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SOME PROPERTIES OF THE PROJECTIVE TENSOR PRODUCT $U\hat{\otimes}X$ DERIVED FROM THOSE OF U AND X

PATRICK N. DOWLING

Let X be a real or complex Banach space and let U be a Banach space with an unconditional basis. We show that the projective tensor product of U and X, $U\hat{\otimes}X$, has the complete continuity property (respectively, the analytic complete continuity property) whenever U and X have the complete continuity property (respectively, the analytic complete continuity property). More general versions of these results are also obtained. Moreover, the techniques applied here to the projective tensor product, can also be used to establish some Banach space properties of the Fremlin projective tensor product.

1. INTRODUCTION

The mathematics literature abounds with results relating to the following question: What properties that are shared by the Lebesgue space $L^p[0,1]$, $1 < p < \infty$, and a Banach space X are also enjoyed by the Lebesgue-Bochner spaces $L^p([0,1],X)$? Some well known examples of such results are those of

> Kwapien [**21**] – $L^p([0,1],X)$ does not contain c_0 if X does not contain c_0;
> Pisier [**23**] – $L^p([0,1],X)$ does not contain ℓ^1 if X does not contain ℓ^1; and
> Turett and Uhl [**31**] – $L^p([0,1],X)$ has the Radon–Nikodym property if X has the Radon–Nikodym property.

Many of these results have been extended to the more general setting of Köthe-Bochner spaces, $E(X)$, where E is a Köthe function space. In 1992, Hensgen [**19**] developed a method, based on semi-embeddings, that allowed him to conclude that the Köthe-Bochner space $E(X)$ has property (P) whenever both E and X have property (P), where property (P) is a Banach space property with certain characteristics.

Less well known are Banach space properties that lift from X to $L^p[0,1]\hat{\otimes}X$, the projective tensor product of $L^p[0,1]$ and X. The reason for this is twofold; in general, Banach space properties are very unstable under the projective tensor product – for example, Bourgain and Pisier [**1**] gave an example of a Banach space X with the Radon–Nikodym property and such that $X\hat{\otimes}X$ contains a copy of c_0. Secondly, the projective tensor

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product norm can be difficult to compute with. However, Bu and Diestel [**5, 3**] were able to give a representation of the elements of $L^p[0,1]\widehat{\otimes}X$ as elements of a vector-valued sequence space, and using this representation, they were able to prove that $L^p[0,1]\widehat{\otimes}X$ has the Radon–Nikodym property whenever X has the Radon–Nikodym property and $1 < p < \infty$. This result was extended by Bu and Dowling [**7**] and Dowling [**13**] to show that $L^p[0,1]\widehat{\otimes}X$ has the analytic Radon–Nikodym property (respectively, the near Radon–Nikodym property, the complete continuity property, the analytic complete continuity property, contains no copy of c_0) if X has the same property and $1 < p < \infty$. All of these results use the Bu-Diestel representation of $L^p[0,1]\widehat{\otimes}X$, which in turn depends heavily on the fact that $L^p[0,1]$, for $1 < p < \infty$, has a boundedly complete unconditional basis. Later on Bu, Diestel, Dowling and Oja [**6**] were able to extend the results in [**3, 5, 7**] to the setting of $U\widehat{\otimes}X$, where U is a Banach space with a boundedly complete 1-unconditional basis.

In this paper we shall show, using techniques similar to those of Hensgen [**19**], that many Banach space properties that lift from X to $E(X)$ can also be shown to lift to $E\widehat{\otimes}X$, where E is a Banach space with a boundedly complete unconditional basis. Of course, a Banach space with a 1-unconditional basis is a Köthe function space [**22**]. It is interesting to note that if X is an infinite dimensional Banach space and $1 < p < \infty$, then $L^p([0,1],X)$ and $L^p[0,1]\widehat{\otimes}X$ are not isomorphic [**10**] (see [**17**] for extensions of this result). The key ingredient that we shall use in getting our results is the fact that $E\widehat{\otimes}X$ semi-embeds in $E(X)$ whenever E is a Banach space with a boundedly complete unconditional basis [**6**].

