3.5 The Dust Coma of Comets

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Abstract: Analytical expressions for the dust entrainment in the limits of low and high gas production from cometary nuclei are derived. The resulting volume density is used to evaluate the dust production of Comets Encke and Kohoutek from IR-measurements. It is predicted, that the dust in solar direction was diffuse for Comet Kohoutek during approach to the Sun, but sharp for Comet Encke and nearly independent of heliocentric distance. The dust production rate of Encke ist not sufficient to sustain the Zodiacal Cloud of micron sized particles. Particle impact probabilities and hazards are estimated for a space probe in a fly-by mission to Encke.

Introduction: Whereas the visible coma of a comet is largely determined by the fluorescence of radicals, i.e. the photolysis fragments of parent molecules, the infrared brightness results from light scattering and thermal emission of dust grains, entrained by gases vaporizing from the surface of the comet's nucleus. The production rate of dust is of significance for an understanding of the surface of the nucleus, and thus simple expressions are desirable which relate the dust production rate to the infrared brightness, obtained in a number of recent measurements (Ney, 1974; Rieke and Lee, 1974; Barbieri et al., 1974; Noguchi et al., 1974). The dust in the coma is also at the same time a hazard for fly-by mission as well as a scientific objective of particle impact detectors on such a space probe. These problems require the column density of dust particles in the vicinity of the nucleus, whereas the density in the far tail has been studied extensively by Finson and Probstein (1968).

The grain density of given size distribution is obtained from their trajectories, which can for the present purposes with sufficient accuracy be taken as parabolic, the parameters being determined by the ejection velocity v_a from the inner coma and the solar radiation pressure $\pi a^2 S q / c R_s^2$ (a = particle radius, S = solar constant, q = relative light-pressure cross-section integrated over the solar spectrum, R_s = heliocentric distance). The grain ejection velocity depends on the entrainment by gases of number density n, molecular weight m, radial velocity v and mass production rate per solid angle $\dot{M}_g = mnv/r^2$, where r is the distance from the comet's nucleus. v is given by the initial enthalpy of the gas $H = \gamma kT/(\gamma - 1)$ and by the degree of conversion of H into radial energy of motion $1/2\, mv^2$. H again depends on the transport processes near the comet's surface.

The Comet's Surface:

The nucleus of old comets like P/Encke or of young ones after perihelion seems to be covered by a crust of grains, too heavy to be entrained by vaporizing gases (mostly H_2O and CO_2). For P/Encke the thickness of this dust layer can be estimated from the heliocentric distance, where the vaporization becomes steady, viz. where $\dot{M}g \sim R_s^{-2}$, which is $R_s \lesssim 1$ AU (Mumma, 1975). If this comet of aphelion temperature $T_{aph} = (S/4\sigma)^{1/4} R_s^{-1/2}$ approaches the sun, the surface of the spinning nucleus, reradiating energy only at the light absorbing surfaces, rises according to $T_s = (S/2\sigma)^{1/4} R_s^{-1/2}$, and a temperature wave travels into the interior on the basis of radiative diffusivity $\varkappa_r = 16\sigma T_{eff}^3 a_e/D\, C_d$, where $D \approx 3$ g/cm^3 the specific weight of dust, $C_d = 0.8 \times 10^7$ erg/g x degr. its specific heat and $T_{eff} = \frac{1}{2}(T_s + T_{aph})$. For P/Encke at heliocentric distances $2 > R_s > 1$ the increase of surface temperature may well be described by $T_s - T_{aph} = 14.6 \times 10^6$ t, such that $t_1 = 10^7$ sec at 1 AU, where the thermal wave has reached the icy core at distance $\Delta r = \sqrt{4\varkappa_r t_1}$ from the surface. The effective particle size a_e in the dry dust layer is obtained by matching the steady heat flux density $16\sigma a T_s^4/\Delta r$ to the power necessary to explain the vaporization of the observed mass production rate $\dot{M}_g = 3.6 \times 10^3/R_s^2$ at a radius of the nucleus $r_N = 1$ km (Mumma, 1975). The result is $a_e = 10^{-2}$ cm and $\Delta r = 10$ cm (Michel and Nishimura, 1975).

