SEISMOLOGY OF SUNSPOT ATMOSPHERES*

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Abstract. The present work deals with the theory of oscillations with periods of about 3 min observed in the chromosphere above sunspot umbrae. The model of these oscillations (slow mode magneto-acoustic waves trapped in a chromospheric resonant cavity) provides an independent method of checking empirical models of the chromosphere above sunspots.

Making use of this method, we investigate sunspot models which have been derived from spectroscopic data; the calculated periods of the oscillations fit well the observed periods.

1. Introduction

Empirical models of the thermodynamic structure of sunspot umbrae seem to be well established at photospheric levels. For lack of reliable observations and difficulties in their interpretation uncertainties arise, however, at larger heights starting from the temperature minimum (T_{min}). Recently, the situation improved because EUV sunspot observations with high spatial and spectroscopic resolutions became available, including hydrogen Lα line contours from HRTS (Basri *et al.*, 1979) and OSO-8 (Kneer *et al.*, 1981); and a unified working model of sunspot structure from the subphotosphere (that is, the upper part of the convective zone) up to the base of the transition layer between chromosphere and corona has been derived using these data (Staude, 1981). Such models cannot be defined unambiguously, however, as long as the EUV data are available only for a small number of sunspots and spectral lines.

Oscillations in velocity and brightness observed at photospheric and chromospheric layers of sunspots could provide an independent method of investigating the atmospheric structure. In a recent paper, Žugžda and Locāns (1981) proposed such a type of sunspot seismology assuming a model for a chromospheric resonant cavity which is forced from below by acoustic noise producted in the convective zone.

In the following section we shall summarize observations and different efforts towards its interpretation. The subsequent sections will give a description of our models

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of sunspot structure and chromospheric oscillations, followed by a discussion of the results and conclusions for further work.

2. Observations and Suggestions for the Interpretation

Intensity and velocity oscillations in chromospheric lines of sunspot umbrae are observed with periods P between about 2 and 3 min. Beckers and Tallant (1969) first discovered oscillatory-type phenomena in the Ca II H and K lines and the infrared triplet, which were called 'umbral flashes'. Havnes (1970) suggested that such flashes could be produced by compression waves. Later umbral velocity oscillations were also observed in Hα and other chromospheric lines, e.g. by Giovanelli (1972) and Giovanelli *et al.* (1978).

At photospheric levels of umbrae, oscillations occur in a much broader range of P between 2 and 8 min. Most of these observations concern velocity oscillations, but there are also reports on oscillations of the magnetic field vector (Mogilevsky *et al.*, 1973). A clear correlation with the chromospheric oscillations does not exist (Beckers and Schultz, 1972; Bhatnagar *et al.*, 1972). In the present paper we shall mainly deal with the chromospheric oscillations.

Recently umbral oscillations in velocity and partly also in brightness have been discovered, even in the transition region at a temperature of $T \approx 10^5$ K. These oscillations observed on the SMM spacecraft (Gurman *et al.*, 1982; Tandberg-Hanssen *et al.*, 1981) show periods of $129 \lesssim P \lesssim 173$ s similar to those in the chromosphere; they seem to represent upward propagating compression waves. Clear correlations between the parameters P, oscillation amplitude, umbral area, and the magnetic field $\mathbf{B}$ do not exist.

In theoretical efforts to explain the umbral oscillations, most authors looked for the resonant response of the umbral atmosphere to forcing from below. Different wave modes have already been considered:

Uchida and Sakurai (1975) assumed Alfvén waves already being downward reflected at the layers around $T_{\min}$ by the strong increase of the Alfvén speed v_A with increasing height and decreasing mass density ρ (see Figures 3 and 4). However, there is no effective upward reflection of downward propagating waves from the subphotosphere (Thomas, 1978; Žugžda and Locāns, 1980, 1982), hence, a resonant cavity cannot form.

