

PART 1:
PLANET FORMATION

The occurrence and the distribution of masses and radii of exoplanets

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Abstract. We analyze the statistics of Doppler-detected planets and *Kepler*-detected planet candidates of high integrity. We determine the number of planets per star as a function of planet mass, radius, and orbital period, and the occurrence of planets as a function of stellar mass. We consider only orbital periods less than 50 days around Solar-type (GK) stars, for which both Doppler and *Kepler* offer good completeness. We account for observational detection effects to determine the actual number of planets per star. From Doppler-detected planets discovered in a survey of 166 nearby G and K main sequence stars we find a planet occurrence of $15_{-4}^{+5}\%$ for planets with $M \sin i = 3\text{--}30 M_E$ and $P < 50$ d, as described in Howard *et al.* (2010). From *Kepler*, the planet occurrence is 0.130 ± 0.008 , 0.023 ± 0.003 , and 0.013 ± 0.002 planets per star for planets with radii 2–4, 4–8, and 8–32 R_E , consistent with Doppler-detected planets. From *Kepler*, the number of planets per star as a function of planet radius is given by a power law, $df/d \log R = k_R R^\alpha$ with $k_R = 2.9_{-0.4}^{+0.5}$, $\alpha = -1.92 \pm 0.11$, and $R = R_P/R_E$. Neither the Doppler-detected planets nor the *Kepler*-detected planets exhibit a “desert” at super-Earth and Neptune sizes for close-in orbits, as suggested by some planet population synthesis models. The distribution of planets with orbital period, P , shows a gentle increase in occurrence with orbital period in the range 2–50 d. The occurrence of small, 2–4 R_E planets increases with decreasing stellar mass, with seven times more planets around low mass dwarfs (3600–4100 K) than around massive stars (6600–7100 K).

Keywords. planetary systems: formation, stars: statistics

1. Introduction

The occurrence of gas-giant exoplanets has been quantitatively studied from careful counting of planets detected by the Doppler technique within well-defined samples of stars. Cumming *et al.* (2008) found that 10.5% of Solar-type stars host a gas-giant planet in the mass range, $M \sin i = 100\text{--}3000 M_E$, and orbital period range, $P = 2$ d – 5.5 yr. These gas-giants occur around solar-type stars with a frequency that depends on planet mass and orbital period as, $df \propto M^{-0.31 \pm 0.2} P^{0.26 \pm 0.1} d \log M d \log P$. The number of planets rises with smaller masses and larger orbital distances (in logarithmic intervals). This distribution as a function of planet mass and orbital-period reveals important information about planet formation and migration as shown by Ida & Lin (2010), Mordasini *et al.* (2011), Raymond *et al.* (2011), Bromley & Kenyon (2011) and Wittenmyer *et al.* (2011). Similarly, the clear dependence of planet occurrence on stellar mass and metallicity, shown for example by Johnson *et al.* (2010), is consistent with formation by rocky-core-nucleated accretion of H and He gas in a protoplanetary disk as shown by Ida & Lin (2008b), Mordasini *et al.* (2011) and Alibert *et al.* (2011).

Here we present two recent studies of the planet occurrence for smaller planets that have masses and radii less than those of Saturn. We first consider the small planets discovered by the Doppler technique, which is sensitive to planets having masses as low

