DYNAMICAL SCREENING OF INTERACTIONS IN GRAVITATING SYSTEMS AND THE EPHEMERIS TIME

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The concept of the screening of interparticle interactions has its origin in electrolyte and plasma theories. The most known example is the Debye-Hückel screening of the potential of the resting test charge provided with the availability of charges of the opposite sign and the total electroneutrality of the plasma. When this charge is moving, the static Debye screening decreases and an anisotropic dynamic screening develops due to excitation of waves of charge density. As a result, an effective potential of the positive moving ion becomes alternating with a characteristic length of space oscillations of order of the Debye length (Peter, 1990). Such dynamic screening is due to perturbations of charges of both signs by the varying field of moving test charge. This effect does not call for an electroneutrality of the system and is associated with the long-range character of the Coulomb interaction potential only.

As a result of the similar long-range character of the Newtonian gravitational potential, analogous effects are to be expected for a gas of gravitating bodies (Binney and Tremaine, 1987; Saslaw, 1987). According to Saslaw (1987, Ch.15), gravitational screening is caused by waves of the gas density generated by a moving gravitating test body. A gravitational potential of the unperturbed constant density of the homogeneous gas can be neglected (the so-called "Jeans Swindle"). Zones of reduced densities imitate negative mass densities and, as a result, the negative layers screen effectively outer bodies from the test moving body. In the same manner, zones of augmented densities generate additional gravitational interactions which strengthen the action of an effective force field of the test body on outer bodies. So, we get space oscillations of the effective potential of a moving body with a characteristic length of order of the Jeans length.

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Here, we discuss briefly a more rigorous analysis (Bashkirov and Vityazev, 1996) of the above scenario and the observable consequences of such effect for Solar planetary System.

The formal analysis is based on the linearized Vlasov equation for the time-dependent perturbation $f_1(\mathbf{r}, \mathbf{v}, t)$ of the distribution function of the gas of gravitating bodies over coordinates $\mathbf{r}$ and velocities $\mathbf{v}$

$$\frac{\partial f_1(\mathbf{r}, \mathbf{v}, t)}{\partial t} + \mathbf{v}\frac{\partial f_1(\mathbf{r}, \mathbf{v}, t)}{\partial \mathbf{r}} + \nabla \Phi \frac{\partial f_0(\mathbf{r}, \mathbf{v}, t)}{\partial \mathbf{v}} = 0 \tag{1}$$

where $f_0(\mathbf{v}) = \rho_0/(2\pi\tilde{v}^2)^{3/2} \exp\{-v^2/2\tilde{v}^2\}$, ρ_0 and $\tilde{v}$ are the mean mass density and thermal velocity, correspondingly.

The self-consistent gravitational potential taking into account the mass density perturbation $\int d^3v f_1(\mathbf{r}, \mathbf{v}, t)$ and a test gravitating body of mass m_1 inserted into the system at the instant t_0 at the point $\mathbf{r}$, with the velocity $\mathbf{u}$, is determined by the Poisson equation:

$$\nabla^2 \Phi(\mathbf{r}, t) = -4\pi G \int d^3v f_1(\mathbf{r}, \mathbf{v}, t) - 4\pi G m_1 \delta(\mathbf{r} - \mathbf{u}(t - t_0)) \tag{2}$$

For the sake of simplicity, the masses of all bodies are taken to be identical. Such a set of equations is widely used for the description of relaxation processes and stability analysis of gravitating systems (Binney and Tremaine, 1987; Saslaw, 1987).

Then, we solve these equations and get the Fourier transform of the perturbed gravitational potential in the form (Bashkirov and Vityazev, 1996)

$$\Phi(\mathbf{k}, \omega) = \frac{8\pi^2 G m_1}{k^2 \varepsilon_g(\mathbf{k}, \omega)} \delta(\omega - \mathbf{k}\mathbf{u}). \tag{3}$$

Here,

$$\varepsilon_g(\mathbf{k}, \omega) = 1 - \frac{k_J^2}{k^2}\left[1 + F\left(\frac{\omega}{\sqrt{2}k\tilde{v}}\right)\right] \tag{4}$$

where $k_J = \left(4\pi G\rho_0/\tilde{v}^2\right)^{1/2}$ is the Jeans wave number and $F(z) = i(\pi)^{1/2} z \times \exp\{-z^2\} \operatorname{erfc}(-iz)$.

The function $\varepsilon_g(\mathbf{k}, \omega)$ is the gravitational susceptibility of the system. This function determines the response of a gravitating system to a gravitational perturbation(Binney and Tremaine, 1987; Saslaw, 1987; Kukharenko *et al*, 1994), and its zeroth value gives the dispersion equation for waves of density of the system. The complex form of the function F accounts for the Landau damping of the collective excitations (Binney and Tremaine, 1987; Saslaw, 1987; Kukharenko *et al*, 1994).

