Neutron Stars and Pulsars: Challenges and Opportunities after 80 years
Proceedings IAU Symposium No. 291, 2012
J. van Leeuwen, ed.

doi:10.1017/S1743921312024386

Recoil velocity of pulsar/magnetar induced by magnetic dipole and quadrupole radiation

Yasufumi Kojima

Department of Physics, Hiroshima University, Higashi-Hiroshima, 739-8526, Japan
email: kojima@theo.phys.sci.hiroshima-u.ac.jp

Abstract. Recoil velocity is examined as a back reaction to the magnetic dipole and quadrupole radiations from a pulsar/magnetar born with rapid rotation. The model is extended from notable Harrison-Tademaru one by including arbitrary field-strength of the magnetic quadrupole moment. The process is slow one operating on a spindown timescale. Resultant velocity depends on not the magnitude, but rather the ratio of the two moments and their geometrical configuration. The model does not necessarily lead to high spatial velocity for a magnetar with a strong magnetic field. This fact is consistent with the recent observational upper bound. The maximum velocity predicted with this model is slightly smaller than that of observed fast-moving pulsars.

Keywords. magnetic fields, stars: neutron, pulsars

---

1. Introduction

Formation of black holes and neutron stars is one of violent energetic events in astronomy. Proper motion of the compact stars may be a relic of the dramatic events at birth. This kind of idea has been discussed since the beginning of relativistic astrophysics in the 1970s. Manchester, Taylor & Van (1974) for the first time determined the proper motion of PSR 1133+16, using timing observation over a four-year period. Since then, the observational progress is remarkable. The statistical property becomes better by increasing the sources (e.g., Hobbs *et al.* 2005). Observation of magnetars may become a key. They have much strong magnetic field strength $B_s \sim 10^{14-15}$G, so that some of them might have larger velocity, if the kick is caused by some magnetic effects. At moment, the upper limit of the transverse velocity $v_\perp$ has been reported, although there is uncertainty in the value. For example, $v_\perp \sim 210$ km s^{-1} for AXP XTEJ1810-197 (Helfand *et al.* 2007), $v_\perp < 1300$ km s^{-1} for SGR 1900+14 (Kaplan *et al.* 2009; De Luca *et al.* 2009) and $v_\perp < 930$ km s^{-1} for AXP 1E2259+586 (Kaplan *et al.* 2009). On the other hand, the magnetic fields of the fast moving pulsars are quite ordinary, $B_s \sim$2-3 $\times 10^{12}$G. Thus, there is no clear correlation between the field strength and the velocity in the present sample.

Theoretically, a number of the pulsar kick mechanisms have been proposed; see e.g., Lai, Chernoff & Cordes (2001) for a review. Among several kick mechanisms operative at the supernova explosion, a strong magnetic field ($> 10^{15-16}$G) is assumed to produce one preferable direction. The origin of the strong fields is also a problem, fossil or dynamo action. Kick mechanisms at birth end on a dynamical timescale of the order of milliseconds or the cooling timescale of ~ 10 s. If the strong magnetic fields are generated on a longer timescale, some natal kick mechanisms involved the magnetic-field-driven anisotropy do not work effectively.

Recoil driven by electromagnetic radiation, which is operative on a longer spindown timescale of $\sim 10^3 (B/10^{15}\text{G})^{-2} (P_i/1\text{ms})^2$ s, has been proposed as a post-natal kick mechanism (Harrison & Tademaru 1975; and Lai, Chernoff & Cordes 2001 for the corrected expression). The model is revisited by taking into account arbitrarily large quadrupole moment (Kojima & Kato 2011). Some results and comparison with previous model are given here.

2. Radiation of energy and linear momentum

We consider a rotating object with angular frequency Ω; the object has a magnetic dipole moment μ and quadrupole moment Q. Each direction of the moment is inclined from the spin axis by $\chi_i (i = 1, 2)$. The energy L and linear momentum F radiated per unit time are calculated as

$$L = \frac{2\mu^2\Omega^4}{3c^3}\sin^2\chi_1 + \frac{Q^2\Omega^6}{160c^5}\sin^2 2\chi_2 + \frac{2Q^2\Omega^6}{5c^5}\sin^4\chi_2, \tag{2.1}$$

$$F = \frac{\mu Q\Omega^5}{20c^5}\sin\chi_1\sin 2\chi_2\sin\delta. \tag{2.2}$$

Three terms in eq. (2.1) are the magnetic dipole radiation $M_{l=1,m=1}$, quadrupole radiation $M_{2,1}$ and $M_{2,2}$ respectively. The linear momentum flux arises from the interference between $M_{1,1}$ and $M_{2,2}$. The angle δ in eq. (2.2) is an azimuthal angle between the dipole and quadrupole directions.

These expressions are compared with those in the off-center dipole model. They are written in term of the magnetic dipole moment (μ_R, μ_ϕ, μ_z) in cylindrical coordinate and distance s from the spin axis as follows:

$$L = \frac{2\Omega^4}{3c^3}\left(\mu_R^2 + \mu_\phi^2\right) + \frac{4\Omega^6}{15c^5}s^2\mu_z^2, \tag{2.3}$$

