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Galactic pulsars have been discovered by their radio emission. The emission has the common feature of being periodic, polarized, weakly frequency dependent (in the frequency range $100 \text{ MHz} \lesssim f \lesssim 10 \text{ GHz}$). Their energy is large for radio emission but still negligible as compared with the observed emission in the γ -ray range. They are also characterized by a very high temperature of brightness ($10^{27} - 10^{31}$ Kelvin) which implies a coherent emission mechanism (this temperature is much larger than the possible particle energy). The only radiation mechanism which may produce such emission seems to be some kind of relativistic plasma instability. Unfortunately, available plasma models for pulsar magnetospheres are up to now too preliminary to make any definite statement. Nevertheless we will adopt the most popular scenario which relates the radio emission to the field aligned currents (relativistic electrons, positrons, ions) produced in the vicinity of the neutron star. These currents may result from a spark gap mechanism with pair creation (as proposed by Ruderman and Sutherland (1975) and others) or implied by the star's neutrality (as a return current system).

In this paper we limit our investigation to the stability of beams flowing along curved magnetic field lines, in the vicinity of the star. With the magnetic field in the range of a few 10^{12} Gauss we may forget quantum effects and assume that the particles are constrained to move along the B field. Even in this restricted domain of plasma physics the available literature is rather unsatisfactory. Two lines of research have been followed: one is due to Goldreich and Keeley (1971) who proposed a radiative electromagnetic instability which is produced by one relativistic beam streaming along a curved path. The other one is a two-stream electrostatic instability (i.e. between a positron and an electron beam). In both cases these instabilities have been mainly used as a bunching mechanism which could increase the vacuum curvature radiation. In fact, outside the beam the Goldreich-Keeley instability is fully electromagnetic. It has also been shown recently by Asséo et al. (1980) that a two-stream electrostatic instability can produce a vacuum electromagnetic radiation if the beams have finite transverse

dimensions. In this paper the mathematics was limited to a thick beam for which the finite field line curvature does not modify the dispersion relation.

To make the connection with the Goldreich and Keeley mechanism we have extended our analysis to a system of field aligned currents of arbitrary thickness. To simplify the analysis we take cylindrical magnetic field lines ($\tilde{B} = B \tilde{e}_\theta$ with $B \rightarrow \infty$). The current system is also cylindrical ($\tilde{j} = j \tilde{e}_\theta$) and radially limited: $r_1 < r < r_2$, with $(r_1 - r_2) \ll r_1, r_2$. The different species are streaming relativistically: $V_\theta(r_2) \lesssim c$. The Goldreich-Keeley analysis can be simplified by using standard techniques of plasma physics which allow more realistic investigations. For example their dispersion relation is valid in the limit of a zero beam thickness and with cylindrical symmetry.

Let us introduce the parallel dielectric constant, $\epsilon_{||}$:

$$\epsilon_{||} = 1 - \sum_j \frac{q_j^2}{M_j \epsilon_0} \int \frac{f_j(p_\theta) dp_\theta}{\gamma_j^3 \left(\omega - \frac{m V_\theta j}{r}\right)^2}$$

where j stands for the different possible particle beams and γ_j is the standard relativistic factor and M_j the particle rest mass, m_j is the azimuthal wave number. If $\epsilon \approx$ constant inside the beam(s), the electric and magnetic wave field components are solutions of Bessel type equations: $J_m(\beta r)$ in vacuum and $J_m \epsilon^{1/2}(\beta r)$ inside the beam where $\beta \equiv (\omega^2/c^2 - k_z^2)^{1/2}$, k_z being the wave number along the cylindrical axis. For the expected unstable frequencies $\omega \sim mc/r_2$ and a thin beam ($|r_1 - r_2| \ll r_2$) it is convenient to use the Airy approximation. Two simple limits appear:

$$|m^{2/3} \epsilon_{||}^{1/2} [(r_1 - r_2)/r_2 + (\omega r_2/c - 1)]| \leq 1 \quad \text{or} \quad \geq 1$$

In the first case one recovers the Goldreich-Keeley type instability driven by the radiative term ($r > r_2$), the eventual two-stream instability being negligible. The dispersion relation reads in this case:

$$i(\epsilon_{||} - 1) \approx$$

$$\frac{3^{2/3} \Gamma^2(1/3) m^{4/3} (1 - 3^{1/2} i)}{2^{1/3} \pi r_1 (r_2 - r_1)} \left\{ \frac{\omega^2}{c^2} + k_z^2 \left(1 - \frac{m^2}{\beta^2} \right) \left(\frac{1 + 3^{1/2} i}{2} \right) \left[\frac{m^{1/3} \Gamma(1/3)}{2^{1/3} 3^{1/3} \Gamma(2/3)} \right]^2 \right\}^{-1}$$

In the limit of one cold beam of zero thickness ($|r_2 - r_1| \rightarrow 0$, $|r_2 - r_1| \omega_p^2 \rightarrow$ finite) and vanishing k_z one recovers the Goldreich-Keeley instability. Our dispersion relation provides two new informations: the maximum growth rate is not modified for $k_z r_0 \leq m^{2/3}$ which is a very mild

condition. Its interest for pulsar radio emission is to demonstrate that, in a more realistic geometry, finite transverse dimensions of the currents in the direction $\vec{M} \times \vec{\Omega}$ do not modify the cylindrical result. The second information is the possibility of convectively amplified wave packets along the field lines: the growth is not modified if the wave packet has an extension $\Delta \ell \gtrsim r_0 m^{-1/3}$, which is sufficient to include inhomogeneities along the field lines. The Poynting flux along the outside B field is larger than across the field by a large factor $m^{1/3}$.

In the opposite limit, for a standard two-stream situation and thickness larger than a few meters (with standard parameters) the radiation term is negligible in the dispersion relation and one recovers the results of Asséo et al. (1980). As far as the total radiated power is concerned it is possible to obtain the experimental figures only if the beams are cold enough, energetic ($\gamma \sim 10^3, 10^4$) and the density fluctuations of order 10^{-1} (the beam density being related to the observed frequency by the dispersion relation). The observed frequency spectrum may be related to the variation of the plasma frequency of the dominant beam due to the divergence of the magnetic field lines (Pellat 1979).

REFERENCES

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