SOLUTIONS

P19. Let $a_1, \ldots, a_n; b_1, \ldots, b_n$ be real numbers and

$$\alpha = n^{-\frac{1}{2}} \sum a_i, \quad \beta = n^{-\frac{1}{2}} \sum b_i .$$

Then

$$\sum a_i^2 \sum b_i^2 - (\sum a_i b_i)^2 \geq \alpha^2 \sum b_i^2 - a\alpha\beta \sum a_i b_i + \beta^2 \sum a_i^2$$

with the sign of equality if and only if the three row vectors

$$a = (a_1, \ldots, a_n),\ b = (b_1, \ldots, b_n),\ e = n^{-\frac{1}{2}} (1, \ldots, 1)$$

are linearly dependent.

H. Schwerdtfeger

Outline of solution by the proposer. Consider the determinant $A'A$ where the prime indicates transposition and A is the $n \times 3$-matrix with the columns a', b', e'. (Also solved by E.L. Whitney and M. Harrow)

P21. It is possible to metrize an affine plane (with preservation of its natural topology) in such a way that on every affine line the metric is Euclidean, but that the whole plane does not become Euclidean, viz. by a Minkowski metric. Similarly, is it possible to metrize an affine space A^n $(n>2)$ in such a way that in every affine plane the metric is Minkowskian, but that the whole space is not Minkowskian?

H. Helfenstein

Solution by the proposer. The required metric has the property that on every line in A^n a Euclidean metric is induced (or, in particular, every affine midpoint would become a metric midpoint). It is well known that this property together with the topological equivalence causes the metric to be Minkowskian. Hence the answer is negative.

P22. For the polynomial $x^{13} + x + 90$, find a quadratic factor with rational coefficients.

Ron Graham

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