2. Preliminaries and Results

Let U be a Banach space with a boundedly complete 1-unconditional normalised basis $(e_i)_{i=1}^{\infty}$. The basis $(e_i)_{i=1}^{\infty}$ being 1-unconditional means that, for all $n \in \mathbb{N}$, and scalars $a_1, a_2, \dots, a_n$ and $s_1, s_2, \dots, s_n$ with $|s_i| = 1$ for each $1 \leqslant i \leqslant n$,

$$\left\|\sum_{i=1}^{n} s_i a_i e_i\right\| \leqslant \left\|\sum_{i=1}^{n} a_i e_i\right\|.$$

It is easy to show that for each $n \in \mathbb{N}$,

$$\left\|\sum_{i=1}^{n} a_i e_i\right\| \leqslant \left\|\sum_{i=1}^{n} b_i e_i\right\|$$

whenever $a_1, a_2, \dots, a_n$ and $b_1, b_2, \dots, b_n$ are scalars with $|a_i| \leqslant |b_i|$ for each $1 \leqslant i \leqslant n$. Hence the 1-unconditionality of U ensures that U is a Köthe function space [**22**].

The vector-valued sequence space $U(X)$ is defined by

$$U(X) = \left\{\overline{x} = (x_i)_i : x_i \in X, \sum_i \|x_i\| e_i \text{ converges in } U\right\},$$

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and the norm on $U(X)$ is given by

$$\|\overline{x}\|_{U(X)} = \left\| \sum_{i=1}^{\infty} \|x_i\| e_i \right\|_U .$$

Now suppose that $\mathcal{P}$ is a Banach space property satisfying the following conditions;

(1) $\mathcal{P}$ is a *separably determined property*; that is, a Banach space X has $\mathcal{P}$ if every separable closed subspace of X has $\mathcal{P}$.

(2) $\mathcal{P}$ is *separably semi-embeddably stable*; that is, if X is a separable Banach space which semi-embeds in a Banach space Y and Y has $\mathcal{P}$, then X has $\mathcal{P}$.

REMARK 1. Conditions (1) and (2) imply that if a Banach space X has $\mathcal{P}$, then so does every closed subspace of X. Conditions (1) and (2) also imply that if X and Y are isomorphic Banach spaces and Y has $\mathcal{P}$, then X has $\mathcal{P}$.

One of the main results in [6] is the following

THEOREM 1. *If U is a Banach space with a boundedly complete unconditional basis and X is a Banach space, then $U\widehat{\otimes}X$ semi-embeds in $U(X)$.*

We are now in a position to state our main result.

THEOREM 2. *Let U be a Banach space with a boundedly complete unconditional basis and X is a Banach space. Let $\mathcal{P}$ be a Banach space property that is separably determined and separably semi-embeddably stable. If $U(X)$ has $\mathcal{P}$, then $U\widehat{\otimes}X$ also has $\mathcal{P}$.*

PROOF: As $\mathcal{P}$ is a separably determined property, we only need to show that every separable subspace of $U\widehat{\otimes}X$ has $\mathcal{P}$. To this end, let S be a separable subspace of $U\widehat{\otimes}X$. Then, by [5], there is a separable subspace X_0 of X such that S is a subspace of $U\widehat{\otimes}X_0$. $U(X)$ has $\mathcal{P}$ by assumption, so $U(X_0)$ also has $\mathcal{P}$ as it is a subspace of $U(X)$. Note also that $U\widehat{\otimes}X_0$ is separable. Moreover, since $U\widehat{\otimes}X_0$ semi-embeds in $U(X_0)$ by Theorem 1, and $\mathcal{P}$ is separably semi-embeddably stable, $U\widehat{\otimes}X_0$ has $\mathcal{P}$. Thus S has $\mathcal{P}$ since it is a subspace of $U\widehat{\otimes}X_0$ and $U\widehat{\otimes}X_0$ has $\mathcal{P}$. This completes the proof. ∎

3. APPLICATIONS

The applications that we consider will be to Banach space properties that lie between the Radon–Nikodym property and the property of not containing a copy of c_0. It is important to note that a Banach space U with a boundedly complete basis has the Radon–Nikodym property [10]. Many (though not all) of these properties we consider can be phrased in terms of properties related to subsets of compact Abelian metrisable groups.

Throughout this section G will denote a compact metrisable Abelian group, $\mathcal{B}(G)$ is the σ-algebra of Borel subsets of G, and λ is normalised Haar measure on G. The dual group of G will be denoted by Γ.

Let X be a real or complex Banach space. We denote by $L^1(G, X)$ (respectively, $L^\infty(G, X)$) the Banach space of (all equivalence classes of) λ-Bochner integrable functions on G with values in X (respectively, (all equivalence classes of) λ-measurable X-valued functions that are essentially bounded).

