

# Is the Vorticity Vector of the Galaxy Perpendicular to the Galactic Plane?

*Precessional Correction and Equinoctial Motion Correction to the FK5 System and Kinematics of the Galactic Warp*

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**ABSTRACT.** The modern astrometric catalogue ACRS invites attempts to re-examine the systematic stellar motions, together with the luni-solar precessional correction and the fictitious equinoctial motion correction to the FK5 system, and gives encouraging results.

On the basis of the three-dimensional Ogorodnikov-Milne model for the overall pattern analysis of the proper motions, the systematic stellar velocity field of about 30000 K–M giants chosen from the ACRS is first examined in the heliocentric distance interval 0.5 to 1.0 kpc. We have found in an iterative fashion a solution for the K–M giants that yields neither deformation- nor vorticity-field other than the classical ones (the Oort constants  $A$  and  $B$ ). The important point to note here is that the generally accepted idea such that the K–M giants are a steady-state constituent of the galaxy is compatible with the luni-solar precessional correction proposed by the VLBI and LLR observations. The K–M giants give the rational set of corrections to the FK5 system: the luni-solar precessional correction  $\Delta p = -0''.27 \pm 0''.03$  / cent and the equinoctial motion correction including the planetary precessional correction  $\Delta e + \Delta \lambda = -0''.12 \pm 0''.03$  / cent. Thus, the precessional correction previously proposed with the modern techniques has been confirmed by the pattern analysis of the proper motions.

Next, applying the corrections obtained above, we have performed the overall pattern analysis of the proper motions of about 3000 O–B5 stars, supergiants, and bright giants, which are chosen again from the ACRS, and considered as an entity of the galactic warp. It is found that the kinematics of these stars is quite different from that of K–M giants. These stars show additional shears and rotations around two mutually orthogonal axes lying in the galactic plane, besides the classical ones. The present finding implies that the young stars are streaming around the galactic center in a tilted sheet (the warp) with the velocity of 225 km/s, and the sheet itself is simultaneously rotating around the nodal line of the warp (galactic center – sun – anticenter line) with the angular velocity of 4 km/s/kpc in increasing sense of the present inclination of the warp.

## 1. Introduction

In this review talk, I would like to report on the proper motion analysis, which has been recently carried out together with M. Sôma and M. Yoshizawa:

There has been a persistent demand in astronomy for accurate stellar positions and proper motions, which are represented by an inertial reference system constructed on the basis of a set of consistent astronomical constants. In the reference system the precessional constant plays a primary role. In a series of papers Fricke (1967a,b, 1977a,b) has determined the luni-solar precessional correction to Newcomb's value and the fictitious motion of the equinox, which have been adopted in the "IAU (1976) System of Astronomical Constants". Based on the precessional correction and the equinoctial motion thus established, the fundamental reference system, the FK5 system (Fricke *et al.* 1988) for positions and proper motions, has been constructed.

However, for several years geodetic VLBI (McCarthy & Luzum 1991) and LLR (Williams *et al.* 1991) observations have been suggesting an additional correction to the luni-solar precessional constant of the IAU (1976) System. That is, these observations indicate the precessional correction of  $\Delta p \approx -0''.30$  / cent to the FK5 system. But, the observational period of the earth orientation is considered to be still insufficient to separate unambiguously the precessional change of the earth orientation from the nutation with the longest period of 18.6 years.

In the mean time, there have appeared two large astrometric catalogues, both of which contain about 320000 stars and provide an accurate net of astrometric reference stars in the FK5 system at higher stellar densities and fainter magnitudes. One of the catalogues is the PPM (Catalogue of Positions and Proper Motions) compiled at the Astronomisches Rechen-Institut, Heidelberg (Röser & Bastian 1989, Bastian & Röser 1990), while another is the ACRS (Astrographic Catalogue Reference Stars) compiled at the U. S. Naval Observatory (Corbin & Urban 1991). These catalogues contain a sufficient number of stars to yield meaningful statistics of the proper motions. Moreover, our cross-identification of these catalogues with the SKYMAP (Warren & Kang 1987) shows that they cover a large variety of stars in the MK classification.