'Fast' mode magneto-atmospheric waves were considered by Antia and Chitre (1979) as well as by Scheuer and Thomas (1981), and Thomas and Scheuer (1982). Downward reflection of these waves can occur due to the increase of v_A, as in the case of the Alfvén mode, but now also an upward reflection from the subphotosphere is possible due to the increase of the sound speed c_S, and a photospheric resonant cavity will form. This mode could explain the umbral oscillations observed in the photosphere. At higher levels, a pure acoustic wave will propagate along the magnetic field; its energy is much smaller than that of the mainly horizontal oscillations in the photosphere, but the amplitude is large enough to explain observed chromospheric oscillations. However, in this model the oscillations in the chromosphere should show a clear correlation to those in the photosphere which contradicts existing observations. Moreover, of course, the model cannot explain the oscillations observed in the transition region if horizontal and vertical motions are assumed to be zero there.

Žugžda and Locāns (1981) proposed a model for slow-mode magneto-acoustic waves which will be described in more detail in the following section. In the limiting case of a strong magnetic field, hence $v_A^2 \gg c_S^2$, the mode degenerates to a sound wave travelling along the magnetic field. Downward reflection occurs at the transition region where c_S rapidly increases, while a lower boundary exists at T_{min}: a chromospheric resonant cavity is formed due to the reflection of propagating waves at the frequency $\omega_0 = \gamma g/2c_S$, where $\gamma = c_P/c_V$ is the ratio of specific heats and g is the gravity. For frequencies $\omega < \omega_0$ the transmission of the layers around T_{min} is strongly reduced, while for $\omega > \omega_0$ we have only little reflection and therefore a lot of leakage from the cavity above T_{min}.

Hollweg and Roberts (1981) studied the resonance of 'tube waves' in a strong magnetic field decreasing with height. Till now only a rough model for the atmosphere above the sunspot was assumed; the consideration of a sufficiently shallow sunspot chromosphere would hardly result in a strong decrease of the magnetic field with height. The model is likely to fit magnetic tubes at the supergranular boundaries rather than sunspots.

Most of the hitherto existing models used very simple and schematic assumptions concerning the atmospheric structure in the umbra, e.g., a constant $\beta = -\partial T/\partial z$ (z is the vertical coordinate increasing upward, i.e., in the opposite direction to the gravity $\mathbf{g}$) or an isothermal layer limited by a subphotosphere with constant gradient β and an infinite gradient at the transition region were assumed. However, a detailed comparison with observations requires the use of a more realistic model of the umbral atmosphere which will be described in the following section.

3. Physical Assumptions and Methods of the Analysis

3.1. Models for the Atmospheric Structure of Umbrae

Details of our methods to derive a sunspot working model were published in a recent paper (Staude, 1981), therefore only a short summary will be given here. From the literature we depended as much as possible upon observed spectroscopic data and as little as necessary upon model parameters. The set of model parameters was completed by our own procedures to obtain a rather self-consistent model with uniform assumptions concerning the chemical composition, the methods to solve the equations of state, of hydrostatic equilibrium, of deviations from LTE, to calculate the absorption coefficients, the conversion of different height scales, etc.

The procedures are partly based on the LINEAR code by Auer *et al.* (1972), hence the states H, H^-, H^+, H_2, H_2^+ are considered for hydrogen, moreover, He and nine other elements are taken into account with two states of ionization, and the ground state of H is permitted to depart from LTE. All thermodynamic quantities for the analysis in the following subsection were calculated using the procedures mentioned above; this concerns, e.g., the effective molecular weight μ, γ, ρ, c_S, and v_A.

