

It may not be enough just to look the book through. Give it a try in class and you, too, will be delighted.

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Computation: Finite and infinite machines, by Marvin Minsky. Englewood Cliffs, N. J., Prentice Hall. xvii + 317 pages. \$12.00.

"The main goal of this book is to introduce the reader to the concept of effective procedure". The concepts effective procedure, algorithm, computability and decidability are studied using three different approaches: machines performing certain actions, recursive functions and symbol manipulation (Post) systems.

The book is self-contained and includes a number of exercises with partial solutions to the harder problems. It provides a good textbook at the undergraduate level. Formal language is reduced to a minimum throughout. This is admirably achieved without apparent loss of conciseness or rigor. Thus the beginning student is introduced to a surprising variety of results and proofs in a leisurely but convincing manner.

Summarizing, the book is an excellent introduction to computability theory. A more detailed discussion of the contents of the book follows.

Part I (Finite-state machines): Finite-state machines are discussed from three points of view: as deterministic devices with a finite amount of memory whose behaviour is characterized by a transition function and a response function, as neural networks (1) built from a small number of basic parts, and as defining sets of words which can be described using regular expressions. The equivalence of the three approaches is demonstrated. The author discusses various alternatives for the basic parts which can be chosen to form a neural network. Part I is not an introduction to finite-state machines as such (thus basic ideas such as minimal state machines are not mentioned) but provides a foundation for further examination of the concept of effective procedure.

Parts II and III (Infinite machines and symbol-manipulation systems): Turing machines are introduced followed by an explanation and discussion of Turing's (Church's) Thesis that Turing machines can be used for adequately defining the concept of algorithm. A universal Turing machine is constructed. The halting and related problems for Turing machines are proved undecidable. Primitive recursive functions are introduced. The reason for their limitation is pointed out, leading in a natural manner to general recursive functions. McCarthy's conditional expressions are discussed as an alternative approach. A simple computer-like machine, the program machine (having only 4 types of instructions and 4 registers, each able to hold one positive integer)

is introduced, giving an alternative and possibly more intuitive formulation of the concept of effective procedure. The discussion of the symbol manipulation systems of Post (2) sheds new light on the importance of the concept of computability.

An elegant proof is given for the difficult Post normal form (canonical systems) theorem, also yielding a short proof of the undecidability of Post's correspondence problem (3). Finally, additional universal (i. e. Turing machine equivalent) devices are defined: two register program machines (obtained from the 4-register machines using a simple Goedel encoding), one-register program machines, two-tape non-writing Turing machines and Post tag-systems. Following Shannon's suggestion to measure the size of a universal Turing machine by its state-symbol product, the book incorporates Prof. Minsky's own construction of a 4-symbol 7-state universal Turing machine, the smallest such machine known to date. The various formulations of effective procedure are proved equivalent by a chain of theorems, partly in Part II, partly in Part III: Turing machine computability implies general-recursive computability, this implies program-machine computability, this implies two-register machine computability, this implies Turing machine computability; Turing machine computability implies Post-tag-system computability, and vice versa.

- (1) The terminology of "A logical calculus of the ideas immanent in nervous activity", McCulloch W.S., and Pitts W., is used. In: Bulletin of Mathematical Biophysics, 5 (1943), pages 115-133.
- (2) Introduced originally for investigating properties of proofs in formal systems.
- (3) A direct and short proof of this theorem, based on the equivalence of production systems and Turing machines, is found in "New proofs of old theorems in logic and formal linguistic", Floyd R. W., Carnegie Inst. of Technology, Pittsburgh 1966. 13 pages.

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Oeuvres mathématiques, by Raphael Salem. Hermann, Paris, 1967. 648 pages. 90 F.

Raphael Salem (1898-1963) was a mathematician of consummate artistry, deep insight and great prowess. His remarkable expositions of the broad, yet unified, range of ideas he considered make his work a pleasure to read.

He was also a warm, kindly and lively personality.

On both scientific and human grounds the mathematical community will welcome the issuance of his writings. It is very useful to have available again the (out of print) monograph which was his doctoral