OBSERVATIONAL ASPECTS OF MACROTURBULENCE IN EARLY TYPE STARS

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I. Introduction

The basic premise of this paper is that the atmospheres of supergiants of spectral type O, B, and A are not homogeneous static and stable. Observations show effects in the spectrum which seem to indicate velocity fields of many different scales, which probably are variable with depth and are certainly variable with time. Some effects can be understood in terms of global, steady, symmetric velocities, namely rotation and expansion. Those effects which still cannot be accounted for are generally attributed to turbulent velocities.

The concept of curve of growth microturbulence dates back to Struve and Elvey (1934), but will not be discussed at length here. A wide ranging review was presented by Wright (1955) and lengthy discussions of problems of interpretation are found in Thomas (1960), and Mihalas (1979). The concept of spectroscopic macroturbulence may also have originated with Struve (1952), and was developed and clarified by Huang and Struve (1953). Theoretical arguments, based on the outward force of radiation pressure, led Underhill (1949) to predict that supergiant atmospheres would be mechanically unstable. In fact, irregular, semi-periodic radial velocity variations were discovered in A type supergiants by Abt (1957). Indirect evidence for turbulent velocities was discussed by Slettebak (1956), and Abt (1958). Reviews of these arguments were presented by Underhill (1960) and Rosendhal (1970).

I will summarize the results of observational studies since then, which have been carried out with new and higher quality data. For the most part these programs incorporated more sophisticated statistical and analytical techniques, and often made reference to modern model atmosphere calculations. I will discuss primarily OBA supergiants, and say little about main sequence stars, Wolf-Rayet stars, or binary systems. Theoretical and computational techniques are discussed in the volume by Cayrel and Steinberg (1978), and in other papers in this colloquium.

II. Photometric Results

During the past decade photometric studies have provided a great deal of new information about the magnitudes, colors, and polarization properties of the continua of early type stars. Time resolved studies have detected and analyzed variability of several interesting and complicated types. The most extensive study of O type stars was that by Morrison (1975), in which u-v-b-y-Hβ photometry of over one hundred stars was analyzed and compared to the predictions of the static non-LTE model atmospheres of Mihalas (1972). Maeder and Rufener (1972) surveyed the photometric properties of 80 non-supergiant OB stars, and 34 supergiants of type B2-G8, and described the variability of their color indices. Sterken (1977), Burki et al. (1978), and Feitzinger (1978) reported the results of statistical studies of the variability of a smaller number of extremely luminous galactic and large Megallanic Cloud supergiants. Schild and Chaffee (1975) investigated the spectrophotometric properties of the Balmer discontinuity in O and B type supergiants, and Serkowski (1970) and Hayes (1975) have studied the intrinsic linear polarization of the continua of O type stars. The results of these and other similar reports can be summarized as follows:

The most luminous stars of all spectral types are variable in both light and color. A rough lower limit to the luminosity at which variability is detected is at an absolute visual magnitude of $M_V \sim -5$. Included therefore are most of the O stars earlier than about O6, all O supergiants and Of stars, and all Ia and Iab supergiants of spectral types B, A, and F. Amplitudes of variation range from the limit of certain detection ($\lesssim$ .1 mag.) in color indices (U-B or v-y). The amplitude correlates fairly well with spectral type and luminosity class. For a given luminosity, the hottest stars show the largest range of variation. Similarly, at a given spectral type, the more luminous stars vary with a larger amplitude.