Thus, the PPM and the ACRS invite attempts to re-examine the systematic stellar motions, together with the luni-solar precessional correction and the fictitious equinoctial motion. The pattern analysis *all over* the sky of the proper motions is indispensable for determining the rotation parameters of the proper motion system as well as the galactic rotation parameters. Therefore, the homogeneous accuracy *all over* the sky of the proper motions is of primary importance for avoiding an unfavourable biased result. In this context, the final version of the PPM-South is not yet complete and the accuracy of the proper motions provided by the PPM-South is lower than that given by the PPM-North. For this reason, the present analyses rely exclusively upon the proper motions given by the ACRS Part 1.

So long as the overall pattern analysis of the proper motions cannot determine the three-dimensional systematic stellar motion at the solar neighbourhood and the rotation left in the reference system all together, it is rational to define the luni-solar precessional motion and the fictitious equinoctial motion with reference to a group of representative stars, which exhibit the steady state of the galaxy. Thus, applying the two-dimensional Oort-Lindblad model, Fricke assumed implicitly that the vorticity vector of the group of his selected stars (512 FK4 and FK4 Sup stars) had only a component (the Oort constant  $B$ ) perpendicular to the galactic plane. But, the selected stars were an inhomogeneous

mixture of stars with a wide range of spectral types and ages, and moreover embedded in the domain of the Gould-belt. Therefore, if the assumption of the perpendicularity of the vorticity vector does not hold true in the group of the selected stars, the precessional correction and the equinoctial motion thus derived from the Oort-Lindblad model might be affected by vorticity components lying in the galactic plane.

In the present work, taking the luni-solar precessional correction  $\Delta p \approx -0''.30$ /cent suggested, independently of the stellar kinematics, by the VLBI and LLR observations as an initial guide, we shall show first, in the framework of the three-dimensional Ogorodnikov-Milne model that the group of K-M giants exhibits indeed a steady-state constituent of the galaxy. Then, applying the two-dimensional Oort-Lindblad model to these stars thus confirmed as the steady-state constituent, we determine the precessional correction and the fictitious equinoctial motion correction to the FK5 system. The present determination of these corrections will make now possible to reveal a delicate systematic motion of the young stars inherent in the galactic warp.

We shall use 29718 K-M giants and 3021 O-B5 stars, supergiants, and bright giants chosen from the ACRS Part 1 for determining the precession and equinox motion corrections, and for detecting the warping motion, respectively. In order to free the solution from effects due to localized stellar velocity field (star streams, Gould-belt, etc.), the lower limit of the heliocentric distance is set to 0.5 kpc. The upper limits in the former and the latter cases are set to 1 kpc and 3 kpc, respectively. A full detail of these investigations are given by Miyamoto and Sôma (1993) and Miyamoto *et al.* (1993), which are hereafter referred to as Paper I and Paper II, respectively.

## 2. Analysis Model of the Systematic Stellar Velocity Field and Equations of Condition

In Fig. 1, the heliocentric rectangular coordinate system  $(x_1, x_2, x_3)$  and the galactocentric cylindrical coordinate system  $(R, \theta, z)$  are explained. The former system is defined by the right-handed triad of the mutually orthogonal unit vectors  $(\mathbf{i}, \mathbf{j}, \mathbf{k})$ , which point to the galactic center ( $l = 0^\circ, b = 0^\circ$ ), the direction of the galactic rotation ( $l = 90^\circ, b = 0^\circ$ ), and the north galactic pole ( $b = 90^\circ$ ) from the sun, respectively. In the latter coordinate system, the  $z$ -axis points to the north galactic pole from the galactic center, and the azimuthal angle  $\theta$  is reckoned from the  $x_1$ -axis counterclockwise around the  $z$ -axis. The galactocentric distance of the sun is given by  $R_0 = 8.5$  kpc (IAU (1985)).