If the test body is one of the system bodies, the Fourier transform (3) of its interaction potential is to be averaged over all the possible values

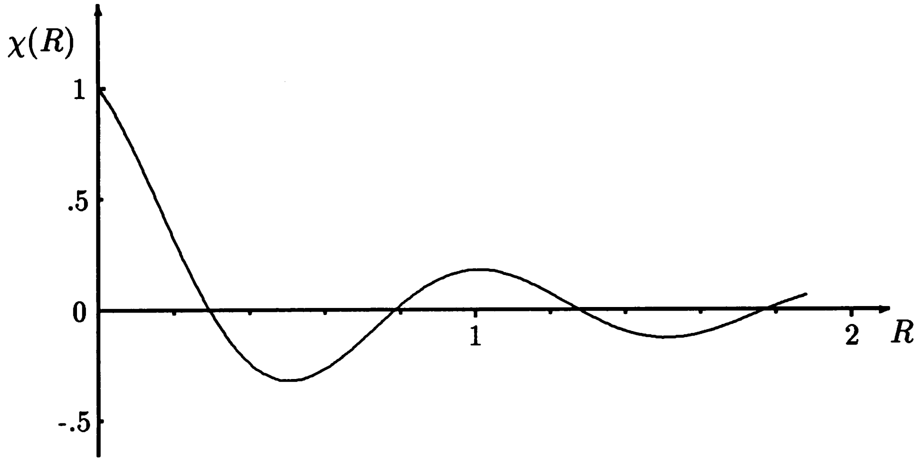

Figure 1. Screening factor $\chi(R)$ of the effective interaction potential $\tilde{\Phi}(r)$

and directions of the velocity $\mathbf{u}$ with the use of the Maxwellian distribution function $f_0(\mathbf{u})$. Then, as a result of the averaging and reverse Fourier transformation of Eq.(3) we get (Bashkirov and Vityazev, 1996)

$$\tilde{\Phi}(\mathbf{r}) = \frac{Gm}{r}\chi(R) \tag{5}$$

where $R = r/\lambda_J$, ($\lambda_J = 2\pi/k_J$). The function $\chi(R)$ has been obtained as a result of analytical and numerical integrations (Bashkirov and Vityazev, 1996). It is represented in the graphical form in Fig. 1.

Thus, we have found that the renormalized (or effective) interaction potential in the Maxwellian gravitating system not only decays faster than Newtonian potential, but it also becomes oscillating.

The final computational results for $\chi(R)$ are represented at Fig.1. The resulted graphic can be approximated as

$$\chi(R) = e^{-\alpha R}\cos(\beta R), \tag{6}$$

where the constants α, β take the values $\alpha = 1.9$ and $\beta = 6.0$. Notice that such approximation is impaired after some oscillation periods as the periods are scaled down and the decay constants α decrease slightly with the distance R.

The dynamic screening effect is manifested itself in such systems as the Solar planetary system as well. Among other things, the attractive Sun potential's variation from the Newtonian potential is to cause an increase of the Keplerian periods of the planets. In this regard, the Earth is a sensitive detector of the Sun potential's properties, as inhabitants of the Earth have determined to date the Keplerian period of their planet with a high degree of precision.

As the Keplerian period $T \propto 1/\sqrt{\chi}$, the correction for non-newtonian potential is $\Delta T/T = (1-\sqrt{\chi})/\sqrt{\chi}$.

We estimate now the value χ for the Sun in the system of gravitating stars of the Galaxy. The Jeans length is estimated as

$$\lambda_J = \sqrt{\pi}\left[\frac{\tilde{v}}{1.55\cdot 10^6}\right]\left[\frac{G}{6.7\cdot 10^{-8}}\right]^{-1/2}\left[\frac{\rho_0}{1.9\cdot 10^{-23}}\right]^{-1/2}\text{cm} = 2.5\cdot 10^{21}\,\text{cm} \tag{7}$$

where we use the known values of the thermal velocity $\tilde{v}$ and the average matter density ρ_0 in the vicinity of the Sun.

For the Earth orbit $\Delta T_\oplus/T_\oplus \simeq 5.7\cdot 10^{-9}$ or $\Delta T_\oplus \simeq 0.18$ sec/year.

The sign and the order of this value and its annual variation correspond to the correction for the ephemeris time of the Earth revolution.

Account for the screening of the Sun potential cause to much greater corrections to Keplerian periods of giant planets. In particular, we get for Jupiter $\Delta T_J \simeq 11$ sec/period, for Saturn $\Delta T_S \simeq 50$ sec/period, for Uranus $\Delta T_U \simeq 290$ sec/period, for Neptune $\Delta T_N \simeq 890$ sec/period and, for Pluto $\Delta T_P \simeq 1700$ sec/period.

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