ponents of the velocity at any point due to such a system can be at once obtained from the principles laid down.

Before leaving the subject, I would remark that our theory asserts that a cyclone could travel from east to west only if a strong anti-cyclone were to the north of it, or a second cyclone to the south of it.

On the expression of a symmetric function in terms of the elementary symmetric functions.

By R. E. ALLARDICE, M.A.

The theorem that any rational symmetric function of n variables $x_1, x_2, \dots x_n$ is expressible as a rational function of the n elementary symmetric functions, Σx_1, $\Sigma x_1 x_2$, $\Sigma x_1 x_2 x_3$, etc., is usually proved by means of the properties of the roots of an equation. It is obvious, however, that the theorem has no necessary connection with the properties of equations; and the object of this paper is to give an elementary proof of the theorem, based solely on the definition of a symmetric function.

It is obvious that only integral symmetric functions need be considered.

Let ${}_np_1$, ${}_np_2$, ${}_np_3$, … stand for Σx_1, $\Sigma x_1 x_2$, $\Sigma x_1 x_2 x_3$ … , when there are n variables. If x_n vanishes, ${}_np_1$, ${}_np_2$, ${}_np_3$, … evidently become ${}_{n-1}p_1$, ${}_{n-1}p_2$, ${}_{n-1}p_3$, …

Now assume that all integral symmetric functions involving not more than $(n-1)$ variables can be expressed rationally in terms of the elementary symmetric functions.

Let $f(x_1, x_2, \dots x_n)$ be any integral symmetric function of n variables. Then $f(x_1, x_2, \dots x_{n-1}, 0)$ is a symmetric function of $(n-1)$ variables, and, by supposition, may be expressed in terms of ${}_{n-1}p_1$, ${}_{n-1}p_2$ … . Let its expression be $\phi({}_{n-1}p_1, {}_{n-1}p_2, \dots {}_{n-1}p_{n-1})$.

Assume now

$$f(x_1, x_2, \dots x_n) = \phi({}_np_1, {}_np_2 \cdots {}_np_{n-1}) + \psi(x_1, x_2, \dots x_n),$$

where ψ is obviously a symmetric function.

Put $x_n = 0$, on both sides of this identity; then

$$f(x_1, x_2, \dots x_{n-1}, 0) = \phi({}_{n-1}p_1, {}_{n-1}p_2, \cdots {}_{n-1}p_{n-1}) + \psi(x_1, x_2, \dots x_{n-1}, 0);$$

and hence $$\psi(x_1, x_2, \dots x_{n-1}, 0) = 0,$$

https://doi.org/10.1017/S0013091500030339 Published online by Cambridge University Press

and therefore x_n is a factor in $\psi(x_1, x_2, \dots x_{n-1}, x_n)$. Since ψ is a symmetric function, $x_1, x_2, \dots x_{n-1}$, must also be factors; and therefore $x_1x_2 \dots x_n$, which is equal to ${}_np_n$, is a factor. If this factor be divided out, the quotient will be a symmetric function, the degree of which will be less by n that of the given function. The above process may then be repeated with this quotient; and so on, till the degree is reduced to zero.

Since every (symmetric) function of a single x_1 is a function of ${}_1p_1(=x_1)$, it follows by induction that every symmetric function of n variables is expressible in terms of the n elementary symmetric functions.

The ordinary propositions about the weight and order of symmetric functions may easily be obtained from the above.

On laboratory work in electricity in large classes.

By Messrs A. Y. Fraser, J. T. Morrison, and W. Wallace.

Seventh Meeting, May 10th, 1889.

George A. Gibson, Esq., M.A., President, in the Chair.

Solutions of two geometrical problems.

By J. S. Mackay, LL.D.

The two problems are :—

1. *To divide a given straight line internally and externally so that the ratio between its segments may be equal to a given ratio.*

https://doi.org/10.1017/S0013091500030339 Published online by Cambridge University Press