

VI. PULSE TIMING

RADIO TIMING OBSERVATIONS

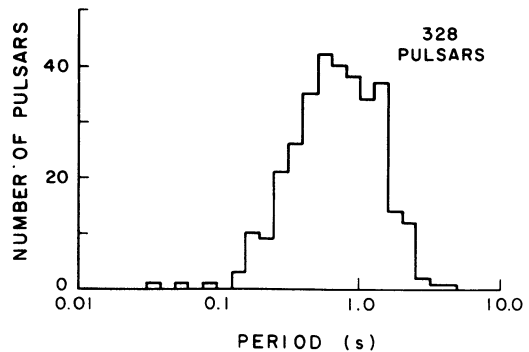
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1. INTRODUCTION

The most recent review of pulsar timing observations (and pulsar astronomy generally) is contained in the monographs by Manchester and Taylor (1977) and Smith (1977). This paper reviews developments in radio timing observations since the time of writing these monographs (~ 1976). In Section 2 results based on the secular variation of pulsar period are reviewed and timing irregularities are discussed in Section 3. Results for binary pulsars, three of which are now known, are reviewed by Taylor (1981).

Two extensive surveys for new pulsars have been made since 1976. The second Molongo survey (Manchester et al. 1978) surveyed the whole sky south of $\delta = +20^\circ$ and detected 154 new pulsars. High galactic latitudes in the northern part of the sky have been surveyed by Damashek et al. (1978, 1980) resulting in the discovery of 25 pulsars. These surveys more than doubled the number of known pulsars, from 149 to 328. This large increase significantly improves the data base available for statistical studies. The distribution in period of all known pulsars is shown in Fig. 1. This distribution is not significantly affected by observational selection except, perhaps, for short-period pulsars (say $P < 0.1$ s), especially those with high dispersion measure. Searches are now relatively complete over the whole sky to a

Fig 1. Observed distribution of pulsar periods



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given flux density level (~ 15 mJy mean). This level is unlikely to be substantially lowered, except for relatively small regions of the sky, for a number of years.

Observed periods range from $0^{\text{s}}.033$ to $4^{\text{s}}.308$ with a median value of $0^{\text{s}}.670$. With the increased data sample, the "1-second gap" evident in earlier distributions (eg. Manchester and Taylor 1977) has essentially disappeared.

2. SECULAR PERIOD VARIATIONS

Using observations of pulse arrival time with a data span in excess of one year, the pulse phase can be fitted with a model yielding an accurate pulse period, period first derivative, possibly also higher order derivatives and the pulsar position. With a data span of several years, proper motion terms can also be included. Recent observations at Arecibo (Gullahorn and Rankin 1978, 1980), the Five College Radio Astronomy Observatory (Helfand et al. 1980), Green Bank (Backus et al. 1980) and Parkes (Newton et al. 1980) have resulted in full timing solutions (period, period derivative and position) for 160 pulsars.

Pulsar positions obtained from timing analyses are relatively accurate and often have an error dominated by uncertainty in orientation of the solar-system ephemeris coordinate system ($\sim 0''.1$ arc). Recently, positions have been measured by direct interferometry using the U.S. Very Large Array (VLA) for 24 pulsars north of $\delta = -10^\circ$ having relatively long timing data spans (Fomalont et al. 1980). The results of a comparison of the VLA and timing positions are summarized in Table 1. These results indicate that the errors quoted for timing

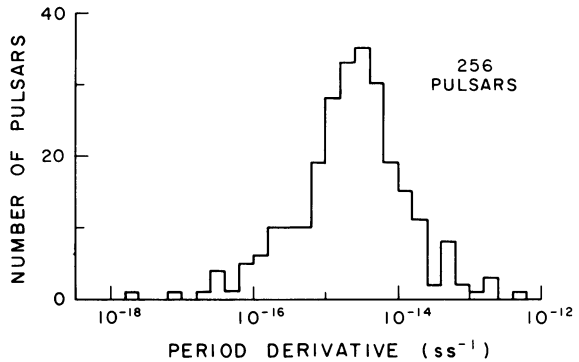
Table 1. Comparison of VLA and Timing Positions

	R.A.	Dec.
VLA r.m.s. error	$0''.36$	$0''.51$
Timing r.m.s. error	$0''.20$	$0''.47$
VLA-Timing: r.m.s. residual	$0''.36$	$0''.66$
Mean Ratio: Residual/Combined Error	0.76	0.57

positions are in general realistic. The observed residual vectors are not consistent with a simple rotation of the timing coordinate system.