Now the systematic stellar velocity field in the solar neighbourhood to the first order in  $\mathbf{r}$  can be expanded as

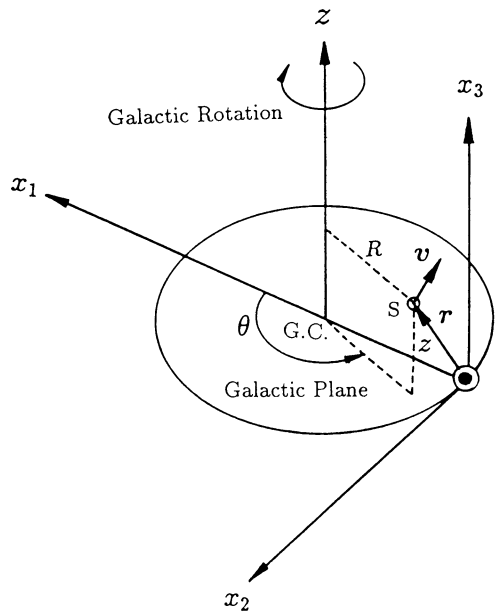


FIGURE 1. The heliocentric rectangular coordinate system  $(x_1, x_2, x_3)$  and the galactocentric cylindrical coordinate system  $(R, \theta, z)$ .

$$\mathbf{v} = \mathbf{S} + \nabla Q + \boldsymbol{\omega} \times \mathbf{r}, \tag{1}$$

where  $\mathbf{S}(S_1, S_2, S_3)$  is the mean flow at the sun of a group of stars considered, which is caused by the solar motion ( $-\mathbf{S}$ ) with respect to the centroid defined by the stars, and

$$Q = \frac{1}{2} \mathbf{r}^T D^+ \mathbf{r} = \frac{1}{2} \sum_{i,j} D_{ij}^+ x_i x_j \tag{2}$$

and

$$\boldsymbol{\omega} = D_{32}^- \mathbf{i} + D_{13}^- \mathbf{j} + D_{21}^- \mathbf{k} = \frac{1}{2} \text{rot } \mathbf{v} \tag{3}$$

with  $D_{ij}^+ = \frac{1}{2} \left( \frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right)$  and  $D_{ij}^- = \frac{1}{2} \left( \frac{\partial v_i}{\partial x_j} - \frac{\partial v_j}{\partial x_i} \right)$  ( $i, j = 1, 2, 3$ ). (4)

The quadratic  $Q$  characterizes the deformation (shear + dilatation) of the group of stars considered, and the vector  $\boldsymbol{\omega}$  describes the vorticity of the velocity field (Ogorodnikov 1932, Milne 1935). In the cylindrical coordinate system, the matrix elements given by Eq. (4) are expressed as

$$\left. \begin{aligned} D_{12}^+ &= \frac{1}{2} \left( \frac{\partial V_\theta}{\partial R} - \frac{V_\theta}{R} + \frac{1}{R} \frac{\partial V_R}{\partial \theta} \right) \\ D_{21}^- &= \frac{1}{2} \left( \frac{\partial V_\theta}{\partial R} + \frac{V_\theta}{R} - \frac{1}{R} \frac{\partial V_R}{\partial \theta} \right) \end{aligned} \right\}, \tag{5}$$

$$\left. \begin{aligned} D_{13}^+ &= -\frac{1}{2} \left( \frac{\partial V_R}{\partial z} + \frac{\partial V_z}{\partial R} \right) \\ D_{13}^- &= -\frac{1}{2} \left( \frac{\partial V_R}{\partial z} - \frac{\partial V_z}{\partial R} \right) \end{aligned} \right\}, \tag{6}$$