The variations have been observed with a wide range of timescales. Long term trends are present in monthly and yearly mean magnitudes for all stars, but owing to a paucity of observations, few stars have been studied in detail. At the other extreme, only in the most luminous Ia supergiants, and those with the largest amplitudes, can statistically significant variations within a single night be confirmed. On the other hand, all of the supergiants vary significantly in several days. Unique periods do not exist, but characteristic timescales, sometimes called "semi-periods" can be found in a well defined manner. A harmonic analysis is performed in which a model light curve is represented by the lowest order terms of a Fourier series. A number of trial periods are tried, each time adjusting the amplitudes until the χ^2 deviation between the model and the observations is minimized. The correlation coefficient is computed for each model and that with the highest correlation is taken to be the best singly periodic representation of the observations. In general, these solutions are good, but not perfect for B8 Ia - A2 Ia stars, with correlation coefficients of about .95, and semiperiods of 20-30 days. Among the earlier types, B0 Ia - B2 Ia, a singly periodic solution is a much less satisfactory model, with correlations of .5 to .6. The semi-periods are shorter, with values of 5-10 days being typical. Among samples of stars with similar luminosity, say $-8.2 \leq M_V \leq -7.2$, the semi-period expressed as log P increases nearly linearly with spectral type, in the sense that the later type stars have longer periods. Similarly, at a given spectral type, the more luminous stars have longer periods - see Figure 1.

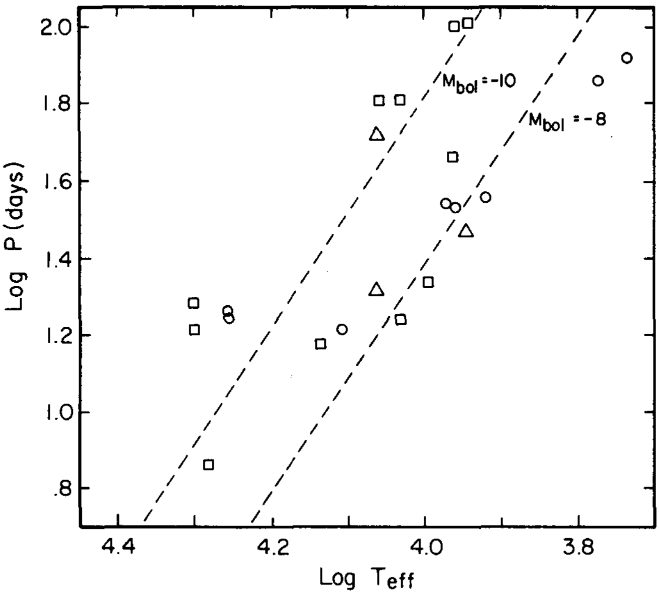

Fig. 1 - The semi-periods of photometric variation compiled from Maeder and Rufener (1972) Δ, Sterken (1977) □, and Burki *et al.* (1978) O, are plotted as a function of temperature. The temperature calibration of Barlow and Cohen (1977) was used. The dashed lines represent equation 4 with Q = .1 day.

It is thought that the observed variability is basically a pulsation phenomenon, possibly the simultaneous excitation of two or more non-radial modes. Despite the lack of strict periodicity the characteristic timescales and their correlation with temperature and luminosity are suggestive of pulsation. For gravitational oscillations, the period is related to the mean density as

$$P = Q(\rho/\rho_\odot)^{-\frac{1}{2}} , \quad \text{Log } P = \text{Log } Q - .5 \text{ Log } (\frac{M}{M_\odot}) + 1.5 \text{ Log } (\frac{R}{R_\odot}) \tag{1}$$

Taking the radius from the definition of effective temperature

$$\frac{L}{L_\odot} = (\frac{R}{R_\odot})^2 (\frac{T}{T_\odot})^4 , \quad \text{Log } (\frac{L}{L_\odot}) = 2 \text{ Log } (\frac{R}{R_\odot}) + 4 \text{ Log } (\frac{T}{T_\odot}) \tag{2}$$

the mass from the supergiant mass-luminosity relation

$$\text{Log } (\frac{L}{L_\odot}) \simeq 2.5 \text{ Log } (\frac{M}{M_\odot}) + 1.38 \tag{3}$$

and the solar values $M_\odot = 4.77$ and $T_\odot = 5760$ K, gives

$$\text{Log } P = \text{Log } Q - .22 \, M_{\text{bol}} - 3 \text{ Log } T_{\text{eff}} + 12.61. \tag{4}$$

Even this simple expression demonstrates the observed correlations as can be seen in Figure 1. The slopes of the relationships are approximately correct when a vlue of Q ~ .1 day is adopted.