If μ is a countably additive X-valued measure on $\mathcal{B}(G)$, we say that it is of bounded variation if $\sup \sum_{A\in\pi} \|\mu(A)\| < \infty$, where the supremum is taken over all finite measurable partitions of G. The measure μ is said to be of bounded average range if there is a positive constant c so that $\|\mu(A)\| \leqslant c\lambda(A)$, for every $A \in \mathcal{B}(G)$.

We shall denote by $\mathcal{M}^1(G, X)$, the space of all X-valued measures on $\mathcal{B}(G)$ that are of bounded variation, and $\mathcal{M}^\infty(G, X)$ will denote the space of all X-valued measures on $\mathcal{B}(G)$ that are of bounded average range.

Let Λ be a subset of Γ. For $\gamma \in \Gamma$ and $f \in L^1(G, X)$, we define the Fourier coefficient of f at γ by

$$\widehat{f}(\gamma) = \int_G f(t)\overline{\gamma}(t)d\lambda(t).$$

Similarly, if $\mu \in \mathcal{M}^1(G, X)$, we define the Fourier coefficient of μ at γ by

$$\widehat{\mu}(\gamma) = \int_G \overline{\gamma}(t)d\mu(t).$$

A measure $\mu \in \mathcal{M}^1(G, X)$ will be called a Λ-measure if $\widehat{\mu}(\gamma) = 0$ for all $\gamma \notin \Lambda$.

Definition 3: ([**14, 11**]) Let G be a compact metrisable Abelian group, let Λ be a subset of Γ, and let X be a Banach space.

(1) X is said to have type I-Λ-Radon–Nikodym property (I-Λ-Radon–Nikodym property) if every Λ-measure μ in $\mathcal{M}^\infty(G, X)$ is differentiable; that is, there is a function $f \in L^1(G, X)$ such that $\mu(E) = \int_E f\,d\lambda$ for all $E \in \mathcal{B}(G)$.

(2) X has type II-Λ-Radon–Nikodym property (II-Λ-Radon–Nikodym property) if every λ-continuous, Λ-measure in $\mathcal{M}^1(G, X)$ is differentiable.

Definition 4: ([**29**]) Let G be a compact metrisable Abelian group, let Λ be a subset of Γ, and let X be a Banach space.

(1) X is said to have type I-Λ-complete continuity property (I-Λ-complete continuity property) if every Λ-measure μ in $\mathcal{M}^\infty(G, X)$ has a relatively compact range.

(2) X is said to have type II-Λ-complete continuity property (II-Λ-complete continuity property) if every λ-continuous, Λ-measure in $\mathcal{M}^1(G, X)$ has relatively compact range.

REMARK 2. If $G = \{-1,1\}^{\mathbb{N}}$, the Cantor group, then $\Gamma = \{-1,1\}^{(\mathbb{N})}$ and Fourier coefficients of measures on $\mathcal{B}(G)$ with values in a real Banach space are well-defined. If $\Lambda = \Gamma$, then I-Λ-Radon–Nikodym property and II-Λ-Radon–Nikodym property are equivalent, and equivalent to the usual Radon–Nikodym property. Similarly, I-Λ-complete continuity property and II-Λ-complete continuity property are equivalent and equivalent to the usual complete continuity property. Furthermore, since Γ is infinite and discrete, it contains an infinite Sidon subset [**30**, page 126]. If Λ is such an infinite Sidon set, then by [**12**], a real Banach space has II-Λ-Radon–Nikodym property if and only if it does not contain a copy of c_0. In particular, I-Λ-Radon–Nikodym property, II-Λ-Radon–Nikodym property, I-Λ-complete continuity property and II-Λ-complete continuity property are all equivalent to not containing a copy of c_0.