$$\left. \begin{aligned} D_{32}^+ &= -\frac{1}{2} \left( \frac{1}{R} \frac{\partial V_z}{\partial \theta} + \frac{\partial V_\theta}{\partial z} \right) \\ D_{32}^- &= -\frac{1}{2} \left( \frac{1}{R} \frac{\partial V_z}{\partial \theta} - \frac{\partial V_\theta}{\partial z} \right) \end{aligned} \right\}, \tag{7}$$

and

$$\left. \begin{aligned} D_{11}^+ &= \frac{\partial V_R}{\partial R} \\ D_{22}^+ &= \frac{V_R}{R} + \frac{1}{R} \frac{\partial V_R}{\partial \theta} \\ D_{33}^+ &= \frac{\partial V_z}{\partial z} \end{aligned} \right\}, \tag{8}$$

where  $V_R$ ,  $V_\theta$ , and  $V_z$  are the velocity components in the cylindrical coordinate system. It is noticed here that the elements  $D_{12}^+$  and  $D_{21}^-$  are identical with the familiar Oort constants  $A$  and  $B$ , respectively, in the classical stellar kinematics under the proviso that the stellar velocity field is axisymmetric or  $V_R = 0$ .

In what follows, we concentrate exclusively on evaluating the elements  $D_{ij}^+$  and  $D_{ij}^-$  with  $i \neq j$ . This is because, first, the present main subject is to examine a coupling of the rotation parameters adopted in the FK5 system with the galactic vorticity in the solar neighbourhood and, second, the elements  $D_{ii}^+$  ( $i = 1, 2, 3$ ) characterizing a uniform



### 3. Solutions Derived from K–M Giants

The main constitution of the galaxy can be considered to be in a steady state with some symmetries, so long as some drastic dynamics of the galaxy is not introduced. Thus, the old and well-relaxed populations of stars such as K–M giants are supposed to have reached already a steady state. Such a state of a stellar system means that the system exhibits only the familiar plane-parallel galactic rotation derived from the Oort constants  $A$  and  $B$ . That is, in the case of the well-relaxed system, the least squares for Eqs. (9) and (11) should yield non-zero solutions only for  $D_{12}^+$  ( $= A$ ) and  $D_{21}^-$  ( $= B$ ), and other matrix elements should be nearly zero. Therefore, the two-dimensional Oort-Lindblad model suffices for describing the issue, only if a group of stars chosen is guaranteed to be a constituent of the well-relaxed system. This notion does not hold true, however, in the case of young stars such as O–B stars, supergiants, etc., since these stars are considered to be not yet well-relaxed, and may show other systematic motions than the familiar galactic rotation.

In fact, there is the possibility that the previous solutions derived from the Oort-Lindblad model have defined a *fictitious* steady-state for the sampled stars, since it seems that the above apparently natural proposition on the steady state for the sampled stars has not been clearly proved from the kinematical point of view. This is because the amount of ambiguity in the precessional and equinoctial motions adopted in the reference system was still too large to disentangle the coupling between the galactic vorticity and the rotation left in the reference system. Recently, the situation has been changed from the past: First, the geodetic VLBI and LLR observations suggest, independently of the stellar kinematics, the luni-solar precessional correction  $\Delta p \approx -0''.30$ /cent to the FK5 system. Second, we have a modernized proper motion catalogue of stars, the ACRS, which is an accurate representation of the FK5 system at higher star densities and fainter magnitudes. With this large amount of the accurate proper motions, the reliability of the statistical analysis is expected to be remarkably increased.

Thus, as an initial trial we adopt  $\Delta p = -0''.30$ /cent for the precessional correction to the FK5 system. Even if  $\Delta p$  is fixed, however, there still remain more than three unknowns in Eq. (11). This case is, of course, not tractable. Therefore, we carry out trial solutions for two cases, in which either one of the unknowns  $D_{32}^-$  and  $D_{13}^-$  is put equal to zero separately in Eq. (11), and concordantly either one of the symmetric counterparts  $D_{23}^+$  and  $D_{13}^+$  is also put equal to zero in Eq. (10). The results are presented in Table 1 for the two height ranges  $|z| \leq 0.5$  kpc and  $|z| < 1.0$  kpc of the K–M giants. In each row of the table, the first line gives the solutions and the second line the standard error of the solutions.

