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BOOK REVIEWS

ACZÉL, J. (ed.), *Functional equations: history, applications and theory* (D. Reidel, Dordrecht 1984), pp. ix + 244, Dfl. 125.

All professional mathematicians, of whatever persuasion, have come across functional equations, even although their exposure may have been restricted to the simplest cases such as those connected with logarithms, exponentials or gamma functions. However, by the time the functions concerned are defined on some abstract algebraic structure or are set-valued, many of us would be in a foreign land. Likewise, the variety of areas in which functional equations are being applied may come as a surprise to the uninitiated.

The object of the present book is to bring together some of the history of the subject, some of the most recent developments in the theory (including some work unpublished elsewhere) and a few of the applications of functional equations. According to the first contribution of the editor, the book "originated from talks and from a panel discussion at the Functional Equations section of the Second World Conference on Mathematics at the Service of Man(kind). It contains selected talks (as 'papers') and all panel contributions (as 'essays')." The essays come first and their authors form a subset of the authors of the papers. Indeed, some of the essays give a brief description of material which is covered in a later paper. On the other hand, some of the essays are little more than short anecdotes or expressions of opinion while some go to the opposite extreme and are indistinguishable in format from the papers. In this connection, possibly the most daunting page of all for a casual reader would be found among the essays where page 38 consists almost entirely of a single system of equations and an explanation of the plethora of symbols appearing in the system. For the reviewer, the most interesting essay is that by Dhombres entitled "On the historical role of functional equations", which discusses the work of Cauchy and d'Alembert among others. Those parts of the essays (and also the papers) which are in the nature of surveys contain a number of extracts from the writings of the masters including Abel, Cauchy, Hilbert, Daniel Bernoulli and "the great and vocal 'pure' mathematician G. H. Hardy".

As regards the papers, possibly the best approach in the space available is to indicate the various subject areas covered. Dhombres leads off and gives a good survey of some recent applications to topics such as chronogeometry (a subfield of relativity theory!), turbulence in water pipes, ideal gases and the geometry of Banach spaces. Analysis is represented further by two papers. The first, by Tsutsumi and S. Haruki, describes how the concept of hypoellipticity, which is of great importance in the theory of partial differential equations, can be used to obtain information about the regularity of solutions of a class of functional equations. The second, by H. Haruki, generalises a theorem of Nevanlinna and Pólya relating two families of analytic functions of a complex variable via a unitary matrix of constants. Algebra is represented by papers on groupoids with Δ-kernels by Krapež (in a similar area to his earlier essay) and Kurepa's functional equation on Gaussian semigroups by Ebanks. However, the major contribution in this area is a long paper by Reich and Schwaiger concerned with the linear or algebraic independence of families of additive and multiplicative functions, the latter being solutions of the Cauchy equations $f(s+t)=f(s)+f(t)$ and $f(s+t)=f(s)f(t)$ respectively. There is a brief note by Alsina on the truncation of probability distribution functions before we are plunged into the realms of information theory and mathematical economics. Aczél's paper was the main paper of the conference and reviews various measures of inequality and some of the areas in which these measures are used. Particularly important is the so-called Shannon entropy which is characterised axiomatically in the paper by Gehrig. Here and in his earlier essay, Gehrig discusses the concept of a concentration measure and shows that the only such measure satisfying a certain collection of "economically meaningful assumptions" is a multiple of the Shannon entropy. The last three papers, by Thibault, Sklar and Clerc and Hartmann, take us into the area of dynamical systems and consider such matters as invariant curves and flows.

Inevitably in a collection of this nature, the styles differ and the level of mathematical background required to appreciate the contents also varies considerably from author to author. A few misprints were detected and the English language takes a battering on odd occasions, notably in the appearance of words such as "modellized". In conclusion, it can be said that this is a book for dipping into, with something for everyone. If the reader's appetite for a particular topic is whetted particularly, it should be possible to delve further by using the references cited liberally at the end of many of the the contributions.

ADAM C. McBRIDE

BARNES, B. A., MURPHY, G. J., SMYTH, M. R. F. and WEST, T. T., *Riesz and Fredholm theory in Banach algebras* (Research Notes in Mathematics 67, Pitman, 1982), 123 pp, £7.95.

Of the various classes of Banach space operators which have been singled out for special study, the compact operators are arguably the most important and best understood. The classical Riesz–Schauder theory shows that their spectral structure parallels that of finite matrices while, from an algebraic point of view, they form a natural ideal in the algebra of all bounded linear operators, the corresponding quotient algebra being the so-called Calkin algebra. Closely related to the compact operators are the Fredholm operators which, although originally defined spatially, may alternatively be characterized as having invertible images in the Calkin algebra. Another cognate class of operators, less central but still of interest, is that consisting of the Riesz operators. They may be defined either as having the same spectral structure as compact operators or, equivalently, as having images in the Calkin algebra with zero spectrum.

The aim of the present monograph is to examine how the ideas of Fredholm and Riesz theory can be developed in the context of a general Banach algebra A. This strategy is to find appropriate definitions for Fredholm and Riesz elements of A and then show that the results of operator theory have analogues in this algebraic setting. The first difficulty encountered in such a programme is that there is no satisfactory definition of a compact element in a general Banach algebra. However, by first considering invertibility modulo the socle when A is semi-simple and then extending to the general case, it is possible to obtain a sensible definition of a Fredholm element of A. Counterparts to the main theorems of classical Fredholm theory, including the index and punctured neighbourhood theorems, can now be proved. After developing their version of algebraic Fredholm theory, the authors turn to Riesz theory in Banach algebras. Elementary results can be obtained by considering elements with zero spectrum modulo an arbitrary closed ideal K but, to take things further, additional restrictions on K are needed. Not surprisingly, these ideas have some special features when A is a C^*-algebra. In that case, it is possible to give satisfactory definitions of finite rank and compact elements and, reflecting the situation for operators on Hilbert space, each Riesz element can be decomposed as the sum of a compact and a quasinilpotent element. Whether such a decomposition is always possible for Riesz operators on Banach spaces is the main problem left open in the subject at an operator-theoretic level. Unfortunately, the algebraic techniques developed here do not appear to shed much light on it.

The book has been well organized and each chapter ends with an extensive section of notes and comments. The authors have made a point of giving plenty of examples to illustrate their results, many of which appear here for the first time, and they include a chapter of applications. Finally, although the main aim is to develop the theory at an algebraic level, the book begins with an account of the necessary background material from operator theory. This in itself is a good survey, which in fact contains both classical results and some interesting new material.

T. A. GILLESPIE

SMITH, K. T., *Primer of modern analysis* (Springer-Verlag, 2nd ed. 1983), xv+446 pp. DM 97.

The original version of this book was published by Bogden & Quigley in 1971; this slightly extended version now appears in the Springer series "Undergraduate Texts in Mathematics". It is