REMARK 3. If $G = \mathbb{T}$, the circle group, then $\Gamma = \mathbb{Z}$ and Fourier coefficients of measures on $\mathcal{B}(G)$ with values in a complex Banach space are well-defined. If $\Lambda = \mathbb{Z}$, then I-Λ-Radon–Nikodym property and II-Λ-Radon–Nikodym property are equivalent, and equivalent to the usual Radon–Nikodym property. Also I-Λ-complete continuity property and II-Λ-complete continuity property are equivalent, and equivalent to the usual complete continuity property. If $\Lambda = \mathbb{N} \cup \{0\}$, then I-$\Lambda$-Radon–Nikodym property and II-Λ-Radon–Nikodym property are equivalent, and equivalent to the analytic Radon–Nikodym property (see [**11**]). Similarly, I-Λ-complete continuity property and II-Λ-complete continuity property are equivalent, and equivalent to the analytic complete continuity property (see [**28**]). If Λ is an infinite Sidon subset of Γ, then I-Λ-Radon–Nikodym property, II-Λ-Radon–Nikodym property, I-Λ-complete continuity property and II-Λ-complete continuity property are all equivalent, and equivalent to the property of not containing a subspace isomorphic to c_0 (see [**12, 28**]).

Another property that can be considered in our scheme is the near Radon–Nikodym property.

DEFINITION 5: ([**20**]) A bounded linear operator T from $L^1[0,1]$ to a Banach space X is said to be near representable if for each Dunford–Pettis operator $D : L^1[0,1] \to L^1[0,1]$, the composition operator $T \circ D : L^1[0,1] \to X$ is Bochner representable. A Banach space X is said to have the near Radon–Nikodym property if every near representable operator from $L^1[0,1]$ to X is Bochner representable.

To be able to apply Theorem 2 to a property, we first need to determine whether the property is both separably determined and separably semi-embeddably stable. It is well known that I-Λ-Radon–Nikodym property, II-Λ-Radon–Nikodym property, I-Λ-complete continuity property and II-Λ-complete continuity property are all separably determined and separably semi-embeddably stable properties for all Λ [**2, 11, 29**]. Similarly, the near Radon–Nikodym property is a separably determined and separably semi-embeddably stable property [**20**].

If U is a Banach space with a boundedly complete unconditional basis, then U

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has the Radon–Nikodym property and hence it has I-Λ-Radon–Nikodym property, II-Λ-Radon–Nikodym property, I-Λ-complete continuity property, II-Λ-complete continuity property and the near Radon–Nikodym property for all Λ. However, it is unknown in general whether all of these properties lift from X to $U(X)$. Some of the known lifting results are;

(1) When $\Lambda = \Gamma$, then II-Λ-Radon–Nikodym property is equivalent to the Radon–Nikodym property, and $U(X)$ has the Radon–Nikodym property by a result of Bukhvalov [**8**]. Following Randrianantoanina and Saab [**26**], and independently Hensgen [**19**], we see that when Λ is a Riesz set, then $U(X)$ has II-Λ-Radon–Nikodym property whenever X has II-Λ-Radon–Nikodym property.

(2) For complete continuity properties, the results are similar. When $\Lambda = \Gamma$, then II-Λ-complete continuity property is equivalent to the complete continuity property and $U(X)$ has the complete continuity property whenever X has the complete continuity property [**25**]. When Λ is a Riesz set, then $U(X)$ has II-Λ-complete continuity property whenever X has II-Λ-complete continuity property [**24**].

(3) $U(X)$ has the near Radon–Nikodym property whenever X has the near Radon–Nikodym property [**27**].

An application of Theorem 2 now yields

COROLLARY 6. *Let G be a compact metrisable Abelian group, let Λ be a subset of Γ, let U be a Banach space with a boundedly complete unconditional basis, and let X be a Banach space. Suppose that $\Lambda = \Gamma$ or that Λ is a Riesz set. Then $U\widehat{\otimes}X$ has II-Λ-Radon–Nikodym property (respectively, II-Λ-complete continuity property, near Radon–Nikodym property) whenever X has the same property.*

In particular, Corollary 6 yields the following results from [**7**]

COROLLARY 7. ([**6**]) *Let U be a Banach space with a boundedly complete unconditional basis. If X is a Banach space with the Radon–Nikodym property (respectively, analytic Radon–Nikodym property, near Radon–Nikodym property, contains no subspace isomorphic to c_0), then $U\widehat{\otimes}X$ has the same property.*

From Corollary 6 we also obtain the following new result

COROLLARY 8. *Let U be a Banach space with a boundedly complete unconditional basis. If X is a Banach space with the complete continuity property (respectively, analytic complete continuity property), then $U\widehat{\otimes}X$ has the same property.*

One final property that does not exactly fit in to our scheme is property (P) of Costé and Lust–Piquard [**9**].