It is noticed that the four solutions in Table 1 show a remarkable concordance, even though we impose two different modelings of the systematic stellar motion, and that only a little discordance is found between the solutions for  $|z| \leq 0.5$  kpc and  $|z| < 1.0$  kpc. The important point to note here is that in each case the two unknowns are determined to be zero within the standard error. This finding demonstrates that each separate solution for the K–M giants all together constitute fortunately the self-consistent resultant  $D_{13}^+ = D_{13}^- = D_{23}^+ = D_{32}^- = 0$  in the present least squares. This is the reason why the four solutions in Table 1 show the remarkable concordance. Of course, this argument does not hold true in the case that the unknowns to be determined gave non-zero values. Thus, present results indicate that the generally accepted idea that the K–M giants are a steady-state constituent of the galaxy is compatible with the luni-solar precessional correction  $\Delta p \approx -0''.30$ /cent proposed by the VLBI and LLR observations (McCarthy & Luzum 1991; Williams *et al.* 1991). In a similar fashion, we have tried to apply the Ogorodnikov-Milne model to the

TABLE 1. Results derived from the proper motions of K–M giants ( $0.5 \text{ kpc} \leq r < 1.0 \text{ kpc}$ ) in ACRS Part 1. The precessional correction  $\Delta p = -0''.30 / \text{cent}$  to the FK5 system is adopted.

Unknown	K1 $ z  \leq 0.5 \text{ kpc}$	K2 $ z  \leq 0.5 \text{ kpc}$	K3 $ z  < 1.0 \text{ kpc}$	K4 $ z  < 1.0 \text{ kpc}$
$S_1$	+13.6 km/s 0.3	+13.6 0.3	+13.8 0.3	+13.8 0.3
$S_2$	+23.3 km/s 0.3	+23.3 0.3	+24.5 0.3	+24.5 0.3
$S_3$	+11.9 km/s 0.3	+11.9 0.3	+11.7 0.3	+11.7 0.3
$S_{\text{total}}$	29.5 km/s	29.5	30.4	30.5
$D_{12}^+ (A)$	+0''.265 / cent 0.012	+0.263 0.012	+0.253 0.011	+0.253 0.011
$D_{21}^- (B)$	-0''.178 / cent 0.011	-0.163 0.013	-0.184 0.010	-0.169 0.012
$D_{13}^+$	0'' / cent	+0.001 0.016	0	+0.032 0.013
$D_{13}^-$	0'' / cent	+0.023 0.020	0	+0.036 0.017
$D_{23}^+$	+0''.012 / cent 0.015	0	-0.002 0.012	0
$D_{32}^-$	+0''.014 / cent 0.013	0	+0.028 0.011	0
$\Delta e + \Delta \lambda$	-0''.156 / cent 0.015	-0.123 0.022	-0.168 0.014	-0.126 0.019
$V_\theta$	-178.3 km/s 5.0	-171.6 7.0	-176.2 6.1	-170.2 6.5
Total Number	24220	24220	29718	29718
Rejected Number	3928	3927	5529	5534

TABLE 2.

Total Number	24220
Rejected Number	3928
$S_1$	+13.6 km/s 0.3
$S_2$	+23.3 km/s 0.3
$S_3$	+11.9 km/s 0.3
$S_{\text{total}}$	29.5 km/s
$D_{12}^+ (A)$	+0."263 / cent 0.012
$D_{21}^- (B)$	-0."176 / cent 0.010
$\Delta p$	-0."267 / cent 0.028
$\Delta e + \Delta \lambda$	-0."116 / cent 0.026
$V_\theta$	-177.1 km/s 6.2

TABLE 3.