The resulting photospheric part of our model 1 is similar to that of Stellmacher and Wiehr (1975), the lower chromosphere for $T < 11\,000$ K is similar to the model of Teplitskaya *et al.* (1978), while the further extrapolation up to $T = 40\,000$ K was carried out to explain EUV observations such as the Lα data mentioned in Section 1. In addition to this sunspot model 1 (Staude, 1981) two sunspot models with steeper gradients β, and therefore larger densities in the chromosphere, were calculated for comparison: the extent of the chromosphere between $T_{\text{min}} = 3000$ K and $T = 22\,000$ K is 1400 km for model 1, but 1050 and 700 km for models 2 and 3, respectively. At a temperature of $T = 20\,000$ K where the Lα line core is formed, the related electron densities n_e are 4.9×10^{10}, 1.9×10^{11}, and 7.8×10^{11} cm^{-3} for spots 1, 2, and 3, respectively. For comparison, models of the mean undisturbed solar atmosphere and a plage were derived from the data of Basri *et al.* (1979); at 20 000 K the corresponding n_e values are 2.2×10^{10} and 4.0×10^{11} cm^{-3}. These values should be compared with observed data, e.g., with the contrast φ, that is the intensity ratio in the Lα line core between spot and undisturbed Sun: Kneer *et al.* (1981) measured $\varphi \approx 3$, while Basri *et al.* (1979) obtained $1.7 \leq \varphi \leq 6.9$ and an average value of 2.5. Following Kneer *et al.* (1981) we roughly estimate φ from the n_e ratios of the models assuming the limiting cases of optically thick and thin radiations: φ values between 1.5 and 5, 3 and 80, and 6 and 1300 were obtained for spots 1, 2, and 3, respectively. Hence, all three models are compatible with the observed data in the physically more probable case of optically thick radiation. Considering further spectroscopic data, however, spot model 1 as derived by Staude (1981) has evidently the most realistic structure while spot 3 seems unlikely. Figures 1 to 3 show the z-dependence of T, ρ, and n_e for the spot models 1 and 2 as well as for the quiet Sun and a plage for comparison. Figure 4 gives c_S and v_A for spots 1 and 2 , and Figure 6 includes $T(z)$ for the chromospheres of all three spot models.

3.2. Model for the oscillations

The model of a chromospheric resonant cavity for slow mode magneto-acoustic waves was proposed by Žugžda and Locāns (1981) and outlined already in Section 2 above. The theory is based on an analytic solution of the equations of ideal magnetohydrodynamics derived by Syrovatsky and Žugžda (1967) with the aim of studying oscillatory convection in a sunspot. Additional assumptions are: a compressible stratified medium with constant values of β, γ, $\mathbf{B}$, and $\mathbf{g}$, parallel directions of magnetic field and gravity, and a strong magnetic field satisfying $c_S^2 \ll v_A^2$. Linearizing the equations about the basic state gives the following solution for the vertical velocity:

$$v_z = \left(\frac{\rho}{\rho_0}\right)^{-1/2} Z_{\pm n}(\xi) \exp(-i\omega t + ik_\perp r_\perp),$$

$$\xi = \frac{2\omega}{|\beta|} \left(\frac{\mu T}{\gamma R}\right)^{1/2}, \qquad n = \frac{\mu g}{\beta R} - 1, \qquad \omega = 2\pi/P, \tag{1}$$

where $Z_{\pm n}$ are the general solutions of Bessel's equation, $k_\perp$ and $r_\perp$ are the horizontal wave and radius vectors, respectively, and R is the gas constant.

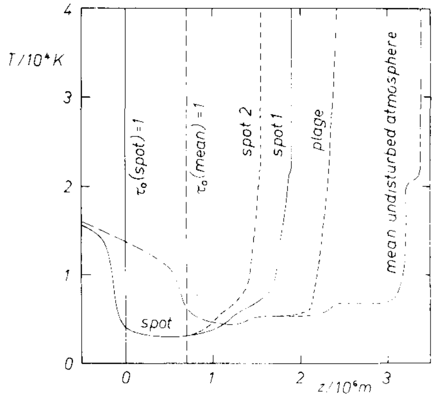

Fig. 1.