III. Radial Velocities

The measurement of precise absorption line positions, expressed as radial velocities, provides the most direct evidence that large scale velocity fields are present in the line forming regions of early type supergiants. The observations can be grouped into two basic categories; those that indicate a radial velocity gradient through the atmosphere (almost always an expansion velocity), and those that show velocity fluctuations.

The radial velocity study of O and Of stars by Conti, Leap, and Lorre (1977) showed that expansion velocities of 20–30 km sec^{-1} are present in the deepest visible layers of Of stars. Bohannon and Garmany (1978) found a strong gradient present in the Balmer line velocities of 25 O type stars. Ebbets (1979) measured strong gradients in O type supergiants, but found no such effect in main-sequence stars (Figure 2). The effect found in all of these studies was that the more opaque Hydrogen lines, Hα, Hβ, etc., have systematically more negative radial velocities than the higher Balmer lines H10–H15, or the helium, silicon, and CNO lines. Hβ is typically 20–30 km sec^{-1} more negative, and Hα may be shifted by from 50 to 200 km sec^{-1} in the same sense.

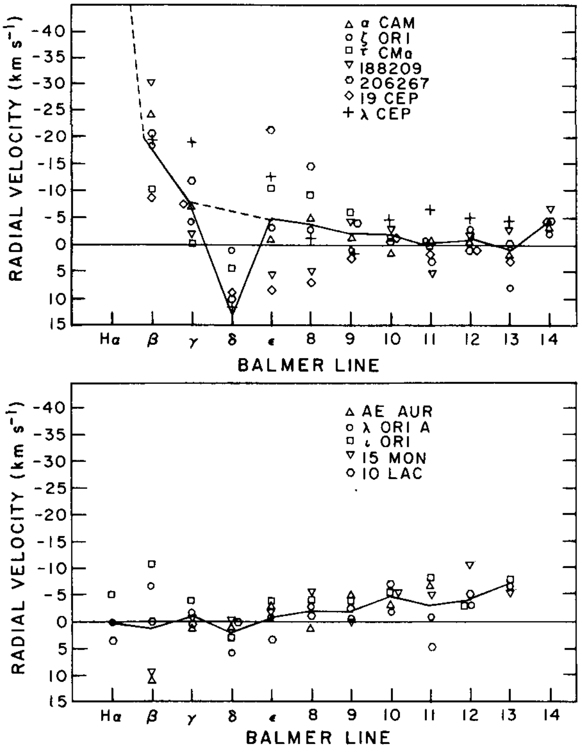

Fig. 2 – The Balmer lines in O supergiants show a systematic trend in velocity which indicates that the lines are formed in an expanding atmosphere which accelerates over the line forming region. The non-supergiants show no such effect. The zero point is the mean velocity of lines of C, N, O, and Si. These figures were adopted from Ebbets (1979), courtesy of Ap. J.

This phenomenon is not restricted to O stars but has been well studied in B types by Hutchings (1976) and in the A type supergiants by Aydin (1972), and Wolf et al. (1974). In these stars, the lines of many different ions can be measured, and a strong correlation between ionization potential and radial velocity is found. The sense of the correlation is almost always that the lower excitation lines have persistent and systematically more negative velocities.

The radial velocity variability of hot supergiants has been known for over twenty years, but has been the subject of few thorough studies. The range of variations is usually small, from the limit of confident detection (~ 1 km sec^{-1}) to about 40 km sec^{-1} in extreme cases. As was the case with photometric variations, the largest amplitudes tend to be found in the most luminous stars. The timescale of variations ranges from days to years. Despite careful statistical searches (Lucy 1975) significant changes within one night have not been found. Long term changes - timescales longer than a year - are similarly difficult to confirm. The best documented variations occur on timescales of 4 to 30 days. Often the changes are fairly regular, but again no stable, single period is ever present. No convincing correlation has yet been demonstrated between the characteristic timescale, and spectral type or luminosity class, since most of the studies to date have analyzed a limited range of spectral types.