Total Number	1892
Rejected Number	524
$S_1$	+ 8.7 km/s 0.8
$S_2$	+15.9 km/s 0.8
$S_3$	+ 9.1 km/s 0.7
$S_{\text{total}}$	20.3 km/s
$D_{12}^+ (A)$	+0."285 / cent 0.019
$D_{21}^- (B)$	-0."260 / cent 0.015
$D_{13}^+$	-0."059 / cent 0.011
$D_{13}^-$	+0."059 / cent 0.011
$D_{23}^+$	+0."039 / cent 0.010
$D_{32}^-$	+0."039 / cent 0.010
$V_\theta$	-219.9 km/s 9.8

TABLE 2. Standard solution derived from the proper motions of K–M giants ( $0.5 \text{ kpc} \leq r < 1.0 \text{ kpc}$  and  $|z| \leq 0.5 \text{ kpc}$ ) in ACRS Part 1, under the imposed conditions  $D_{13}^+ = D_{13}^- = D_{23}^+ = D_{32}^- = 0$ .

TABLE 3. Results derived from the proper motions of O–B5 stars, supergiants, and bright giants ( $0.5 \text{ kpc} \leq r < 3.0 \text{ kpc}$  and  $|z| \leq 0.25 \text{ kpc}$ ) in ACRS Part 1, under the imposed conditions  $D_{13}^+ = -D_{13}^-$  and  $D_{23}^+ = D_{32}^-$ .



A–F giants as well chosen from the ACRS Part 1. But, we could not prove clearly the consistent relation  $D_{13}^+ = D_{13}^- = D_{23}^+ = D_{32}^- = 0$  for the A–F giants.

Encouraged by the above results, in order to refine further the precessional and equinoctial motion corrections to the FK5 system, we now try to solve once again the least squares for the K–M giants under the conditions  $D_{13}^+ = D_{13}^- = D_{23}^+ = D_{32}^- = 0$ . Namely, we determine the seven unknowns  $S_1, S_2, S_3, D_{12}^+ (= A), D_{21}^- (= B), \Delta p$ , and  $\Delta e + \Delta \lambda$ , on the basis of the two-dimensional Oort-Lindblad model. The results (hereafter referred to as the standard solution) are given in Table 2. Thus, we have the refined values of the luni-solar precessional correction  $\Delta p = -0''.27 \pm 0''.03$ /cent and the equinoctial motion correction including the planetary precessional correction  $\Delta e + \Delta \lambda = -0''.12 \pm 0''.03$ /cent to the FK5 system. The precessional correction thus obtained confirms the one suggested by the VLBI and LLR observations. It should be especially noticed that the present determination of the precessional correction to the FK5 system coincides completely with the one derived from the LLR observations (Williams *et al.* 1991) of two decades which are marginally longer than the longest nutation period.

As for the equinox motion correction, Morrison (1982) obtained from an analysis of lunar occultation observations a correction  $+1''.08 \pm 0''.03$ /cent to the FK4 equinox motion. Since the correction  $\Delta e + \Delta \lambda = +1''.25$ /cent was applied to the FK4 equinox motion to construct the FK5 system, his value corresponds to a correction  $-0''.17 \pm 0''.03$ /cent to the FK5 equinox motion. Thus, the present result coincides with the occultation result as well within the formal standard errors.

It is particular interest to give here a reasoning why the FK5 system is still rotating: In the previous works based on the two-dimensional Oort-Lindblad model, it has been implicitly assumed that the group of stars considered is a steady-state constituent of the galaxy so that the group ought to manifest the galactic vorticity ideally perpendicular to the galactic plane. However, if the selected stars as a whole manifest also the vorticities other than the familiar vorticity, the Oort constant  $B$ , the vorticity components lying in the galactic plane are, in the framework of the Oort-Lindblad model, attributed all together to the luni-solar precessional correction and the equinoctial motion correction to the reference frame. Therefore, the determination of the luni-solar precessional correction, the fictitious equinoctial motion, and the Oort constant  $B$  all together depends closely on the validity of the steady-state assumption.