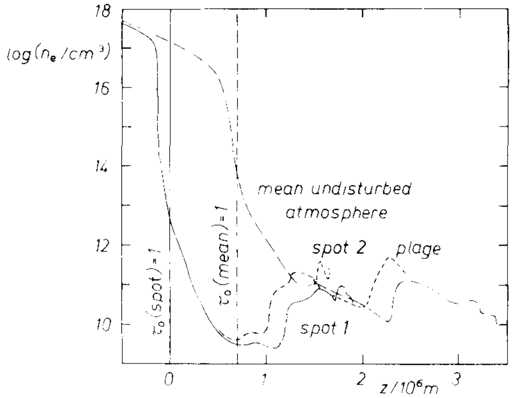

Fig. 2.

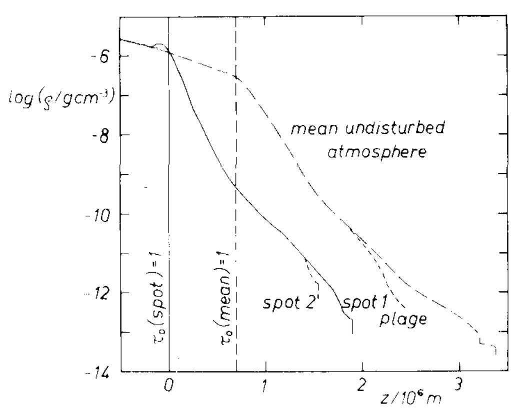

Fig. 3.

Fig. 1, 2, and 3. Temperature T, mass density ρ, and electron density n_e versus the geometrical height z. The sunspot models 1 and 2 as well as models for the mean undisturbed Sun and a plage are plotted for comparison. $z = 0$ corresponds to an optical depth of $\tau_0 = 1$ for $\lambda_0 = 5000$ Å in the sunspots.

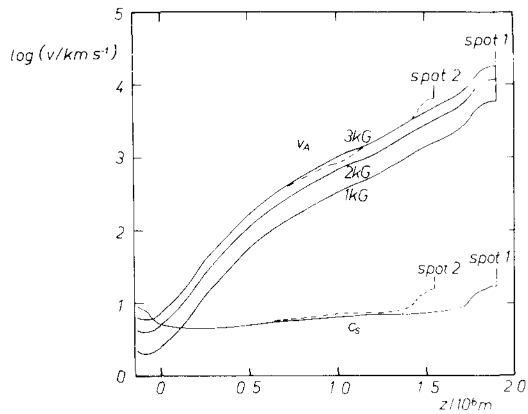

Fig. 4. Alfvén speed v_A and sound speed c_S versus z for the sunspot models 1 and 2, v_A for three different values of B. T_{min} corresponds to a height of z = 500 km.

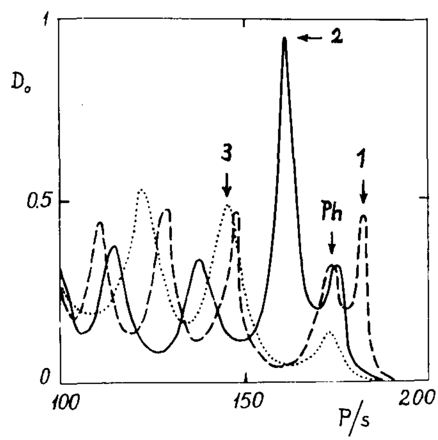

Fig. 5. Transmission D_0 versus period P for waves travelling vertically from the lower photosphere to the upper chromosphere of the three sunspot models. Arrows with numbers indicate the fundamental resonant peaks of the chromospheric cavity in the three models, while Ph marks the photospheric resonance.

The solution (1) does not depend on $k_\perp$. The existence of such a solution is evident from the solution for a strong field in an isothermal atmosphere (Žugžda and Džalilov, 1981); the peculiarity is due to the behaviour of a plasma in a strong magnetic field where motions along the field cannot produce significant transverse displacements ($v_\perp \ll v_z$). Figure 4 shows that the strong field approximation $c_S^2 \ll v_A^2$ is well satisfied in the sunspot chromosphere and partly even in the photosphere. We assume the reflection of waves from the lateral boundaries of the resonator resulting in a standing wave in a horizontal direction. Therefore, we consider only vertically-propagating waves.