The earliest spectral type star for which a comprehensive, modern radial velocity study exists is α Cam -O9.5 Ia. The absorption lines of this massive ($M_* \sim 30\ M_\odot$) and luminous ($M_{bol} \sim -9.75$) supergiant show both line to line, and time to time variations in radial velocity. This complication demands that a careful statistical procedure be used to separate these effects. Such an analysis was initiated by Bohannon and Garmany (1978) in which a two-way analysis of variance, accompanied by an F test, confirmed the highly significant nature of both effects. The gradient across the Balmer line is about 25 km sec^{-1} at Hβ, and itself varies significantly. Systematic variations of the entire spectrum with an amplitude of about 20 km sec^{-1} are also confirmed with a timescale of about thirty days. In a follow-up study with more and better data, Tryon and Garmany (1979) were not able to represent these motions with an orbital solution. The motions are definitely real, with a well established amplitude and timescale, but do not allow a simply periodic description.

An even more thorough analysis of the radial velocity variations of α Cyg - A2 Ia - by Lucy (1975) produced a remarkable insight into the nature of supergiant variability. Lucy was able to represent a long series of 447 radial velocity measurements (Paddock 1935) with a harmonic series containing sixteen terms. Sophisticated statistical tests demonstrated that the periods, amplitudes, and phases of all terms were significant, and stable for at least the six years during which the observations were made. The periods of the individual terms range from seven to one hundred days with no significant power present at shorter timescales. The individual amplitudes range from .4 to 1.0 km sec^{-1}, but add together in such a way as to produce a total range of variation of 10 km sec^{-1}. It is hypothesized that these terms represent many discrete modes of non-radial pulsation, and that other modes are present, but either undetected or unresolved.

IV. Absorption Line Profiles

The inference of turbulent velocities in early type supergiant atmospheres was first made many years ago on the basis of absorption line strengths and shapes. Equivalent widths of all lines in O star spectra were stronger than LTE model atmospheres predicted, and the profiles were much too broad to be attributed to the thermal and stark broadening. Microturbulent velocities in excess of 20 km sec^{-1} were required to account for the observed equivalent widths, and rotation and/or macroturbulence often in excess of 100 km sec^{-1} was implied by the line shapes. Considerable progress has been made in these areas in the past decade, with the availability of static non-LTE model atmospheres, and several new observational studies. For the most part, the predictions of line strengths by the new models are entirely successful at reproducing the observations (Auer and Mihalas 1972, Conti 1973). Most of the hydrogen, neutral helium, and ionized helium equivalent widths are consistent with zero microturbulence, however some important differences do remain. The neutral helium lines at λ5876 Å and λ6678 Å are still observed to be considerably stronger than the models allow. Conti (1974) and Rosendhal (1973b) suggest that some small classical microturbulence included in the calculations would ameliorate these discrepancies. Most of the lines of He II are adequately accounted for by the non-LTE models. A possible discrepancy was found by Snijders and Underhill (1975) for lines having a lower level of n = 4. In a detailed analysis of the He II spectrum of ζ Pup - O4 f - they found these lines to be systematically stronger than expected on the basis of a model atmosphere which describes the other lines well. They suggest that the n = 4 level of He II is overpopulated by absorbing Hydrogen Lyman Alpha photons in the n = 2-4 transition. In a quiescent medium these transitions are not exactly coincident in wavelength (Auer and Mihalas 1972) but if a velocity field (the authors suggest macroturbulence) of about 50 km sec^{-1} were present, the broadened lines could overlap sufficiently to produce the necessary pumping. The only other lines whose characteristics are seriously in disagreement with the static, plane parallel models are Hα and He II λ4686 Å and He II λ10124 Å, whose formation almost surely occurs in the higher, extended and expanding regions of the atmosphere.