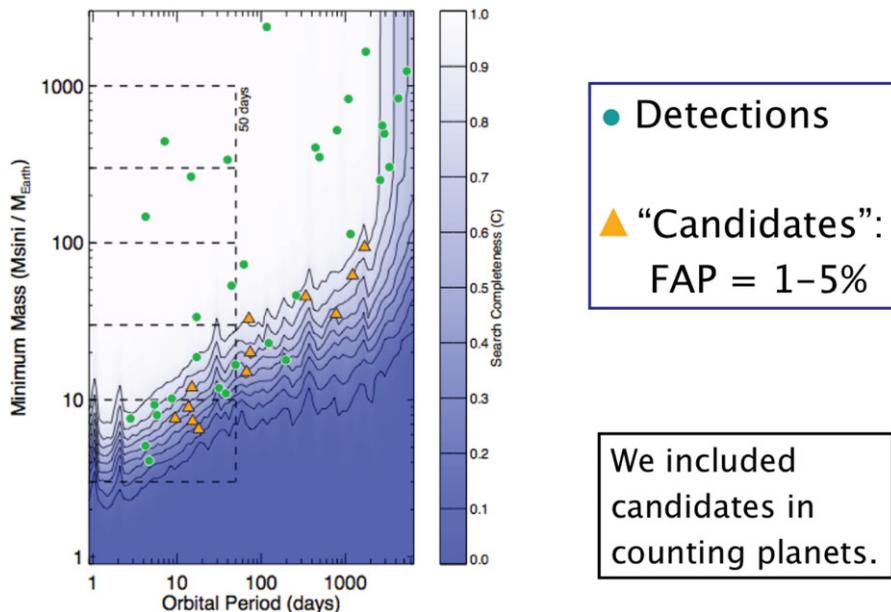


Figure 1. Doppler-detected planets (green circles) and candidate planets (orange triangles) from the survey by Howard *et al.* (2010), in a two-parameter space of orbital period and minimum mass. We consider five mass domains of 3–10, 10–30, 30–100, 100–300, 300–1000 Earth-masses, and orbital periods less than 50 days, as marked by dashed lines. The fraction of stars with sufficient measurements to rule out planets in circular orbits of a given minimum mass and orbital period is shown as blue contours from 0.0 to 1.0 in steps of 0.1. The white regions have 100% detectability and the blue regions have low detectability. Our occurrence statistic corrects for this detectability in each domain, described in Howard *et al.* (2010).

as a few Earth masses for close-in orbits with periods under 50 days, as described by Howard *et al.* (2010). We also carry out an analysis of the epochal *Kepler* results for transiting planet candidates from Borucki *et al.* (2011) with a careful treatment of the completeness. We focus attention on the planets with orbital periods less than 50 days to match the period range for which the Doppler surveys are robust, as described by Howard *et al.* (2011b). Here, we review planet occurrence as a function of planet masses and radii, restricting our attention to planets within 0.25 AU of G and K-type main sequence stars, i.e. solar-type stars. We also consider planet occurrence as a function of stellar mass.

2. The Occurrence of Small Planets from Doppler Surveys

We have conducted a sensitive Doppler survey for planets during the past five years, beginning with a blind sample of 166 G & K-type main sequence stars that are chromospherically quiet. From this sample we detected many planets from our Doppler measurements, and some planets were found by others. We have included all planets here, to be complete. In Fig. 1 we show all of the detected planets in this survey by Howard *et al.* (2010). The Doppler-detected planets are shown as green circles, and those with a false-alarm probability 1–5% are deemed “candidate planets” and shown as orange triangles. We exhibit all planet detections in a two-parameter space of orbital period and minimum mass. We divide this space into five domains of minimum planet mass ($M \sin i$) of 3–10, 10–30, 30–100, 100–300, 300–1000 Earth-masses. We only consider orbital periods less