In order to investigate the effectiveness of the resonant cavity, the transmission D_0

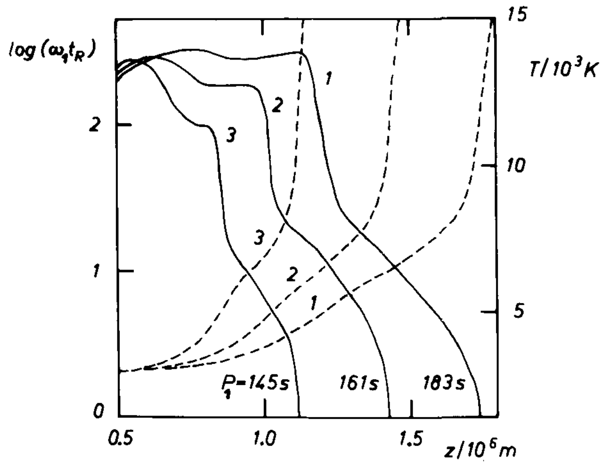

Fig. 6. Height dependence of the radiative decay time t_R of waves normalized to the period $P_1 = 2\pi/\omega_1$ of the fundamental resonant mode of the corresponding chromospheres for the three sunspot models (full lines). For comparison the $T(z)$ functions are also plotted (dashed lines).

for waves propagating through photosphere and chromosphere is calculated. The procedure is similar to that of Uchida (1967), and the algorithm agrees with that which has been used by Žugžda and Locāns (1980, 1982) in a study of Alfvén wave propagation. Such a model in terms of resonant transmission, that is, a filtering of a broad band, incident flux, differs from the approach of some other authors who assumed fixed boundary conditions and looked for the eigenvalues of the problem. Our model atmospheres with given $T(z)$ are subdivided into some 10 layers each of which can be represented by a constant β; the solutions of the equations for neighbouring layers are connected to each other by assuming boundary conditions which guarantee the conservation of mass and momentum.

The eigenvalue approach is successful for complete wave reflection from the lower and upper boundaries of a resonant cavity. For instance, the quiet Sun, photospheric oscillations (300 s) become non-propagating waves above and below the resonance layer in the convective zone; these are trapped waves. For incomplete reflection from the boundaries of such a layer we get complex eigenvalues due to the emission of waves from the resonator; the imaginary part (determining the Q-(quality) factor of the resonator) describes the damping of waves in the resonator due to the leakage of energy across the boundaries. These waves could be called 'semi-trapped waves'. If the characteristic time of wave damping is smaller than the oscillatory period, then we have no real resonance. We get accurate results by means of the eigenvalue approach if there are sharp, strongly reflecting boundaries of the considered layer. For gradual boundaries with a weak reflection we give preference to our method for studying the transfer of waves through the considered atmospheric layer. Here the source of the waves is assumed to be placed far enough from the considered region; no artificial layer of wave reflection is introduced at the starting level. The maximum of the coefficient of wave energy transmission determines the real part of the eigenvalue, while its imaginary part or the quality of the resonance is determined by the halfwidth of the resonant peak. In this way, the calculation of the transmission renders accessible the study of both the resonant filtering of oscillations and the response of the resonator to forcing from oscillatory sources inside the resonant layer.

4. Discussion of the Results

Figure 5 shows the dependence of the transition D_0 on the period P for waves travelling through the photosphere and chromosphere of our three sunspot models. The fundamental resonant modes for models 1, 2, and 3 (indicated by arrows) have values of P_1 = 183, 161, and 145 s, respectively, which are within the range of the observed periods of chromospheric oscillations. The periods of the resonant oscillations decrease with decreasing extent of the chromosphere; this applies also to the other resonant peaks at higher frequencies (harmonics) which are also sometimes detected. But larger values of P are more frequently observed which with the spectroscopic data supports model 1 and perhaps 2 more than 3. Unfortunately no simultaneous observations of EUV line profiles and oscillations in the chromosphere of sunspots are available, therefore our discussion must be limited to such vague statements.