Two studies of the absorption line widths in O stars by Slettebak (1956) and Conti and Ebbets (1977), were intended to measure rotation velocities, but in addition outlined the statistical properties of what is called macroturbulence. Both studies started out with line profiles thought to be free of any macroscopic broadening, either the observed profiles from sharp lined stars, or the theoretical profiles from model atmospheres. These sharp lines were then numerically broadened, using the convolution approximation with a limb darkened rotation function, and the relationship between full width at half maximum vs. V sin i was calibrated. The line widths of about two hundred program stars were then expressed as rotation velocities. Finally, the distribution with spectral type and luminosity class was studied (Figure 3). The conclusions of both studies regarding line broadening were similar, and can be summarized as follows: The O stars show a wide range of apparent rotation velocities, from essentially zero to some in excess of 400 km sec^{-1}. Regardless of how the stars are classified, the mean and modal velocity is near 100 km sec^{-1}. Only the stars of type O9 V have any members which show lines sufficiently sharp to be consistent with zero macroscopic broadening. Stars hotter than type O9, and with luminosity brighter than class V have no members with unbroadened lines. The number of observations is sufficiently large that if the broadening were purely rotation, some sharp lined - i.e. pole on stars - would be expected. This residual broadening increases with increasing temperature and increasing luminosity. Among the earliest types which are well observed - say O6 V - the minimum broadening is equivalent to 50 km sec^{-1} of rotation. Among the Ia supergiants of spectral type O8 and O9, the non-rotational component of line broadening is about 30 km sec^{-1}. These velocities are referred to as macroturbulence. They describe line broadening which cannot be attributed solely to rotation, and refer only to groups of stars in a statistical sense.

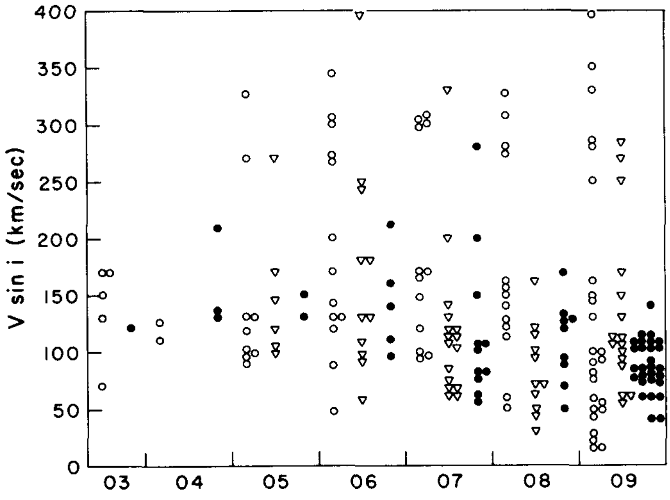

Fig. 3 - Apparent rotational velocities for O stars as catalogued by Conti and Ebbets (1977). Open circles represent main sequence stars, triangles are giants; filled circles indicate supergiants. This figure originally appeared in the Ap. J.

An attempt to separate the effects of rotation and macroturbulence in individual stars was made by this author (Ebbets 1979) using the Fourier Transform analysis described by Smith and Gray (1976). In the wavelength domain the effects of different velocity structures are subtle and difficult to separate, buth given sufficiently precise data, the differences are more pronounced in the Fourier transform domain (Figure 4). Adopting an isotropic Gaussian representation of the macroturbulence, the rotation function of Huang and Struve (1953) and the unbroadened profiles of Auer and Mihalas (1972), I derived rotation and macroturbulence parameters from the He I and He II lines of 16 O supergiants and non-supergiants. The rotation velocities were consistent with the previous values, and macroturbulence ranged from less than the detection threshold of 10 km sec^{-1} in O9 V stars to 30 km sec^{-1} in O9.5 Ia stars. In addition the macroturbulence parameter shows a strong correlation with the Balmer lines' velocity gradient.