Proper motions from timing observations have been published by Gullahorn and Rankin (1978) and Helfand et al. (1980). Unfortunately, except for a few pulsars, the observed proper motions are too small and/or irregularities in the intrinsic pulsar period are too large (Section 3) for these results to be reliable with data spans available at present. Direct interferometric observations of proper motions are given by Lyne (1981).

Fig. 2. Observed distribution of period first derivatives



In addition to the 160 new period derivatives obtained from full timing solutions as mentioned above, Ashworth and Lyne (1980) have obtained period derivatives for a further 30 pulsars from partial timing solutions. This brings the total number of known period derivatives to 256. The distribution of these 256 values, shown in Fig. 2, covers over five orders of magnitude from 2.0×10^{-18} to 4.2×10^{-13} and has a median value of 2.5×10^{-15} . For full timing solutions there is no selection against either small ($> 10^{-13}$) or large period derivatives, so the observed distribution (Fig. 2) represents the actual distribution for the sample of known pulsars. The gap in the distribution at $(2-4) \times 10^{-14}$ discussed by Ferguson (1979) is of reduced significance with the larger sample (see also Fig. 6).

In standard models the evolution of pulsar periods is described by $\dot{\Omega} = -K\Omega^n$ where $\Omega = 2\pi/P$, P is the pulsar period, K is a constant and n is the braking index. For a stable dipole field $n = 3$ and for $\Omega_{\text{birth}} \gg \Omega$, the pulsar age (characteristic age) is given by $\tau = \Omega / 2\dot{\Omega} = P / 2\dot{P}$. The magnetic field strength at the surface of the neutron star $B_0 = K_1(P\dot{P})^{1/2}$ where K_1 is a constant related to the radius and moment of inertia of the neutron star (Manchester and Taylor 1977). The distribution of B_0 for the 256 pulsars with known period derivative is shown in Fig. 3. For most pulsars, B_0 lies within a decade of 10^{12} G; the lowest value (2.3×10^{10} G) is for the binary pulsar PSR 1913+16 and the median value is 1.2×10^{12} G. The distribution of characteristic ages (Fig. 4) covers nearly seven orders of

Fig. 3. Distribution of magnetic field strength at the surface of the neutron star computed assuming a dipolar magnetic field, neutron star radius 10^6 cm and moment of inertia 10^{45} g cm²

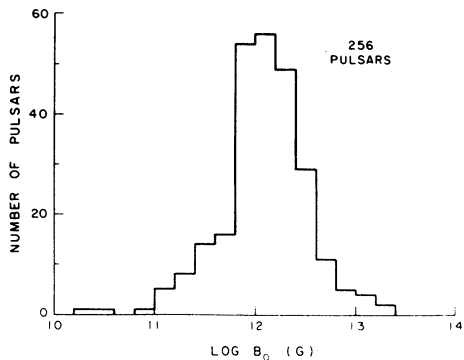
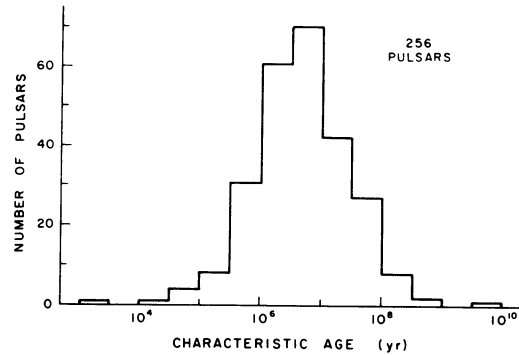


Fig. 4. Distribution of pulsar characteristic age $\tau = P/(2\dot{P})$



magnitude from 10^3 to 3.5×10^9 years with a median value of 4.9×10^6 years. There is good evidence that the larger characteristic ages are gross over-estimates of the true age. The most plausible reason for this is that the magnetic field decays significantly on a timescale short compared to the larger characteristic ages. Alignment of the magnetic axis with the rotational axis may also occur, although it is not clear that this will result in a decreased period derivative.

