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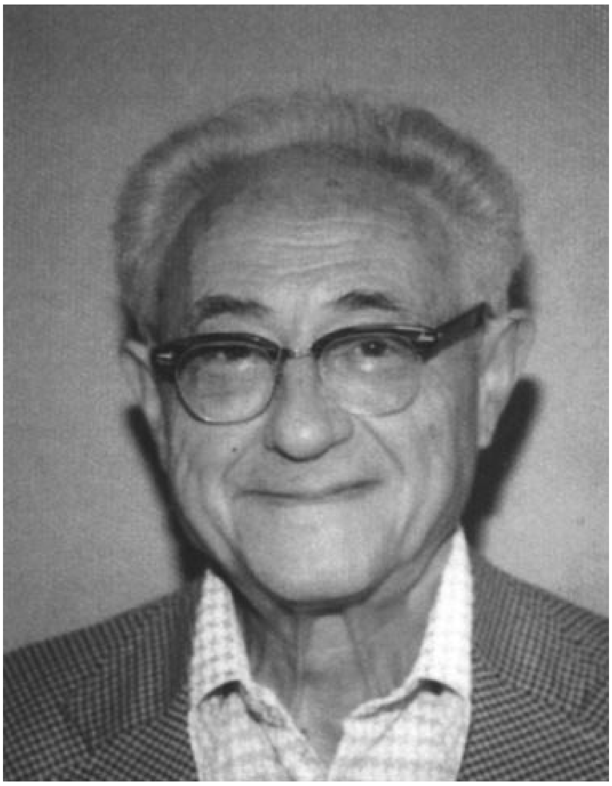
OBITUARY

ARTHUR HAROLD STONE (1916–2000)



Arthur Harold Stone, who died on 6 August 2000, was one of the foremost general topologists of his time, and made significant contributions to a number of different parts of general topology. He had been a member of the Society since 1948. His parents were Simon and Rosa Petrescu who came from Galatz (later Galati), Romania, where his father was a civil engineer, working in Bulgaria. They had two daughters, but when the father lost his job in Bulgaria as a result of the Balkan war of 1912–13, the family decided to emigrate to England, where they anglicised their name to Stone.

Their son, Arthur Harold Stone, was born in London on 30 September, 1916. He grew up in Sherriff Road and attended the local school, but in 1927 won an LCC Scholarship to Christ's Hospital (Horsham). This was a boarding school which had had such successful pupils as Philip Hall, Christopher Zeeman (later Sir Christopher) and D. G. Northcott (Stone's contemporary). The mathematics teaching was in the hands of C. A. J. Trimble, himself a Wrangler. Here, Arthur won prizes in almost all subjects except sports (though he was also good at rugger).

In 1935 he gained a major scholarship to Trinity College, Cambridge. He excelled at the academic subjects, but was also an outstanding violinist and good at chess. At Cambridge he continued with the violin and became leader of the orchestra of the Cambridge University Music Society. He was a Wrangler, and took his BA in 1938, before going to Princeton, to work for a PhD under S. Lefschetz.

Although a single-minded mathematician, he had wide-ranging interests, and this combination often showed up in unexpected ways. To fit the American notebook sheets into his English binder, he had to trim off an inch of paper, and he began to fold these strips in various ways. This led to some intriguing figures, which later became famous as 'flexagons' (see ⟨**6**, **3**⟩). He was both very inventive and also adept with his hands, talents which he used in building a counterclockwise grandfather clock.

Another problem that occupied him and some of his friends, was how to dissect a square into unequal smaller squares. They managed a dissection with 69 squares, and this led to his first (joint) paper [**1**]. The method was criticized by Bouwkamp ⟨**1**⟩, but this was later retracted, and in [**5**] the authors deal with Bouwkamp's criticism and give a formula for their example in Bouwkamp's notation. Later, Stone returned to graphs in [**35**], where he shows how Lichtenbaum's conjecture on the density of an n-dimensional normal space can be reduced to a question in graph theory. In [**60**], he and the authors of [**1**] look at electrical networks as graphs, and give a determinantal expression for the current flow.

A similar problem, but with a more topological flavour, was generalizing the 'sandwich' theorem. Ulam had shown how to bisect three sets in space, of finite outer measure, by a plane. In [**2**], Stone and Tukey generalized the problem to n subsets of any set R with Carathéodory outer measure, where the plane is now replaced by an appropriate real function.

Stone's main interest was to be general point-set topology, where he wrote on metrizable and paracompact spaces, unicoherent and multicoherent spaces, modifications of compact spaces, Borel sets, rectangular tilings and various kinds of continuous functions. A paper with a great and lasting influence is [**7**], where he solves a problem raised by Dieudonné, by proving in a most elegant way that all metrizable spaces are paracompact—by showing that paracompactness is equivalent to full normality, which was known to follow easily from metrizability. This proof was far from trivial, and the methods have played a significant role in later work done by others. Another interesting result proved here is that the product of uncountably many copies of the integers is not normal, which (combined with the previous theorem) shows that a product of metrizable spaces is normal (or paracompact) if and only if at most countably many factors are non-compact. This led him to a study of unicoherent spaces; in [**8**] he established various conditions for unicoherence and proved relations between subsets, their frontiers and intersections, often best possible. He continued this work in [**9**] and [**10**], where he showed that some expected generalizations break down in the multicoherent case, and found valid (but more complex) generalizations, and extensions of the Phragmen–Brouwer theorem [**11**], and studied an infinite degree of multicoherence.