$$F = \frac{8\Omega^5 s\mu_\phi\mu_z}{15c^5}, \tag{2.4}$$

The first term in eq. (2.3) is the magnetic dipole radiation $M_{1,1}$. Correspondence to our expression (2.1) is clear by replacing $\mu_R^2 + \mu_\phi^2 = \mu^2\sin^2\chi_1$. The second term in eq. (2.3) is a sum of $\Omega^6 s^2\mu_z^2/(6c^5)$ by electric dipole radiation $E_{1,1}$, and $\Omega^6 s^2\mu_z^2/(10c^5)$ by magnetic quadrupole radiation $M_{2,1}$. The parameter in the off-center dipole model corresponds to $Q\sin 2\chi_2 = 4s\mu_z$ except for a complex phase factor. There is a constraint on the quadrupole moment Q as $Q\sin 2\chi_2 \leqslant 4\mu R_s\cos\chi_1$, since $s \leqslant R_s$. In the dipole-quadrupole model, it is possible to consider the case of $Q \gg \mu R_s$ in magnitude. Net linear momentum flux (2.4) arises from two types of interference: $\Omega^5 s\mu_\phi\mu_z/(3c^5)$ between $M_{1,1}$ and $E_{1,1}$, and $\Omega^5 s\mu_\phi\mu_z/(5c^5)$ between $M_{1,1}$ and $M_{2,1}$. The latter reduces to eq.(2.2) if $s\mu_z = Q\sin 2\chi_2/4$ and $\mu_\phi = \mu\sin\chi_1\sin\delta$.

Although there is a slight difference in the radiative components between the off-center dipole and dipole-quadrupole models, both formulae for eqs. (2.1),(2.2) and eqs. (2.3),(2.4) are parameterized as

$$L = \alpha\frac{\mu^2\Omega^4}{c^3} + \beta\frac{Q^2\Omega^6}{c^5}, \quad F = \frac{\gamma}{10}\frac{\mu Q\Omega^5}{c^5}, \tag{2.5}$$

where α, β and γ are dimensionless numbers that depend on only the geometrical configuration. The typical values are listed in Table, assuming that $\sin\chi_l, \sin\delta \to 1/\sqrt{2}$, that is, the directional average of $\langle\sin^2\chi_l\rangle = \langle\sin^2\delta\rangle = 1/2$. It is clear that the coefficient β in off-center dipole is considerably smaller than that in the dipole-quadrupole models. This leads to a larger velocity, as discussed below.

Table 1. Comparison of models.

Model	Multipole	α	β	γ	$\gamma/(\alpha\beta)^{1/2}$
Off-center dipole	$M_{1,1}, M_{2,1}, E_{1,1}$	0.33	0.83×10^{-2}	0.47	9.0
Dipole-quadrupole	$M_{1,1}, M_{2,1}, M_{2,2}$	0.33	0.10	0.18	0.97

The angular velocity $\Omega(t)$ is determined by equating the loss rate of rotational energy with the luminosity L, and the velocity $V(t)$ is determined from the momentum emission F. In terms of the mass M and inertial moment $I(= 2MR_s^2/5)$, we have

$$I\Omega\dot{\Omega} = -L, \quad M\dot{V} = -F. \tag{2.6}$$

The maximum velocity ΔV_* gained from the initial angular velocity Ω_i is given with respect to the magnetic moment ratio $Q/(\mu R_s)$:

$$\Delta V_* \approx 9.2 \times 10^{-3} \frac{c\gamma}{(\alpha\beta)^{1/2}} \left(\frac{\Omega_i R_s}{c}\right)^2 \approx 120 \left(\frac{P_i}{1\mathrm{ms}}\right)^{-2} \times \frac{\gamma}{(\alpha\beta)^{1/2}} \text{ km s}^{-1}, \tag{2.7}$$

where present angular velocity is assumed to be much smaller than initial one Ω_i. The actual maximum depends on the combination of parameters $\gamma/(\alpha\beta)^{1/2}$, which is determined by the magnetic field configuration. By optimal choice, the resultant kick velocity increases up to $\sim 10^3(P_i/1\mathrm{ms})^{-2}$ km s^{-1}.

3. Implication

Unlike most magnetic induced kick mechanisms, the electromagnetic rocket mechanism considered in Harrison & Tademaru (1975) and in this paper does not depend on field strength. In our model, the ratio of dipole and quadrupole moments is important. The velocity also depends on the geometrical configuration of the multipole moments, that is, each inclination angle from the spin axis and the angle between the axes of symmetry of the moment. The configuration is quite unknown, and is closely related to the origin of the magnetic field, dynamo or fossil. Nevertheless, interesting results are reported within the mean-field dynamo theory (Bonanno, Urpin & Belvedere 2006): (1) Strong large-scale and weak small-scale fields are generated only in a star with a very short initial period, that is, the Rossy number is small: (2) Maximum strength decreases and small-scale fields become dominant with decrease of the initial period. Thus, magnetars may have an ordered dipole with a strong field, while some pulsars may have rather irregular fields with higher multipoles. Through the superposition of higher multipoles, the pulsars come to have a larger radiation recoil velocity than magnetars.

References

Bonanno, A., Urpin, V., & Belvedere, G., 2006, *A&A*, 451, 1049
De Luca, A., Caraveo, P. A., Esposito, P., & Hurley, K., 2009, *ApJ*, 698, 250
Harrison, E. R. & Tademaru, E., 1975, *ApJ*, 201, 447
Helfand, D. J. *et al.* 2007, *ApJ*, 662, 1198
Hobbs, G., Lorimer, D. R., Lyne, A. G., & Kramer, M., 2005, *MNRAS*, 360, 974
Kaplan, D. L., Chatterjee, S., Hales, C. A., Gaensler, B. M., & Slane, P. O., 2009, *AJ*, 137, 354
Kojima, Y. & Kato, E. Y., 2011, *ApJ*, 728, 25
Lai, D., Chernoff, D. F., & Cordes, J. M., 2001, *ApJ*, 549, 1111
Manchester, R. N., Taylor, J. H. & Van, Y. Y., 1974, *ApJ*, 189, L119

https://doi.org/10.1017/S1743921312024386 Published online by Cambridge University Press