Variations in the absorption line profiles are also detected in some supergiants. Perhaps the most dramatic example is that reported by Wolf et al. (1974) in the A2 Ia

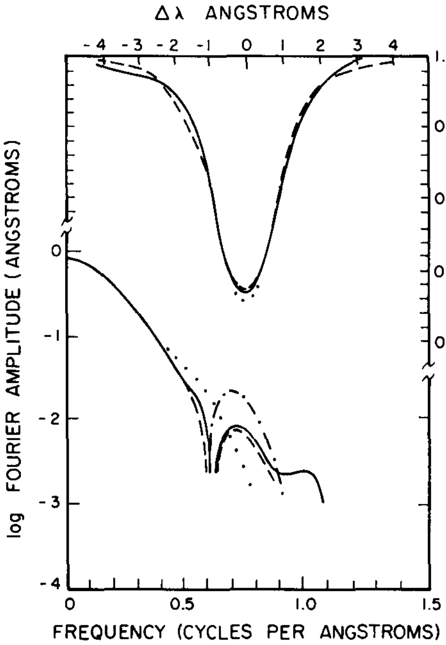

Fig. 4 - A line profile and Fourier transform analysis was performed to measure macroturbulence in O type stars. The line shown is He I λ4471 in 19 Cep. Models with various combinations of rotation and turbulence can reproduce the profile well, but only one set, V sin i = 75 km sec^{-1}, V_{turb} = 20 km sec^{-1}, matches the observation in both domains simultaneously. The dot-dashed transform is from a model with no turbulence but V sin i = 80 km sec^{-1}. The dotted profile and transform model zero rotation, but V_{turb} = 55 km sec^{-1}. From Ebbets (1979).

star HD 160529. In addition to the systematic and variable displacements discussed in the previous section, the metallic lines (Fe II, Si II, etc.) are variable in profile and equivalent width. In extreme instances, the lines actually split into two distinct components sometimes by as much as 40 km sec^{-1}. This splitting is irregular in time, the relative strengths of the components vary, and the effect does not appear to be due to a binary companion. The authors believe that very large sections of the atmosphere moving with quite different velocities are responsible for the phenomenon. If this interpretation is correct, it is a paradigm example of the concept of macroturbulent motions in a stellar atmosphere. Similar, but less dramatic variations are observed in other stars. Smith (private communication 1979) has observed rapid changes (often within one night) in the Si III lines of ρ Leo B1 Ib, and Rosendhal (1972) reports variation of the Si II lines in α Cyg A2 Ia on timescales of days. The interpretation of these observations is unclear at present, but must imply some kind of substantial change in the visible hemisphere on short timescales.

V. Emission Line Profiles

In addition to their absorption lines, the spectra of many O stars, and almost all O, B, and A supergiants, contain emission lines of hydrogen, neutral helium, and ionized helium. It is generally thought (Beals 1951 for example) that the emission lines arise in the atmospheric layers well above the photosphere, in the extended and expanding envelope of the star. The strength, profiles, and variability of emission lines therefore provide information about the state of motion of the gas in layers much higher in the atmosphere than the regions where absorption lines are formed. Not surprisingly, careful observations have revealed that the emission lines are variable in strength and profile on a variety of timescales. The best example is the Hα profile of the B3 Ia star, 55 Cygni, whose changes over about 25 years were sketched by Underhill (1966). At various times the profile was strong and symmetric in absorption, very weakly in absorption, or strongly in emission with a pronounced P-Cygni profile. Since the time coverage was very spotty, a characteristic timescale is difficult to assign to these changes. Rosendhal (1973a) studied the variability of Hα in B and A type supergiants, again showing large changes in spectra taken several years apart. Conti and Niemela (1976) observed a gradual weakening of Hα in ζ Pup over three years, and suggested a global decrease in the envelope density, and therefore the rate of mass loss.