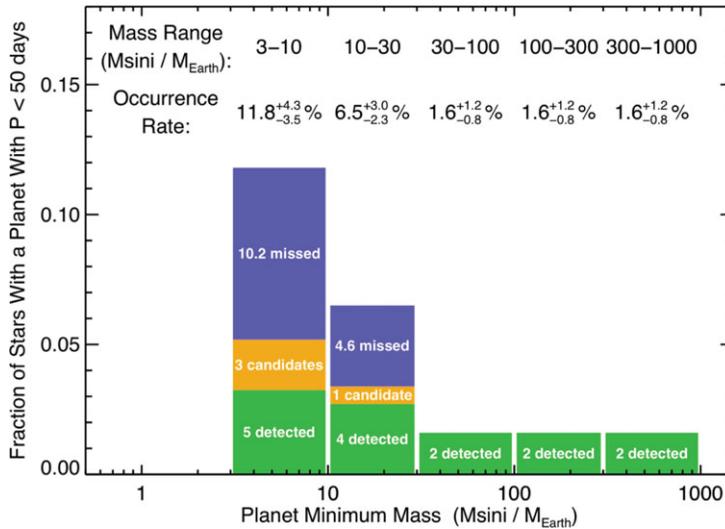


Figure 2. The fraction of solar-type stars with a planet as a function of planet (minimum) mass, $M_{\text{sin}i}$. Only periods less than 50 d are included. The detected (green), candidate (orange), and missed (blue) planet are shown separately. Missed planets represents the correction for detectability due to inadequate sensitivity, based on those stars that did have adequate sensitivity. (From Howard *et al.* (2010).)

than 50 days for which Doppler detection remains strong down to masses of $3 M_E$, as marked by dashed lines. The fraction of stars with sufficient measurements to rule out planets in circular orbits of a given minimum mass and orbital period is shown as blue contours from 0.0 to 1.0 in steps of 0.1. The white regions have 100% detectability of planets and the blue regions have low detectability. Our occurrence statistic corrects for this detectability in each domain.

From the knowledge of the 166 GK stars in the original Doppler survey, one may compute the planet occurrence. We exhibit the planet occurrence as a function of minimum-planet mass ($M_{\text{sin}i}$) in Fig. 2. The occurrence of planets rises rapidly toward smaller masses. For planets having minimum masses $3\text{--}10 M_E$, the occurrence is 11.8%, and for planets of minimum masses $10\text{--}30 M_E$, the occurrence is 6.5%. These two mass bins give the occurrence of “super-Earths” and “exo-Neptunes” for periods less than 50 days around solar type stars, a total of 18.3%.

In Fig. 3 we exhibit the Doppler-detected planets again, but this time overplotted with the planets predicted from population syntheses. The small black dots represent those predicted by the theory of Mordasini *et al.* (2009) for which the results are similar to those from Ida & Lin (2010). We note that the observed planet distribution differs from those of theory in two ways. The predicted planet desert from 3 to $10 M_E$ is in fact well populated with planets. The desert doesn’t actually exist. And the predicted uniform distribution of planets with mass from 100 down to $10 M_E$ actually is populated by an increasing number of planets with decreasing mass. The observed increase in planet number with decreasing mass was not predicted by the simulations. Thus, there seems to be a missing physics in either the planet formation or migration included in the simulations.

3. The Occurrence of Small Planets from *Kepler*

From all 1235 planet candidates announced in Borucki *et al.* (2011), we consider only a subset of target stars and the associated planets. We consider only stars having surface

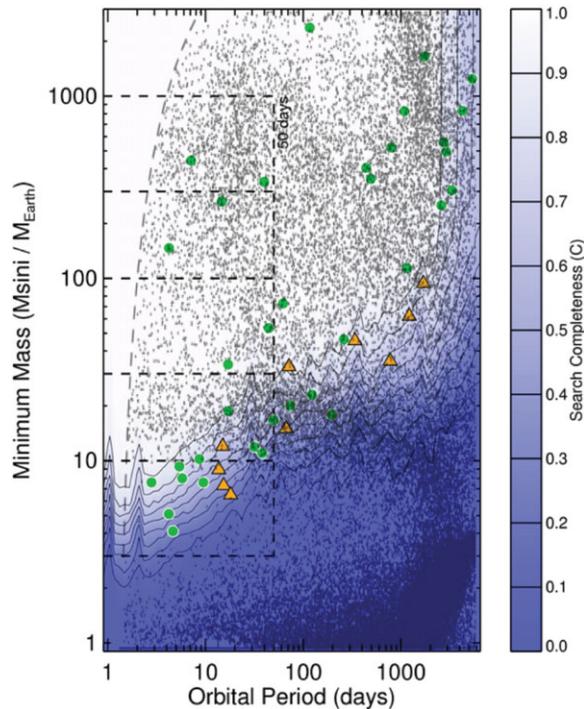


Figure 3. Same as Figure 1, but overplotted is the synthetic population of planets from Morasini *et al.* (2009) and Ida & Lin (2010). The predicted “desert” of planets from 3–10 Earth-masses and $P < 50$ d from theory is quite well populated with actual planets. Thus the theory of planet formation and migration is missing a key ingredient that actually populates the desert as noted by Howard *et al.* (2010).