In 1948 he made a brief excursion into fluid dynamics in [**6**], when he discussed the theoretical basis, validity and uniqueness of calculations by Kopal of flow past yawing cones, and treated second-order effects in [**12**].

Another excursion, into abstract sets, resulted in a paper with G. Higman [**15**], where they constructed an inverse system of non-empty sets with empty inverse limit; it had been known that for compact sets this limit must be non-empty (see ⟨**2**⟩). Stone returned to the topic in [**45**], where he gave conditions for the inverse limit of compact non-Hausdorff spaces to be non-empty, compact or hereditarily compact. Another paper along these lines is [**30**], where he gave a criterion for a preordered set to have a partition into k cofinal subsets: each element must have at least k successors. The proof is an elaborate use of transfinite induction.

In 1942, in Pittsburgh, he married Dorothy Maharam, who was also a mathematician, working in measure theory, where she obtained some notable results and also did some joint work with her husband, resulting in a fruitful blend of general topological methods applied to measure-theoretic questions [**43**, **44**, **46**, **47**, **52**]. After the war he returned with his wife to England, where he was fellow of Trinity College, Cambridge from 1946 to 1948, and then went as lecturer to Manchester University. He was a superb expositor, both in his papers and in his lectures, which were distinguished by their clarity. In 1952 his wife also joined the staff of the University.

In 1953 he made a study of the Boltyanskii density of a space. This was known to be at least 6 for any two-dimensional compactum; Stone proved that for any two-dimensional normal space it did not exceed 7, and gave examples where the value 6 was realized, using the nerve of the covering. He next returned to metrizability problems. Yu. Smirnov had proved in 1956 that a locally countably compact Hausdorff space which is the union of $\aleph_0$ many separable metric subsets is itself metrizable. Stone in [**17**] obtained analogous results without assuming the subsets to be separable. This led him in [**18**] to a study of the number of closed subsets of a metric space of a given cardinality and weight (equal to the least cardinal of a basis). These results were remarkable because in the Čech compactification of Z, a closed set is either finite or of cardinal 2^{c}. He went on, in [**19**], to discuss the existence of universal spaces in the class of all metric uniform spaces of density character m with conditions placed on the uniformity. In [**21**], he studied topological spaces in which each subspace is compact. The interesting examples of such spaces are not T_2; they have been studied in connexion with algebraic constructions.

In 1961, the Stones went back to the USA, where they both obtained professorships at the University of Rochester, NY. Stone now turned to the study of Borel sets and analytic sets in metric spaces. He showed in [**22**] that a metric space is an absolute F_σ if and only if it is locally compact. Since 'locally compact' implies 'σ-compact', this includes the classical characterization of F_σ-spaces by σ-compactness. A detailed study of Borel sets appeared in [**23**], where he attacked the question of whether Borel isomorphisms are equivalent to generalized homeomorphisms, by an examination of cases where this is so. He continued his efforts to classify topological properties of Borel spaces preserved by Borel isomorphisms in [**27**]. For σ-discrete spaces this leads to a solution of the classification problem. Later, in [**39**], he treats the problem of finding properties of absolute analytic spaces invariant under Borel isomorphisms, using the weight of the space. The current progress on absolute Borel sets and k-analytic sets in metric spaces is summarized in [**34**]. There follows a series of papers in which various properties of subsets of topological spaces, such as compactness, measurability, and so forth, are characterized.

In a paper with E. Michael [**32**], which has been described as 'very interesting and significant', he studies continuous images of the space of irrationals. They show that a metrizable space which is a continuous image of the space P of irrational numbers is also a quotient of P; in particular, the space of rationals is a quotient of the space of irrationals.

In 1982, the University of Rochester held a conference in Stone's honour ⟨**5**⟩. Five years later he and his wife retired and went to Northeastern University, Boston, MA, where Arthur became Adjunct Professor in 1988. This was a part-time appointment, and it allowed them to spend each winter in England. He had been in good health (in fact, the couple regularly used to walk the three miles to the University and back home) until 2000, when in March he had an operation to remove an aneurism; he recovered well, but on August 6 died from ideopathic pulmonary fibrosis.

The Stones had a son and a daughter, who both became mathematicians.

I am greatly indebted to his widow and his daughter for providing information about the family. A number of mathematicians and former colleagues have also helped me in writing this obituary: P. J. Hilton, E. A. Michael, D. G. Northcott, S. Rosenbaum, C. A. B. Smith, F. Smithies, W. T. Tutte and J. E. G. Utting. I should like to express my warmest thanks to them here.

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