On the more rapid timescales - several days - Conti and Frost (1974), Conti and Leep (1979), Hutchings and Sanyal (1976) documented variations in the Hα and He II λ4686 Å emission lines in the O6 ef star λ Cep. A possible periodicity (~ 3.3 days) consistent with the rotation of an inhomogeneous envelope was suggested. Similar nightly, and often hourly, changes are present in the Hα profile of α Cam, O9.5 Ia (Figure 5, from Ebbets 1980a). Again, the recurrence of similar profiles at multiples of the estimated

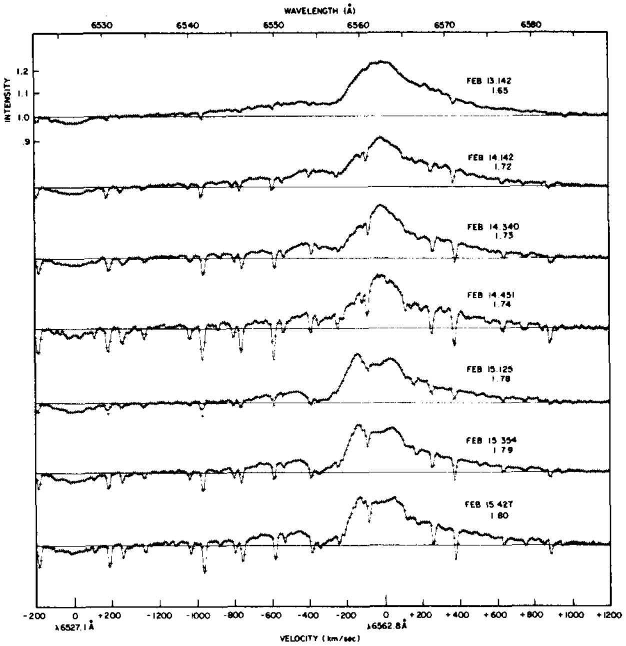

Fig. 5 - The Hα profile of the O9.5 Ia star α Cam shows a Beals Type III P Cygni profile. The width of the line, and the extensive, symmetric wings are due to emission and scattering in the lower regions of the stellar wind. The narrow features are atmospheric water vapor.

rotation period (15 days) is suggestive of a non-axisymmetric envelope. Occasionally a star which ordinarily does not show emission lines in its spectrum experiences a short episode of activity. Such an episode occurred in the O9 V star ζ Oph in 1973. Emission at Hα was first reported by Niemela and Mendez (1974), and its development and dispersal was followed by Barker and Brown (1974). This outburst was presumably related to the extremely rapid rotation (V sin i > 450 km sec^{-1}) but is cited as an example of a transitory large scale velocity field.

A final relevant observation is the presence of extremely broad but faint wings flanking the Hα emission line in most O type supergiants. These features extend to at least ±1500 km sec^{-1} in α Cam and ζ Ori, and to over ±2000 km sec^{-1} in ζ Pup. Their presence was detected in high signal-to-noise coude reticon observations, and are probably due to a combination of emission in the rapidly accelerating lower envelope, plus the effect of non-coherent scattering by free electrons (Ebbets 1980b). Scattering by electrons in the expanding atmosphere may also produce some of the line broadening observed in photospheric absorption lines.