temperature $T_{\text{eff}} = 4100\text{--}6100$ K, surface gravity $\log g = 4.0\text{--}4.9$, and *Kepler* magnitude < 15 mag. This restriction reduces the number of *Kepler* target stars under consideration to $\sim 58,000$. We construct a two-dimensional space of orbital period and planet radius, as shown in Fig. 4. We divide this space into small cells of specified increments in orbital period and planet radius, and we carefully determine the subset of target stars for which the transit depths of planets in that domain would have a signal-to-noise ratio, $\text{SNR} > 10$. In that way, each domain of orbital period and planet size (or mass) has its own subsample of target stars (typically 58,000 or less) that are selected *a priori*, within which the detected planets can be counted and compared to that number of stars. The occurrence of planets within each cell is a simple ratio. In the numerator is the number of planets detected in that cell, with each planet multiplied by the correction for orbital inclination, a/R_{star} , to account for inclined, non-transiting orbits. In the denominator is the total number of stars for which such a detection would have been possible with $\text{SNR} > 10$.

Thus, planet occurrence is simply the number of detected planets having some set of properties (radius and orbital periods) compared to the set of stars from which planets with those properties could have been reliably detected. We include only planet candidates found in three *Kepler* data segments (“Quarters”) labeled Q0, Q1, and Q2, for which all photometry is published in Borucki *et al.* (2011). Planet radii stem from stellar radii which are estimated from T_{eff} and $\log g$ and carry an uncertainty of 35% rms as measured by Brown *et al.* (2011).

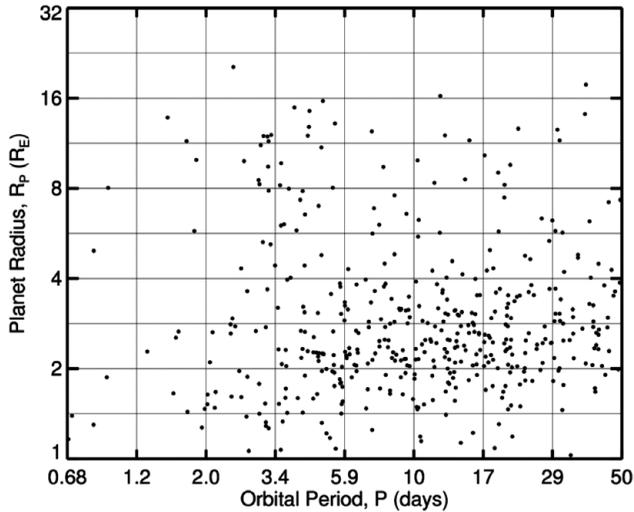


Figure 4. *Kepler* planet candidates plotted in a two-parameter space of orbital period and planet radius from Borucki *et al.* (2011). The space is divided into a grid of equal logarithmically spaced cells within which planet occurrence is computed. The planet candidates detected by *Kepler* having SNR > 10 are shown as black dots. We compute planet occurrence within each cell individually. In each cell there are a certain number of *Kepler* target stars with low enough noise to permit detection of that cell's planets with SNR > 10. The planet occurrence is the number of detected planets, multiplied by factor that accounts for the missed planets due to inclination, namely a/R_{Star} , divided by the number of stars amenable to such detection. The method is described in detail in Howard *et al.* (2011b).

The determination of planet occurrence is carried out *only* among those stars having photometric quality so high that the transit signals stand out easily. We adopt the metric of the signal-to-noise ratio (SNR) of the transit signal integrated over a 90 day photometric time series, setting a threshold, SNR > 10. Fig. 5 shows the parameter space again, but with the number of surviving target stars written in white within each cell. We restrict our study to orbital periods under 50 days.