Fricke's material (the 512 stars) is an inhomogeneous mixture of stars with various spectral types and luminosity classes biased to young ages, so that the mixture is not guaranteed to be a steady-state constituent of the galaxy. Furthermore, to avoid the local irregularities of the stellar systematic motion, he has intended to select stars as distant as possible from the sun, and set up the nearest and remotest limits as 100 pc and 1300 pc, respectively. However, the nearest limit of the distance is still not sufficient to avoid the local irregularities (star streams, Gould-belt, etc.), and on the other hand, the motion of the more distant young stars may be liable to be disturbed by the galactic warp, as described below (see Paper II). These are the reasons why the FK5 system is still rotating with respect to a system defined by the K–M giants which are confirmed as a steady-state constituent of the galaxy in the present work.

#### 4. Kinematics of the Galactic Warp

It has been known since the early HI gas surveys in our galaxy that the HI gas layer in the outer part of the galaxy is distorted systematically above the galactic plane defined by  $b = 0^\circ$  on the northern hemisphere and below it on the southern hemisphere (see a review described by Burton & Deul (1986)). The distortion of the HI layer (galactic warp) starts near at the solar circle ( $R_0 = 8.5$  kpc) and the nodal line of the warp happens to be quite close to the galactic center – sun – anticenter line. More recently, it has become clear that such non-planer distortions in the outer HI gas layer of spiral galaxies are more the rule than exceptions. However, almost no information has not yet been obtained on the kinematics of the warp.

Since young optical objects such as O–B stars, supergiants, bright giants, etc. have very close relation to the HI gas, these objects can be considered as the optical counterpart of the HI warp. Thus, it is expected that these young stars share the same kinematic behaviour with the HI warp. In order to detect the warping motion inherent to the galactic warp, we have chosen 3021 O–B5 stars, supergiants, and bright giants (hereafter, referred to these stars as young stars) as a common set to the three sources, the ACRS Part 1, the SKYMAP, and the UBV Photoelectric Photometry Catalogue (Mermilliod 1987), which provide the proper motions, and the MK classes and the BV magnitudes to estimate the distance. Then, on the basis of the Ogorodnikov–Milne model, we have carried out the overall pattern analysis of the proper motions of the young stars in the domain  $0.5 \text{ kpc} \leq r \leq 3.0 \text{ kpc}$  and  $|z| \leq 0.25 \text{ kpc}$ . Namely, under the condition that the values of  $\Delta p$  and  $\Delta e + \Delta \lambda$  are given by the standard solution in Section 3, we have carried out least square solutions of Eqs. (9) and (11) for the nine unknowns  $S_1, S_2, S_3, D_{12}^+, D_{13}^+, D_{23}^+, D_{21}^-, D_{13}^-,$  and  $D_{32}^-$ .

We should pay attention here to the fact that the layer of the young stars is very thin (about 100 pc) and the tilt of the layer with respect to the galactic plane is very small (about  $1^\circ$ ), as is shown in Paper II, the derivatives of the systematic velocity with respect to  $z$  are hardly detectable. Moreover, the velocity field should be nearly symmetric with respect to the galactic plane in the case of slightly tilted layer. In fact, the correlations among the nine unknowns show that the correlation coefficients between  $D_{13}^+$  and  $D_{13}^-$  and between  $D_{23}^+$  and  $D_{32}^-$  are practically equal to +1 and –1, respectively. Therefore, in the present model of the warping motion we assume  $\partial V_R / \partial z = \partial V_\theta / \partial z = 0$  in Eqs. (6) and (7), that is,

$$D_{13}^+ = -D_{13}^- \quad \text{and} \quad D_{23}^+ = +D_{32}^- \tag{12}$$

The results thus obtained for the nine unknowns are given in Table 3. The standard errors of  $D_{13}^+ = -D_{13}^-$  and  $D_{23}^+ = D_{32}^-$  are even smaller than those of the Oort constants ( $D_{12}^+$  and  $D_{21}^-$ ), so that these parameters are determined fairly well.