VI. Ultraviolet Observations

The most opaque transitions in early type stars are the ultraviolet resonance lines of the dominant ionization stages of carbon, nitrogen, and oxygen. Observations of these lines therefore probe the highest atmospheric regions accessible to the spectroscopist. By now the message of these lines is well known - the outer atmosphere of all O type stars, and most B and A supergiants are expanding at supersonic velocities (often in excess of 2500 km sec^{-1}) and returning mass to the interstellar medium at rates of several times 10^{-6} solar masses per year. (See Snow and Morton 1976, Conti 1978, Cassinelli 1979 for extensive reviews of stellar winds.) Detailed analyses of ultraviolet spectra, in which the resonance line profiles are modeled and interpreted, have investigated the temperature, density, and velocity distribution in the envelope. All studies to date (for example Lamers and Morton 1976, Olson 1978) have assumed spherically symmetric, laminar steady flow, and have been reasonably successful at reproducing the observed profiles. Turbulence was not explicitly included in these studies, and there are no glaring inconsistencies which cry out for its introduction. Two classes of models postulate a temperature in the wind higher than that which strict radiative equilibrium would allow, and may implicitly require the dissipation of some form of mechanical energy as a heating mechanism. The "warm wind" model and the coronal models are reviewed by Cassinelli, Castor, and Lamers (1978).

Since ultraviolet observations can only be made from spacecraft, time series observations have been difficult and rare. There is, however, a growing literature reporting variability in the ultraviolet spectrum. Snow (1977) reobserved fifteen O and B stars which had shown evidence of massive stellar winds in the first Copernicus survey. Most stars showed only marginal changes over timescales of two to four years. At least two stars, ζ Pup, O4 f, and δ Ori A, O9 I, seemed to have less material in the expanding envelope when reobserved, implying a decrease in the mass loss rate. The profiles in ζ Ori A O9 I suggest a slightly more dense stellar wind and thus an increased mass loss. The only observation searching for variations on timescales of days were reported by Conti and Leep (1979), who found little significant change in the spectrum of λ Cep. Very rapid changes in the fine structure of the ultraviolet absorption lines (timescale ~ hours) were reported by York et al. (1977). The nature and duration of the fluctuations were such that these authors suggested density perturbations - waves - propagating upward as a transient disturbance in the atmosphere. Prolonged ultraviolet observations of several hot supergiants with time resolutions of several hours to a day would clearly be an interesting and worthwhile project.

Summary and Conclusions

Observations of many different kinds demonstrate that the outer layers of early type supergiants are variable. Photometric and radial velocity studies indicate semi-regular fluctuations of low amplitude and timescales of days to weeks. The nature of these variations suggests that pulsation contributes at least somewhat to the phenomenon. Brightness and color fluctuations may reflect changes in the depth dependent velocity field. Variations in the electron pressure in the continuum forming layers may contribute to the erratic behavior of the hydrogen Balmer discontinuity. The combined effect of the velocities of many low amplitude modes of oscillation may help explain the width of absorption lines and their occasional asymmetry and splitting. Such turbulent velocities could also allow the n = 4 level of ionized helium to overlap with hydrogen Lyman alpha enough to slightly overpopulate that level in the deeper layers, accounting for the anamolous strength of the absorption lines of that series.

All of these effects occur in the deeper layers of the stellar atmosphere. What effect they have on the properties of the stellar wind is an unsettled question. Observations have not yet provided a strong case for short term variability in the wind. Do photospheric disturbances merely damp out in the lower layers, do they propagate outwards quiescently, or do they shock and act as a source of heat for a chromosphere corona structure? Purely radiative models (Lucy and Solomon 1970, Castor, Abbot, and Klein 1975) indicate that the mass loss can be driven without a mechanically heated coronal push. At the same time, models which do incorporate a thin corona are more successful at explaining the observed ionization balance, while not interfering with the basic radiatively driven wind (Cassinelli, Olson, and Stalio 1978). On the other hand, the imperfect flow model (Cannon and Thomas 1977) contends that the propagation and dissipation of mechanical energy is of fundamental importance in determining the properties of the expanding atmosphere.

Observations leave no doubt that variations are present in even the deepest visible layers. A good deal of progress has been made in characterizing the fluctuations, and identifying possible physical mechanisms. Further progress demands that we seek and clarify the connection between photospheric "turbulence" and the dynamics and energetics of the expanding supergiant envelope.

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