We adopt the *Kepler* planet candidates and their orbital periods and planet radii from Table 2 of Borucki *et al.* (2011). Morton & Johnson (2011) note that the false positive probability depends on transit depth, galactic latitude, and *Kepler* magnitude. Using their model we estimate that 22 planet candidates are actually false positives. The resulting false positive rate is 5–10% for planet radii, $R_P > 2 R_E$.

The above description defines planet occurrence, f , as the number of planets detected within a cell in Fig. 5 augmented by the factor that accounts for all orbital inclinations, a/R_{Star} , divided by the number of a target stars within that same cell in Fig. 5. The planet occurrence within a cell is given by

$$f_{\text{cell}} = \sum_{j=1}^{n_{\text{pl, cell}}} \frac{1/p_j}{n_{\star, j}}, \quad (3.1)$$

where the sum is over all detected planets within the cell that have SNR > 10. In the numerator, $p_j = (R_{\star}/a)_j$ is the probability of a transiting orientation of the orbital plane for planet j . Thus each detected planet is augmented in its contribution to the planet count by a factor of a/R_{\star} to account for the number of planets with similar radii and periods that are not detected because of non-transiting geometries. For each planet, its specific value of $(a/R_{\star})_j$ is used, not the average a/R_{\star} of the cell in which it resides.

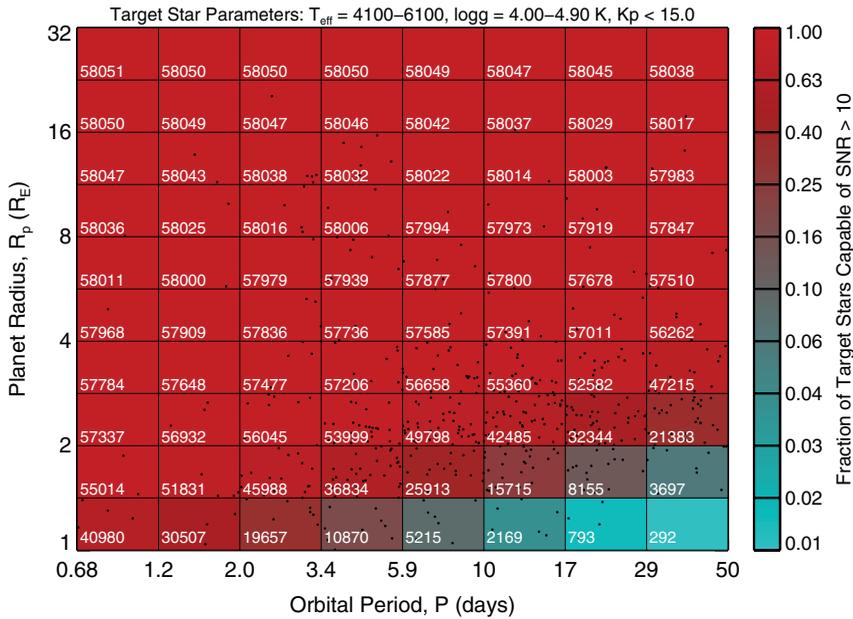


Figure 5. The two-parameter space of orbital period and planet radius. The white number in each cell is the number of *Kepler* target stars under consideration within the cell. The *Kepler* target stars must satisfy these criteria: $T_{\text{eff}} = 4100\text{--}6100$ K, $\log g = 4.0\text{--}4.9$, *Kepler* magnitude < 15 . Also we used the observed photometric noise to include only those stars quiet enough so that planets in that cell would be detected with $\text{SNR} > 10$. The number of stars shown in each cell (typically $\sim 58,000$ for the “red” cells) is the denominator of the planet occurrence calculation. The color code indicates the fraction of target stars capable of achieving $\text{SNR} > 10$, indicating low detectability for small radii and long periods (fewer transits). (See Howard *et al.* (2011b).)

Note that each scaled semi-major axis $(a/R_*)_j$ is measured directly from *Kepler* photometry and is not the ratio of two quantities, a_j and $R_{*,j}$, separately measured with lower precision. In the denominator, $n_{*,j}$ is the number of stars for which a planet of radius $R_{p,j}$ and period P_j would have been detected with $\text{SNR} > 10$.