The results given in Table 3 provide new kinematical information on the young stars: We have

$$2D_{13}^- = -2D_{13}^+ = \frac{\partial V_z}{\partial R} = 5.6 \pm 1.1 \text{ km/s/kpc} \tag{13}$$

and

$$2D_{32}^- = 2D_{23}^+ = -\frac{1}{R} \frac{\partial V_z}{\partial \theta} = 3.7 \pm 1.1 \text{ km/s/kpc} \tag{14}$$

in addition to the solar motion  $20.3 \pm 1.3 \text{ km/s}$  and the *classical* galactic rotation  $-219.9 \pm 9.8 \text{ km/s}$  at  $R = 8.5 \text{ kpc}$ . Eqs. (13) and (14) imply that the thin sheet of the young stars is performing a rotation around the  $x_1$ -axis (pointing to the galactic center) in positive

sense with the angular velocity 4 km/s/kpc and simultaneously another rotation around the  $x_2$ -axis (pointing to  $l = 90^\circ$  and  $b = 0^\circ$ ) in positive sense with the angular velocity 6 km/s/kpc, besides the classical galactic rotation.

In order to simplify the above apparent view of the warping motion, we introduce a concept of the *kinematic warp*, defining the inclination  $\mathcal{E}$  of the kinematic warp with respect to the galactic plane by

$$\mathcal{E} = \tan^{-1} \left| \frac{D_{13}^-}{D_{21}^-} \right|, \tag{15}$$

where  $\mathcal{E} = 12^\circ.7$ . Then, the young stars as the optical counterpart of the HI warp are streaming around the galactic center in the plane of the kinematic warp with the velocity  $-225$  km/s, and simultaneously the plane is rotating around the galactic center – sun – anticenter line in increasing sense of the inclination of the plane with the angular velocity 4 km/s/kpc (Fig. 2).

Finally, a concern left over unsolved should be added. Since the primary purpose of the present work is to obtain *kinematical* informations on the galactic warp, the difference among the inclinations indicated by the HI warp, the optical warp, and the kinematic warp is not discussed here. The difference may be closely related to the origin as well as the dynamics of the galactic warp.

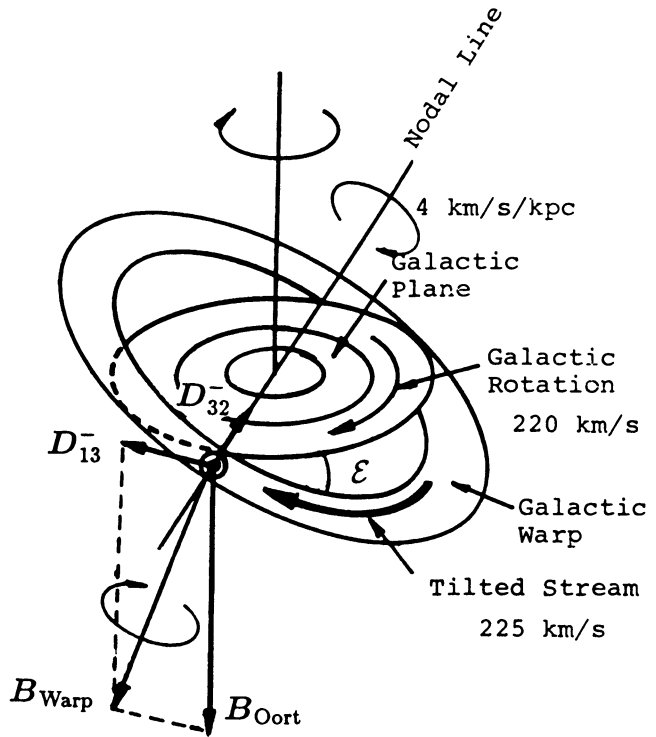


FIGURE 2. A schematic view of the warping motion of the galactic warp, which is derived from the proper motion analysis of the young stars.

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