CORONAL ARCADES IN THE SUN AND THEIR HYDROMAGNETIC STABILITY

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Two ribbon flares on the sun are sometimes preceded by luminous arcade like structures in the corona with fillament activity. The arcades which are associated with the coronal magnetic field are sometimes without these fillaments when they are found to be relatively long lived. The reasons underlying the stability and instability of these structures may be relevant in understanding the basic mechanism of two ribbon flares. Here we present results on the equilibrium structure of coronal arcades and their hydromagnetic stability taking into account the effects of magnetic field line-tying on the photosphere.

The configuration of the arcade structures can be idealized as an elongated tunnel or series of magnetic arches. On the photosphere, Zeeman-splitting measurements show that the base of the active region of this structure has two distinct regions of opposite magnetic polarity separated by an inversion line. Observations of the Hα-fibrils show that the field lines lying lower towards the photosphere become progressively more parallel to the magnetic neutral line while at higher altitudes X-ray observations made from the Skylab imply the luminous arcs (and the underlying magnetic fields) to be more nearly circular with their planes perpendicular to and centered on the neutral line. The equilibrium hydromagnetic structure of the arcade was modeled (Ray and Van Hoven 1982a) in a simple analytic form in two dimensions and was considered to have a cylindrical symmetry in the upper half plane above the photosphere. As the pressure gradients in the corona are expected to be small in quiescent conditions, and the coronal magnetic fields involved are several hundreds of Gauss, the magnetic field configuration within a certain radius (about the neutral line) was modeled by a force free field. The force free field joins smoothly to a potential field outside this radius R. In this model, there are no unrealistic sheet currents in the equilibrium configuration. In cylindrical coordinates about the neutral line, the field structure inside the radius R is described by the Lundquist (1950) solution

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of force-free fields:

$$B_z = B_o J_o(\alpha r); \; B_\theta = B_o J_1(\alpha r) \tag{1}$$

where J_o and J_1 are Bessel functions of order zero and one respectively. The solution has the feature that near the axis of the cylinder, the field is mostly in the z-direction, whereas near the cylinder surface defined by $\alpha R = 2.4$, the field is mostly in the azimuthal θ direction. Outside this particular radius, the force-free field is smoothly joined to a potential field:

$$B_z = o \; ; \; B_\theta = B_o \left(R \, J_1(\alpha R)/r \right) \tag{2}$$

Having modeled the equilibrium structure of the arcades, a linear stability analysis (Bernstein et al 1958) of the solution was performed using a prescription due to Newcomb (1960).

The field lines that protrude from the photosphere into the corona satisfy certain boundary conditions on the surface. Since there is a sudden change in density at the photosphere, the field lines are essentially rigidly tied to the solar surface. The only displacement on the photosphere that is possible is a movement in the direction of the magnetic field at the footpoints (Van Hoven et al. 1981). Thus, there are nodes of the radial and perpendicular (to the B_o-field) components of $\underset{\sim}{\xi}$ but there is a complete freedom for the parallel (to B_o) motions. Each component of the displacement $\underset{\sim}{\xi}$ was expanded in a series in sine and cosine $m\theta$ and appropriate combinations of sine and cosine kz (for detailed expansions see Ray and Van Hoven 1982a) and was used in the energy integral to obtain a form (after integration over the θ and z co-ordinates):

$$\delta W_{m,r}(\underset{\sim}{\xi}) = \int_0^\infty dr \left[f \, \xi_r'^2 + g \xi_r^2 \right] \tag{3}$$

where ξ_r is the radial component of $\underset{\sim}{\xi}$ and f and g are functions of r, m, k, α and R. The detailed forms of f and g inside and outside $\alpha R = 2.4$ are given Ray and Van Hoven (1982a). The Euler-Langrange equation obtained from eq. (3) can be used to provide a necessary and sufficient condition (Newcomb, 1960) for hydromagnetic stability in the plasma pinch. As the obtained Euler-Lagrange differential equations do not have any singular points, (i.e. f does not vanish in the interval 0 to ∞,) to ensure that a given equilibrium mhd structure is stable, one needs to check that the solution ξ_r of the Euler-Lagrange equation which is finite at the origin does not vanish in a finite interval of r. This is a convenient global condition for stability and the Euler-Lagrange equation for $Q = r \, \xi_r$ obtained from eq. (3) was integrated numerically to