We computed planet occurrence as a function of planet radius by integrating the planet occurrence over all orbital periods with $P < 50$ days. The resulting distribution of occurrence with planet radius is shown in Fig. 6, which shows a clear increase with decreasing planet radius. Smaller planets are far more numerous than large planets, for solar-type stars and periods less than 50 d.

We fit the data with a power law, finding:

$$\frac{df(R)}{d \log R} = k_R R^\alpha. \quad (3.2)$$

Here $df(R)/d \log R$ is the mean number of planets per star having $P < 50$ days in a \log_{10} radius interval centered on R (in R_E), with $k_R = 2.9^{+0.5}_{-0.4}$, $\alpha = -1.92 \pm 0.11$, and $R = R_P/R_E$. For comparison, Howard *et al.* (2010) found a power law planet *mass* function, $df/d \log M = k' M^{\alpha'}$, with $k' = 0.39^{+0.27}_{-0.16}$ and $\alpha' = -0.48^{+0.12}_{-0.14}$ for periods $P < 50$ days and masses $M \sin i = 3\text{--}1000 M_E$.

We also computed planet occurrence as a function of orbital period. Fig. 7 shows that planet occurrence increases slowly with increasing orbital period for the logarithmic binning used here, and the increase is fastest for the smallest planets with $R_P = 2\text{--}4 R_E$ (shown in yellow). Finally, we computed the planet occurrence as a function of

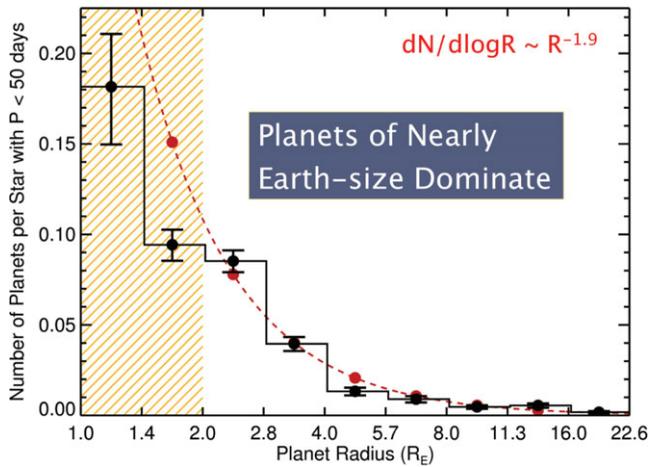


Figure 6. The number of planets within a bin of radius divided by the number of target stars that could have detected such planets by *Kepler*, as a function of planet radius. We include only orbital periods of $P < 50$ days (black filled circles and histogram). We also only include GK main sequence stars consistent with the selection criteria in Figure 5 to compute planet occurrence. A power law fit is shown in red. The estimates of planet occurrence are incomplete for radii below $2 R_E$, shown hatched. Error bars indicate statistical uncertainties and do not include effects of the poorly known stellar radii (35% uncertainty). (From Howard *et al.* (2011b).)

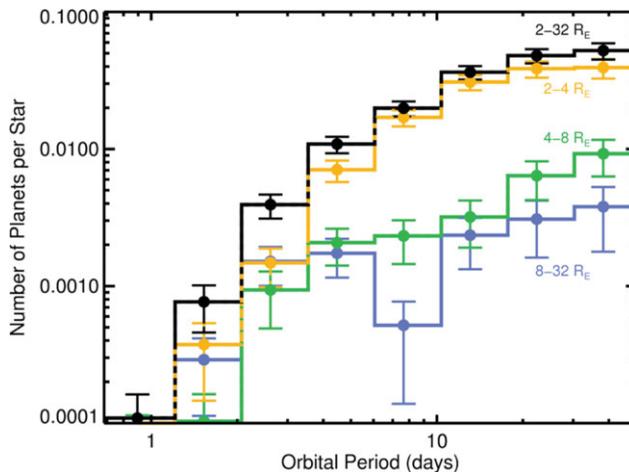


Figure 7. Planet occurrence as a function of orbital period from *Kepler*, for different planet sizes designated by color. Only stars consistent with the target selection in Figure 5 (and their associated planets) were included, namely solar type stars brighter than 15th magnitude. (See Howard *et al.* (2011b).)