A linear histogram of characteristic ages (Fig. 5) shows an approximately exponential decrease in the density of pulsars with increasing τ . This implies that many pulsars live for only a few million years and/or that τ is not a linear function of the true age. Provided significant decay of the magnetic field does not occur on time scales less than 10^6 years, the mean equivalent lifetime (lifetime if all pulsars died at the same age) is given by $(256/45) \times 10^6 \sim 6 \times 10^6$ years. This will be an over-estimate of the true mean lifetime if (as seems likely) significant field decay occurs on timescales of less than 10^6 years. This lifetime estimate is in good agreement with that based on kinematic arguments (Lyne 1981).

The distribution of known pulsars on the $P-\dot{P}$ plane is shown in Fig. 6. This diagram has the same general character as earlier versions of it (eg. Lyne et al. 1975) with a few exceptions. There are now a few pulsars in the lower-left part of the diagram. These

Fig. 5. Distribution of characteristic ages less than 2×10^7 years, plotted on a linear scale

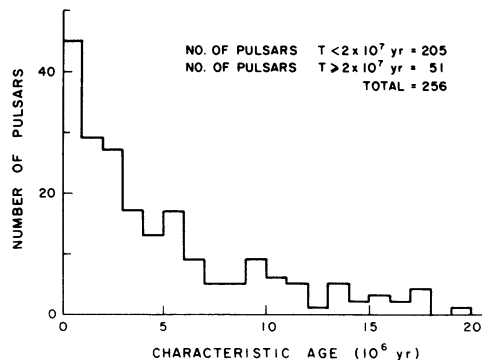
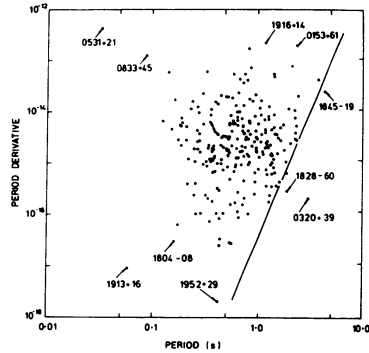


Fig. 6. Distribution in the $P-\dot{P}$ plane of the 256 pulsars with known derivatives. The sloping line is given by $\dot{P}P^{-5} = \text{constant}$ and represents an empirically determined locus for cutoff in pulsed radio emission (Lyne et al. 1975).



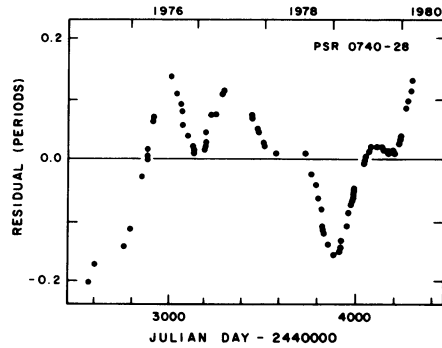
pulsars must have a different evolutionary path from the bulk of other pulsars, probably being born with a weaker magnetic field. If they have typical lifetimes, they must also die well before the cutoff line is reached. A number of pulsars, in particular PSR 0320+39, lie to the right of the cutoff line, also indicating the importance of other factors in determining the cutoff of pulsed emission. Fujimura and Kennel (1980) and Phinney and Blandford (1980) have analysed the $P-\dot{P}$ distribution based on a sample of about 110 pulsars. Both conclude that for $n > 2.5$, the decay time for the magnetic field is less than 10^6 years and maybe as short as a few $\times 10^5$ years.

It is in principle possible to determine the braking index from timing observations: $n = \frac{\ddot{\Omega}}{\dot{\Omega}^2}$. It is clear that, apart from the Crab pulsar, (Groth 1975), published values of n are dominated by random period irregularities and do not represent the underlying secular period decay (Gullahorn and Rankin 1980; Cordes and Helfand 1980). It seems unlikely that significant values of n will be obtained for pulsars other than the Crab pulsar in the foreseeable future.

3. PERIOD IRREGULARITIES

Pulsars are extremely good clocks; the stability of their periods is often in excess of a part in 10^{11} over intervals of a year or more. In most pulsars, however, irregularities in period are readily detectable. These irregularities fall into two distinct classes: (i) apparently continuous random walks and (ii) isolated discontinuities in period (glitches). A typical example of a pulsar exhibiting the random walk type of timing noise is shown in Fig. 7. Cordes and Helfand (1980) have defined an "activity" parameter $A = \log(\sigma_R/\sigma_{R,Crab})$, where σ_R is the r.m.s. phase residual from a fit of period, period derivative and position to a data span of length ~ 1000 days, in order to provide a measure of the strength of the timing irregularities relative to those in the Crab pulsar. In Fig. 8 this activity parameter is plotted with respect to period derivative showing that there is a significant correlation between these two quantities; the correlation coefficient is ~ 0.5 .