From the ensuing balance of energy fluxes follows that practically all of the solar energy absorbed at albedo 0.1 by the comet's nucleus is reradiated into space by the nearly black surface of temperature $T_s = 330/\sqrt{R_s}\ {}^oK$. Only a small fraction of the incident

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power (1 %) penetrates by radiative diffusion into the interior, where it serves for vaporization. The vaporizing gases are slowly heated up when diffusing through the dry dust layer and escape finally at surface temperature T_s into the coma. The escape velocity from the crevices of the surface occurs under free molecular flow conditions and is given by $\dot{r}_s = \sqrt{kT_s/2\pi m} = 0.36$ km/sec at Encke's perihelion $R_s = 0.34$ AU where $T_s = 560^\circ K$. The upper limit of grain sizes, which can be swept out by this rarefied flow against gravitational attraction is given by (Hübner, 1970):

$$a_{max} = \frac{9 \quad \dot{M}_g \dot{r}_s}{16\pi \; GD\varrho \, r_N^3}$$

where G is the gravitational constant and $\varrho = 1$ g/cm^3 the mean density of the entire nucleus. At Encke's perihelion $a_{max} = 1$ cm. The smallest particle size is probably $a_{min} = 3 \times 10^{-6}$cm, corresponding to the smallest interstellar grains (Pecker, 1974).

The Gas Velocity:

Molecules escaping from the nucleus at velocity r into the coma collide with other molecules and are partly backscattered toward the surface where they pick up further energy, which is added to the flow. So, at a distance from the surface which is of the order of a mean free path, local sound velocity $v_l = \sqrt{\frac{2\gamma}{\gamma+1} \frac{kT_s}{m}}$ is established (Michel and Nishimura, 1975). In the subsequent continuum expansion, thermal energy is converted into directed flow energy with an efficiency which is dictated by the dependence of the terminal Mach number on Knudsen number $M_t = 2/\sqrt{Kn}$. The residual temperature after complete expansion is given by $T_t/T_s = [2(\gamma-1)/Kn + 1]^{-1}$, which implies for Encke at $R_s \doteq 1$ AU with $Kn = mv_1 r_N / 2\delta \dot{M}_g < 10^{-2}$, that practically all of the available enthalpy is converted into radial flow energy of terminal velocity $v = \sqrt{\frac{2\gamma}{\gamma-1} \frac{kT_s}{m}} \approx 1.1$ km/sec at $R_s = 1$ AU. From the continuity equation for steady, spherically symmetric and isentropic expansion follows readily that 90 % of the terminal gas velocity is reached within 6.2 r_N from the nucleus for a ratio of specific heats $\gamma = 1.33$ (H_2O and CO_2). Further change in the gas velocity due to heat addition by both thermal conduction from grains and by energetic photolysis products can be shown to be trifle in the vicinity of the nucleus, where also dust entrainment and acceleration by molecular drag takes place. Since grains require longer paths for drag acceleration than the gases themselves, the gas velocity can be considered constant.

Grain Trajectories:

Grains are subject to molecular drag and light pressure. The former acts strongly in the immediate vicinity of the nucleus. The force on dust particles of velocity v_a is under steady conditions

$$\frac{4}{3}\pi a^3 D v_a \frac{dv_a}{dr} = \pi a^2 (v - v_a)^2 mn$$

giving upon integration with v = const. and for $\dot{M}_g/aDvr_N < 0,2$ or $v_a \ll v$

$$v_a^2 = \frac{3}{2} \frac{\dot{M}_g v}{Dar_N} \left(1 - \frac{r_N}{r}\right) .$$

This gives for $r = \infty$ a terminal dust speed in excellent agreement with the more detailed numerical calculations of Finson and Probstein (1968) even for the case of high dust-to-gas ratios (N.B.: the effect of high dust content on the gas velocity has been neglected here). Thus, if dust particles of radius a = (0.2 - 1) x 10^{-4}cm have been ejected from Comet Kohoutek at R_s = 3.8 AU at velocity v_a = 0.5 km/sec (Grün et al., 1976) and nuclear radius r_N = 5 km (Rieke and Lee, 1974), it follows readily that the gas production rate must have been $\dot{M}_g$ = (0,5 - 2,5) x 10^6 g/sec sr, i.e. of the same order as the estimated dust production rate. By comparison, the gas production rate at a preperihelion distance of R_s = 0.64 AU has been estimated to be $\dot{M}_g$ = 2.7 x 10^6 g/sec x sr (Barbieri et al., 1974) and the gas molecules are probably mostly H_2O and CO_2 (corresponding to a formation temperature of the comet of 250 - 400° K in the equilibrium approximation). The high gas loss at large heliocentric distance where T_s = 144°K can only be explained by evaporation of very volatile compounds of low heats of vaporization. Hence, after its formation at T > 250°K, the nucleus was coated probably at T < 100°K by a hoar-frost of CH_4 or CO.