The present model is also compatible with the observed oscillations at $T \approx 10^5$ K because the free boundary conditions in our treatment permit an oscillation of the thin transition layer as a whole. Sometimes several periods are observed simultaneously in one spot which could be explained by the existence of harmonics of low order (without integer multiples of P_1, due to the inhomogeneity of the resonator) in our model, but a detailed comparison with observations requires a more exact consideration of additional effects such as those discussed below.

Another feature (marked by Ph) in Figure 5 is still worth mentioning: A maximum of $P \approx 173$ s for all three sunspot models is clearly due to a photospheric resonant cavity, because the photospheric part is the same in our three models. Note that this period is similar to those in the models of a photospheric resonant cavity for the fast mode (see Section 2) and it could explain the observed photospheric oscillations as well.

Dissipative processes have been neglected in the present analysis. In order to get some information on the possible effect of radiative relaxation we calculated the radiative decay time t_R for continuum radiation in an optically thin medium,

$$t_R = \frac{\rho c_V}{16\, \sigma \varkappa_0 T^3} ,$$

where σ is the Stefan–Boltzmann constant and $\varkappa_0$ is the absorption coefficient per unit length at a wavelength of λ_0 = 5000 Å. Figure 6 shows the resulting values of t_R related to the fundamental resonant periods for our three chromospheric sunspot models. It is clearly demonstrated that $\omega_1 t_R \gg 1$ for the greatest part of the chromospheres, therefore the waves correspond to almost adiabatic perturbations and radiative damping can be neglected in this rough estimate. For the upper chromosphere and transition region, however, a more exact treatment is required, and line radiative losses should be considered also in the lower chromosphere as it has been done by Giovanelli (1978, 1979).

5. Conclusions

Our model of a chromospheric resonant cavity for slow-mode magneto-acoustic waves is compatible with the properties of observed oscillations in sunspot chromospheres and even in the transition layer. This refers to the periods, but also to the observed independence of the period on the magnetic field strength. The oscillations are clearly determined by the atmospheric structure, that is to say, mainly by the temperature gradient or the extent of the chromosphere, thereby providing a method to test sunspot model atmospheres. The proposed sunspot models, especially model 1, result in resonant oscillations the periods of which agree with those from observations. It would be highly desirable to get simultaneous observations of oscillations and of EUV line profiles, e.g., of Lα, from the same sunspots in order to check the proposed models by more reliable data.

At photospheric levels, our model provides a resonant cavity as well, producing periods which are similar to those of a model for the fast mode (Scheuer and Thomas, 1981). This result points out the necessity to consider the linear interaction of waves in a strong magnetic field in order to investigate the photospheric resonator (Žugžda and Džalilov, 1981).

The photosphere is likely to build up a mixed resonance from two oscillatory modes because the linear interaction of the slow and fast modes is very strong there. In our opinion, a better diagnosis of sunspot atmospheres by means of oscillations could be achieved by considering the full system of wave equations, as has been done by Thomas and Scheuer (1982), but by using the method of Žugžda and Locāns (1982). Thomas and Scheuer (1982) replaced the reflection from a flat gradient of temperature by reflection form a sharp gradient; this procedure could result in an inaccurate determination of the resonant frequencies if there would be only a small real reflection of waves from the lower photosphere and convective zone (see Section 3.2).

A more exact analysis should consider the effects of wave absorption which could strongly influence the quality of atmospheric resonances; for Alfvén waves such effects have been demonstrated by Žugžda and Locāns (1982).

The treatment of radiative relaxation has to take into account the influence of lines including non-LTE effects in an optically thick medium. In the transition layer the validity of ionization equilibrium could perhaps be called in question. In order to improve the diagnostic means the heights of formation of the observed lines should be calculated, and the influence of the perturbations introduced by the waves in the thermodynamic quantities and thereby in the line profiles should be considered. Some of these improvements to our model are being prepared.

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