Fig. 7. Plot of phase residuals after a fit of position, period and period derivative to pulse arrival time data for PSR 0740-28 obtained at Tidbinbilla (cf. Manchester et al. 1976)



The observed timing irregularities can be modelled as a random walk process in phase, frequency or frequency derivative (Groth 1975; Cordes 1980). Cordes and Helfand (1980) have analysed observations for 11 pulsars having well-sampled data spans of length > 1500 days in order to determine the character of the timing noise. The results, summarized in Table 2, show that data for two pulsars are consistent with phase noise (P), four and maybe seven pulsars are consistent with frequency noise (F) and two are consistent with derivative noise (D). Activity parameters tend to be large for pulsars with D-type noise and smaller for those with P-type noise. The strength parameters are given by $R \langle \delta\phi^2 \rangle$, $R \langle \Delta\nu^2 \rangle$ and $R \langle \Delta\dot{\nu}^2 \rangle$ for P, F and D noise respectively where R is the occurrence rate of steps in the random walk, ϕ is the pulse phase and $\nu = \Omega/2\pi$. A firm lower limit on R is given by the inverse of the shortest data span analysed $R > (500^d)^{-1}$. The fact that individual steps cannot in general be resolved suggests that $R > (\sim 10^d)^{-1}$, at least for phase and frequency noise. For frequency noise one can place an upper limit $R < (\sim 0^d.1)^{-1}$ if it is assumed that $\Delta\nu > 0$ always.

The second class of timing irregularity, period discontinuities or glitches, are characterized by an abrupt (to date, always unresolved) increase in Ω (decrease in period) followed by an increase in $|\dot{\Omega}|$ or \dot{P} . Unambiguous events have so far been observed in four pulsars; the parameters for these events are summarized in Table 3. Those in the Vela pulsar, PSR 0833-45, are by far the largest and those in the Crab pulsar, PSR 0531+21, the smallest. Weaker events

Fig. 8. Timing activity parameter (see text) plotted against period derivative for 50 pulsars (Cordes and Helfand 1980)

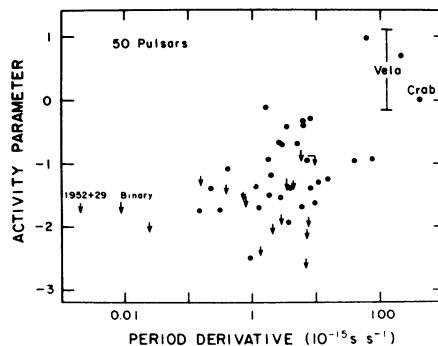


Table 2. Random Timing Noise (after Cordes and Helfand 1980)

PSR	Activity Parameter	Type of Random Walk	Strength
			$(10^{-14} \text{ s}^{-1})$
1133+16	-1.9	P	1.5 ± 0.9
2217+47	-1.5	P	16 ± 9
			$(10^{-27} \text{ Hz}^2 \text{ s}^{-1})$
0329+54	-1.1	F	7 ± 4
0531+21	0.0	F	66000 ± 30000
1508+55	-0.7	F	10 ± 6
2016+28	-1.8	F	0.20 ± 0.12
1915+13	-1.0	(F)	110 ± 70
2002+13	-0.9	(F)	1.0 ± 0.7
2020+28	-1.5	(F)	2.0 ± 6
			$(10^{-40} \text{ Hz}^2 \text{ s}^{-3})$
0611+22	+1.0	D	1300 ± 900
0823+26	-0.1	D	2.0 ± 1.3

$(\Delta\Omega/\Omega \sim 10^{-9}-10^{-10})$ have been reported for several other pulsars (Manchester and Taylor 1974, Gullahorn and Rankin 1980). Cordes and Helfand (1980) have shown that these are all consistent with fluctuations in a random walk process and hence are not distinct events. It is possible, however, that at least in some pulsars, the same basic mechanism is responsible for both random timing noise and glitches, with the glitches representing a long tail on the distribution. The Parkes pulsar timing program (Newton et al. 1980) represents about 180 pulsar-years of timing observations, comparable to that obtained in all previous timing observations. During this program one or possibly two glitches were observed suggesting that in "normal" pulsars (ie. excluding the Crab and Vela pulsars) the rate of these events is $\sim (100 \text{ yr})^{-1}$. For the Vela pulsar, four events have been observed in 12 years, giving an average rate of $(\sim 3 \text{ yr})^{-1}$.