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test this. In order to satisfy the boundary conditions on the photosphere, the radial and perpendicular (to B_0) components of ξ were taken to be proportional to sin mθ but both sine and cosine functions of kz were allowed. The wave numbers k were unrestricted but the angular mode numbers m were taken to be either even or odd integers. The most general decomposition of ξ_r and $\xi_\perp$ in the angular interval $0<\theta<\pi$ would involve both even and odd mode numbers m. Allowing such a general perturbation would however prevent a term by term (m,k) analysis of the stability and would force one to take into account effects of mode coupling (between m and m′). With the somewhat restricted set of perturbations (of only odd or only even m) not only a term by term analysis is possible, but the surface energy integral δW_S which must vanish for the applicability of the stability theorem of Bernstein does indeed vanish.

Newcomb (1960) had argued that the m = 1 mode is the most dangerous mode as far as the possibility of instability in a plasma pinch is concerned. Higher m modes are more stable than m = 1 case. The stability of the equilibrium configuration with respect to the m= 1 mode was tested and found to be stable. The m = 0 mode is not allowed in our equilibrium model which is line-tied on the photosphere at $\theta=o,\pi$.

Thus, in a linear stability analysis of a force-free sheared field surrounded by a potential field, the equilibrium configuration was found to be stable to a reasonably general perturbation summation. With a view to handle mode couplings between even and odd modes encountered in a more general perturbation, the criterion for stability of a plasma configuration given by Newcomb (1960) was extended to find the conditions for positive definiteness of bilinear forms involving cross terms (Ray 1983; Ray and Van Hoven,1982b). With such a generalization of the stability criterion it is possible to test equilibrium configurations with respect to more general mode mixing perturbations.

The condition of field line-tying on the perturbations impose a strong stabilizing influence. Migliuolo and Cargill (1983) have performed a normal mode analysis of the marginal stability of several force-free equilibria and have arrived at similar conclusions. They have also argued that due to line-tying at a reference boundary and the absence of mode-rational surfaces(where($B_0 \cdot \nabla)\xi_r$ could be zero)resistive modes do not contribute to an instability. Dissipation of magnetic energy due to field-line reconnection is thus ruled out unless some of the field lines are not tied to the photosphere. Hence such arcades would not be able to rapidly release a great deal of energy as is observed in X-ray flare activity. Indeed, coronal arcades that are without filaments are generally not observed to erupt. Two-ribbon flares are in general associated with some filament activation and it could be that an imbedded prominence in the coronal arcade is a necessary condition. It is also possible that some arcades may be made unstable through non-linear mhd wave coupling or due to emerging magnetic flux from the photosphere.

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DISCUSSION

Ionson: In order for line tying to hold true, the diffusion time in the photosphere as well as the convective turnover time must be longer than the instability growth time. Are these conditions satisfied?

A. Ray: Both the resistive diffusion timescale and convective turnover timescale are longer than the relevant timescale, which is the Alfvén wave crossing time at the base of the corona.

Heyvaerts: I would like to stress that fact that stability analysis is perhaps not the best way to look at arcade evolution, because it supposes that the equilibrium under study has already been realized. Of more significance, perhaps, is the fact that a sequence of MHD equilibria may cease to exist at some catastrophy point. This is a property of non-linear force free fields, as opposed to constant α force free fields. (See Heyvaerts et al. 1982, Astron. Astrophys. 111, 104 and references therein, and Aly, these proceedings).

A. Ray: The arcades are seen to presist in the corona for long periods of time and it is not unrealistic to investigate an MHD equilibrium. The equilibrium may of course be changed at a later time by a change of conditions at the photosphere, when its subsequent development may lead through a disruptive phase.