03-02*. Dept. of Mathematics and Statistics, U. Alaska, Fairbanks

Bueler E, Lingle CS, Kallen-Brown JA, Covey DN and Bowman LN (2005) Exact solutions and verification of numerical models for isothermal ice sheets. *J. Glaciol.*, **51**, 291–306 (doi: 10.3189/172756505781829449)

Clarke GKC and 6 others (2013) Ice volume and subglacial topography for western Canadian glaciers from mass balance fields, thinning rates, and a bed stress model. *J. Clim.*, **26**, 4282–4303 (doi: 10.1175/JCLI-D-12-00513.1)

Cuffey KM and Paterson WSB (2010) *The physics of glaciers*. Elsevier, Amsterdam

Faraoni V and Vokey MW (2015) The thickness of glaciers. *Eur. J. Phys.*, **36**(055031), 1–10 (doi: 10.1088/0143-198 0807/36/5/055031)

Faraoni V and Vokey MW (2016) Revisiting analytic models of longitudinal glacier profiles, unpublished paper

Glen JW (1955) The creep of polycrystalline ice. *Proc. R. Soc. Lond. A*, **228**, 519–538

Greve R and Blatter H (2009) *Dynamics of ice sheets and glaciers*. Springer, New York

Harrison WD, Elsberg DH and Echelmeyer KA (2001) On the characterization of glacier response by a single time-scale. *J. Glaciol.*, **47**, 559–564 (doi: 10.3189/172756501781831837)

Harrison WD, Raymond CF and Echelmeyer KA (2003) A macroscopic approach to glacier dynamics. *J. Glaciol.*, **49**, 13–21 (doi: 10.3189/172756503781830917)

Hooke RL (2005) *Principles of glacier mechanics*, 2nd edn. Cambridge University Press, Cambridge, UK

Jóhannesson T, Raymond C and Waddington E (1989) Time-scale for adjustment of glaciers to changes in mass balance. *J. Glaciol.*, **35**, 355–369

Leclercq PW, Oerlemans J and Gogley J (2011) Estimating the glacier contribution to sea-level rise for the period 1800–2005. *Surv. Geophys.*, **3**, 519–535 (doi: 10.1007/s10712-011-9121-7)

Lüthi MP (2009) Transient response of idealized glaciers to climate change. *J. Glaciol.*, **55**, 918–930 (doi: 10.3189/002214309790152519)

Meier MF and 6 others (2007) Glaciers dominate eustatic sea-level rise in the 21st century. *Science*, **317**, 1064–1067 (doi: 10.1126/science.1143906)

Mernild SH, Lipscomb WH, Bahr DB, Radić V and Zemp M (2013) Global glacier retreat: a revised assessment of committed mass losses and sampling uncertainties. *Cryosphere*, **7**, 1987–2005 (doi: 10.5194/tc-7-1565-2013)

Ng FSL, Barr ID and Clark CD (2010) Using the surface profiles of modern ice masses to inform palaeo-glacier reconstruction. *Quat. Sci. Rev.*, **29**, 3240–3255 (doi: 10.1016/j.quascirev.2010.06.045)

Nye JF (1951a) The flow of glaciers and ice-sheets as a problem in plasticity. *Proc. R. Soc. Lond. A*, **207**, 554–572

Nye JF (1951b) A method of calculating the thicknesses of the ice-sheets. *Nature*, **169**, 529–530

Nye JF (1952) The mechanics of glacier flow. *J. Glaciol.*, **2**, 82–93

Paterson WSB (1994) *The physics of glaciers*, 3rd edn. Butterworth-Heinemann, Oxford, UK

Pfeffer WT, Sassolas C, Bahr DB and Meier MF (1998) Response time of glaciers as a function of size and mass balance 2: numerical experiments. *J. Geophys. Res.*, **103**, 9783–9789 (doi: 10.1029/98JB00508)

Radić V and Hock R (2011) Regionally differentiated contribution of mountain glaciers and ice caps to future sea-level rise. *Nat. Geosci.*, **4**, 91–94 (doi: 10.1038/ngeo1052)

Raper SBC and Braithwaite RJ (2006) Low sea level rise projections from mountain glaciers and ice caps under global warming. *Nature*, **439**, 311–313 (doi: 10.1038/nature04448)

Raper SBC and Braithwaite RJ (2009) Glacier volume response time and its links to climate and topography based on a conceptual model of glacier hypsometry. *Cryosphere*, **3**, 183–194 (doi: 10.5194/tc-3-183-2009)

Vialov SS (1958) Regularization of glacial shields movement and the theory of plastic viscous flow. In *IAHS Publ. 47 (Symposium at Chamonix 1958, Physics of the Movement of the Ice)*. IAHS Press, Wallingford, UK