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CORRECTION AND COMMENT CONCERNING "ON DERIVATIONS AND HOLOMORPHS OF NILPOTENT LIE ALGEBRAS"

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The examples on p. 44 of metabelian Lie algebras L_1 and L_2 are isomorphic over certain base fields contrary to the claim in the paper. More specifically, the error lies in the incorrect statement that the rank of $\mathrm{ad}\, x$ for any x in L_1 is either zero or is at least 3. This statement does hold over $\boldsymbol{Q}$, but if τ is a scalar satisfying $\tau^2 - \tau - 1 = 0$ then $\mathrm{ad}\,(x_1 + \tau x_2 + \tau^2 x_3 + x_4 + \tau^2 x_5)$ has rank 2.

These examples with holomorphs isomorphic to $H\langle 5, 4\rangle$ were important in that they were to demonstrate the existence over any field of characteristic 0 of non-isomorphic metabelian Lie algebras with isomorphic derivation algebras and holomorphs. Actually, we have since discovered that there is, up to isomorphism, only one Lie algebra over $\boldsymbol{C}$ whose holomorph is isomorphic to $H\langle 5, 4\rangle$.

Thus, although Theorem 4.5 is technically still justified by the examples, we feel it important now to point out that there do exist metabelian Lie algebras with isomorphic derivation algebras and holomorphs which are nevertheless non-isomorphic even under extensions of the base field. We present the following metabelian Lie algebras M_1, M_2, over any field of characteristic 0; these algebras have isomorphic derivation algebras and have holomorphs isomorphic to $H\langle 6, 4\rangle$:

M_1 has basis $x_1, \cdots, x_{10}$ with the multiplication table

$$[x_1, x_3] = x_8, \quad [x_1, x_5] = x_7, \quad [x_1, x_6] = x_{10}\,,$$
$$[x_2, x_4] = x_{10}, \quad [x_2, x_5] = x_8, \quad [x_2, x_6] = x_7\,,$$
$$[x_3, x_4] = x_7, \quad [x_3, x_5] = x_9, \quad [x_4, x_6] = x_9$$

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with $[x_i, x_j] = 0$ for $i < j$ if it is not in the above list.

M_2 has basis $y_1, \cdots, y_{10}$ with the multiplication table

$$\begin{aligned}
&[y_1, y_2] = y_9, \ [y_1, y_3] = y_8, \ [y_1, y_4] = y_{10} \\
&[y_1, y_5] = y_7, \ [y_2, y_3] = y_{10}, \ [y_2, y_5] = y_8 , \\
&[y_2, y_6] = y_7, \ [y_3, y_4] = y_7, \ [y_3, y_6] = y_9
\end{aligned}$$

with $[y_i, y_j] = 0$ for $i < j$ if it is not in the above list.

To see that M_1 and M_2 are non-isomorphic, one may verify that the rank of $\mathrm{ad}\, x$ for x in M_1 is either 0 or at least 3 (and this is valid for all fields) while in M_2 the derivations $\mathrm{ad}\, y_4$, $\mathrm{ad}\, y_5$ and $\mathrm{ad}\, y_6$ each have rank 2.

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