DEFINITION 9: ([**9**]) A Banach space X is said to have property (P) if each operator $T : L^1[0,1] \to X$ is representable if and only if for every weak*-null sequence (φ_n)

in $L^\infty[0,1]$, the sequence $\big(T(\varphi_n)\big)$ defines an element of $c_0\widehat{\otimes}X$.

Property (P) is separably semi-embeddably stable [**9**, Proposition 4]. However, we only know that it is separably determined in spaces with the separable complementation property [**9**, Theorem 4]. Recall that a Banach space has the separable complementation property if for each separable subspace X_0 of X, there is a separable complemented subspace X_1 of X that contains X_0. Hensgen [**19**, Proposition 2.8] proved that if E and X have property (P) and X has the separable complementation property, then $E(X)$ has property (P). It is also easily seen that $E(X)$ has the separable complementation property whenever X has the separable complementation property. Consequently, an application of Theorem 2 yields

COROLLARY 10. *Let U be a Banach space with a boundedly complete unconditional basis. If X is a Banach space with the property (P) and X has the separable complementation property, then $U\widehat{\otimes}X$ has property (P).*

4. A specific application to the Fremlin tensor product

In this section we shall restrict our attention to Banach lattices. Fremlin [**15, 16**] defines an order in $X \otimes Y$, where X and Y are Banach lattices by

$$(X\otimes Y)^+ = \{u \in X\otimes Y : \langle u, f\otimes g\rangle \geqslant 0 \text{ for all } f \in X^{*+}, g \in Y^{*+}\}$$

and the positive projective norm on $X \otimes Y$ by

$$\|u\|_{|\pi|} = \inf\left\{\left|\sum_{k=1}^{n}\varphi(x_k, y_k)\right| : u = \sum_{k=1}^{n} x_k\otimes y_k, \varphi \in M\right\},$$

where M is the set of all positive bilinear functionals on $X \times Y$ of norm less than or equal one. The completion of $X\otimes Y$ with respect to the norm $\|\cdot\|_{|\pi|}$, is denoted by $X\widehat{\otimes}_F Y$ and is called the Fremlin tensor product of X and Y. It should be noted that $X\widehat{\otimes}_F Y$ is a Banach lattice.

In [**4**], Bu and Buskes derive a sequential representation of $\ell^p\widehat{\otimes}_F X$, where X is a Banach lattice and $1 < p < \infty$, by adapting the methods of the earlier representation result for $\ell^p\widehat{\otimes}X$ by Bu and Diestel [**5**]. In particular, they were able to show that if X is a Banach lattice and $1 < p < \infty$, then $\ell^p\widehat{\otimes}_F X$ semi-embeds in $\ell^p(X)$ [**4**, Theorems 7 and 15]. The reader should note that in [**4**], $\ell^p(X)$ is denoted by $\ell_p^{strong}(X)$.

The techniques used in proving Theorem 2 are again available in this special situation of $\ell^p\widehat{\otimes}_F X$, where X a Banach lattice and $1 < p < \infty$, and produce

THEOREM 11. *Let $1 < p < \infty$ and let X be a Banach lattice. Let $\mathcal{P}$ be a Banach space property that is separably determined and separably semi-embeddably stable. If X has $\mathcal{P}$, then $\ell^p\widehat{\otimes}_F X$ also has $\mathcal{P}$.*

REMARK 4. Actually, a little more can be said in Theorem 11. Since Banach lattices have the separable complementation property [18], the *separably determined* condition can be weakened in the obvious manner; namely, a Banach space X has $\mathcal{P}$ if every separable complemented subspace of X has $\mathcal{P}$.

An application of Theorem 11 and the last remark, along with the results of the last section, yields

COROLLARY 12. *Let $1 < p < \infty$. If X is a Banach lattice with the Radon–Nikodym property (respectively, analytic Radon–Nikodym property, near Radon–Nikodym property, contains no subspace isomorphic to c_0, complete continuity property, analytic complete continuity property, property (P)), then $\ell^p \widehat{\otimes}_F X$ has the same property.*

REMARK 5. Qingying Bu recently informed the author that the conclusion of Corollary 12 remain true if ℓ^p is replaced by Banach lattice U with a boundedly complete unconditional basis whose order is defined coordinately by the unconditional basis. The author wishes to thank Professor Bu for pointing out this extension to him.

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Department of Mathematics and Statistics
Miami University
Oxford, OH 45056
United States of America
e-mail: dowlinpn@muohio.edu