The post-jump behaviour is commonly interpreted in terms of the "two-component" model for neutron stars, first proposed by Baym et al. (1969). In this model the pulses are attached to a rigid outer crust and the interior of the star consists (largely) of superfluid neutrons. Because of the weak frictional coupling between these two components, angular momentum transfer takes place on long time scales. The pulsar frequency (post-jump) is given by

$$\Omega(t) = \Omega_0(t) + \Delta\Omega[1 - Q(1 - \exp(-t/\tau_d))]$$

Table 3. Pulsar Timing Discontinuities

PSR	P(s)	J.D.-2440000	$\Delta\Omega/\Omega$	Ref.
0531+21	0.033	494.1±0.9	$(9\pm 4) \times 10^{-9}$	1
		2447.4±0.1	$(37.2\pm 0.8) \times 10^{-9}$	2
0833-45	0.089	280±4	$(2.33\pm 0.02) \times 10^{-6}$	3,4
		1192±7	$(1.97\pm 0.01) \times 10^{-6}$	5
		2683±3	$(1.97\pm 0.01) \times 10^{-6}$	6
		3692±11	$(3.06\pm 0.01) \times 10^{-6}$	7
1325-43	0.532	3590±24	$\sim 1.2 \times 10^{-7}$	8
1641-45	0.455	3390±62	$(1.91\pm 0.01) \times 10^{-7}$	9

References: 1. Boynton et al. (1972). 2. Lohsen (1975). 3. Radhakrishnan and Manchester (1969). 4. Reichley and Downs (1969). 5. Reichley and Downs (1971). 6. Manchester et al. (1976). 7. Downs et al. (1978). 8. Newton et al. (1980). 9. Manchester, Newton et al. (1978).

where $\Omega_0(t)$ is the pre-jump frequency (including the secular slow-down), Q is a parameter which measures the extent to which the pulsar frequency relaxes back toward $\Omega_0(t)$ after the jump $\Delta\Omega$, and τ_d is the time constant for this relaxation. For light neutron stars most of the moment of inertia is thought to be contained in the crust and $Q \sim 0$; for heavy neutron stars the superfluid component dominates and $Q \sim 1$. A fit of this two-component model to about five years of timing data for the Vela pulsar, including two glitches, is shown in Fig. 9. Compared to the model phase contribution, the observed resi-

Fig. 9. Phase residuals from a fit of the two-component model for post-jump timing behaviour to five years of timing data for the Vela pulsar (cf. Manchester et al. 1976). The data span includes two jumps, marked with arrows on the lower axis.

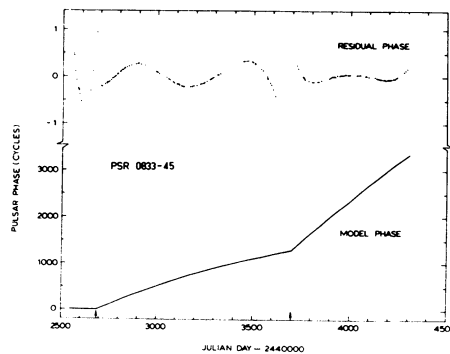


Table 4. Post-jump Decay Parameters

PSR	Event Date (J.D.-2440000)	Fitted Data Span (J.D.-2440000)	Q	τ (d)
0833-45	2690	2563-4440	1.0±0.3	1300±200
"	"	2563-2860	0.188±0.002	381±4
"	3700	2563-4440	0.37±0.07	1400±400
"	"	3564-3892	0.124±0.004	367±16
1641-45	3400	2563-4440	0.91±0.03	36500*

* Not solved for in the least-squares fit.

duals are small, showing that the two-component model represents the data relatively well. Nevertheless there are clear systematic residuals from the fit showing that other noise processes are present. Parameters from this fit, from separate fits to each of the two jumps, and from a fit to the PSR 1641-45 event are given in Table 4. It is clear that the derived parameters depend strongly on the fitted data span demonstrating that the model is not adequate. Fits to the longer data spans are more influenced by the random noise processes and it is likely that the more local fits give a better estimate of the parameters Q and τ_d . If this is the case, the two Vela events have rather similar parameters. Further analysis is clearly required.

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