Since the terminal grain velocity is established effectively at $r_{N/r}$ = 20, where the grains also decouple from the gas, the solar

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radiation pressure results in strictly parabolic trajectories, the envelope of which is a paraboloid, characterized by $X_o - X = Y^2/4X_o$ where the apex distance from the nucleus on the connecting line toward the Sun

$$X_o = \frac{2}{3}\,\frac{a D c R_s^2}{S}\,\frac{v_a^2}{q}\,.$$

For all grain sizes $a > 0.4\ \mu$ the radiation pressure efficiency is $q = 1$, for $0.05 < a < 0.2\ \mu$ one finds $q \approx 15 \times 10^8\ a^2$ (compare Lamy, 1974). Inserting the appropriate expression for v_a in the approximation $v_a < v$, one obtains for P/Encke at $R_s < 1$ AU, i.e. where $\dot{M}_g \sim R_s^{-2}$, that grains of all sizes $a > 0.4\ \mu$ produce the same stand-off distance $X_o = 800$ km and the same enveloping paraboloid independent of heliocentric distance. So Encke should feature a sharp dust front, and a fly-by mission for dust impact studies should pass within 800 km in front of the nucleus. For Kohoutek one calculates with the post-perihelion data of Barbieri et al., (1974) at $R_s = 0.64$ AU and $\dot{M}_g = 2.5 \times 10^5$ g sec^{-1} sr^{-1} a sharp dust front at $X_o = 4500$ km.

This refocussing of initially divergent grain trajectories by radiation pressure takes place only if $v_a \sim 1/\sqrt{a}$, which is not the case if $\dot{M}_g/aDvr_N > 1$. Then all grains, obeying this unequality, are accelerated uniformly to the terminal gas velocity v. In this case, the particle size does not cancel anymore in the expression for X_o and we expect a diffuse dust front, as for Comet Kohoutek during approach to the Sun, where at $R_s = 0.64$ $\dot{M}_g = 2{,}7 \times 10^6$g/sec x sr with $a < 0.3\ \mu$.

The Surface Density of Grains:

The exact density profile, given by the parabolic grain trajectoties yields an untractable expression. For the present purpose, however, the mean surface density averaged over a slab at distance X from the nucleus yields for particles in the size range between a and a + da in the sunward direction from the nucleus

$$\frac{d\bar{N}}{da}\,da = \frac{dQ_a}{da}\,da\ \frac{\pi}{v_a X_o}\ \frac{\ln\left(\sqrt{\frac{X_o}{X}} + \sqrt{\frac{X_o}{X} - 1}\right)}{\sqrt{1 - \frac{X}{X_o}}}$$

where dQa/da is the number production rate in this size range. Polarization measurements (Noguchi et al., 1974 made these on the

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inner dust coma, where alignment of non-spherical particles by solar wind or radiation pressure is negligible) suggest a size distribution $dN/da \sim a^{-4}$, so that the infrared thermal emission for Comet Encke at 3.5 and 4.8 μ (Ney, 1974) can be evaluated with optical constants and grain temperatures from Lamy (1974). The result is that the mass ratio of dust-to-gas production rate is $\dot{M}_d/\dot{M}_g = 0.1$ for this Comet (Michel and Nishimura, 1975). With

$$\frac{dG_a}{da} = \frac{3\sqrt{a_{min}}}{8\pi D}\, a^{-4.5}\, \dot{M}_d$$

and the expression for v_a, one obtains then the mean surface density of dust grains on the sunward side

$$\bar{N} = \int_{a_m} \frac{d\bar{N}}{da}\, da = \frac{1}{8\pi}\, \frac{\dot{M}_d}{\dot{M}_g} \sqrt{\frac{2\dot{M}_g r_N a_{min}}{3D v}}\; \frac{\ln\left(\sqrt{\frac{x_c}{x}} + \sqrt{\frac{x_o}{x} - 1}\right)}{x_o\sqrt{1 - \frac{x}{x_c}}} \cdot \frac{1}{a_m^3} = 0.76\cdot 10^{-10}\, \frac{1}{a_m^3}$$

for Encke at perihelion at a distance of 400 km from the nucleus. Hence, one may expect about 10^5 impacts per cm^2 for particles in the range $0.1 < a < 1\mu$, detectable by an impact detector, but only $10^{-4} cm^{-2}$ for grains with radius $a_m > 0.1$ mm, which can cause serious damage.

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