stellar effective temperature and converted them to stellar masses (approximately). As the targets were restricted to main sequence stars this conversion is straightforward. Fig. 8 shows that planet occurrence of the smallest planets, $2\text{--}4 R_E$, increases with decreasing stellar mass. Apparently the K and M dwarfs have small planets more commonly than the G and F main sequence stars for orbital periods under 50 days considered here.

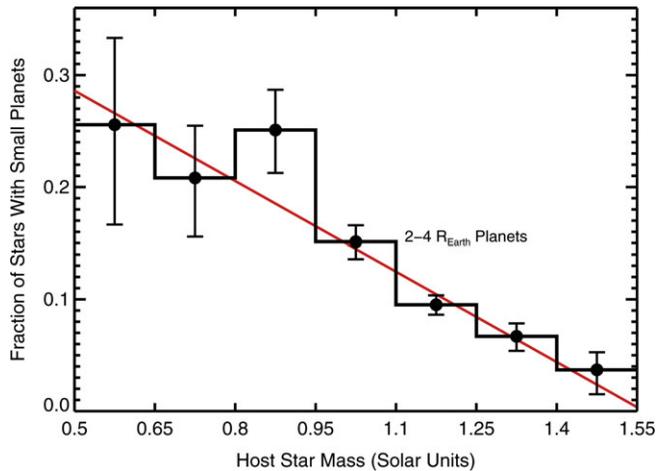


Figure 8. Occurrence of small planets, $2-4 R_E$, as a function of stellar mass on the main sequence. Stellar masses were assigned based in the effective temperatures, T_{eff} , in the *Kepler* Input Catalog (uncertain by $\sim 135\text{K}$), yielding stellar masses uncertain by $\sim 20\%$. We consider only planets with $P < 50$ days and host main sequence stars having $T_{\text{eff}} = 3600-7100\text{K}$. The occurrence of small planets of $2-4 R_E$ rises substantially toward lower mass stars. A similar result is shown in Figure 13 in Borucki *et al.* (2011) for the Neptune-size planets ($2-6 R_E$). The best-fit linear occurrence model for these small planets is shown as a red line.

4. Summary

From the Doppler-detected planets in the Solar neighborhood, we find a planet occurrence of $15_{-4}^{+5}\%$ for the mass range, $M \sin i = 3-30 M_E$ and period range, $P < 50$ d, around main sequence G and K stars. This occurrence of smaller planets is continuous with the occurrence of giant planets found by Cumming *et al.* (2008). From *Kepler* the planet occurrence varies by over three orders of magnitude in the radius-orbital period plane and increases substantially down to the smallest radius ($2 R_E$) and out to the longest orbital period (50 days, ~ 0.25 AU). The distribution of planet radii is given by a power law, $df/d\log R = k_R R^\alpha$ with $k_R = 2.9_{-0.4}^{+0.5}$, $\alpha = -1.92 \pm 0.11$, and $R = R_P/R_E$. The number of planets per star for planet radii of $2-4 R_E$ is 0.13, and the number of planets per star for radii of $4-8 R_E$ is 0.023. Combining these gives an occurrence for planets of $2-8 R_E$ of 0.15, remarkably similar to the 18% occurrence rate found from the Doppler-detected planets in approximately the same range of periods and for GK stars. Thus two independent methods of planet detection yield an occurrence of planets between $2-8 R_E$ of 15–18%, for periods under 50 days. This occurrence stands as a test benchmark for the theory of planet formation, migration, and planet-planet interactions, such as those of Ida & Lin (2010), Mordasini *et al.* (2011), Schlaufman *et al.* (2010) and Wu & Lithwick (2011).

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