(α-2L) TERMS AS OBTAINED FROM PZT OBSERVATIONS

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Abstract

Periodic terms with arguments such as (α-2L) and (2α-2L) etc. which appear via various phenomena in the results of time and latitude observations by PZT are summarized in analytical forms in order to clarify their mutual relation. Numerical values in time and latitude for these terms to be expected at the Tokyo PZT are also given. From the data of time observations obtained by the Tokyo PZT during the past 15 years, amplitudes of the terms, $\cos(\alpha-2L_{☾})$ and $\sin(2\alpha-2L_{☾})$ in time, are obtained as +0.72 and 1.21 ms, respectively. These results give the value of $(1+k-\ell)$ as 1.14, and the values of nutation coefficients for $2L_{☾}$, $\Delta\varepsilon=+0''.0893$ and $\sin\varepsilon\cdot\Delta\Psi=-0''.0818$.

1. Relations among α, 2α, (α-2L), and (2α-2L) terms

Periodic terms in time and latitude with the arguments of α, 2α, (α-2L), and (2α-2L) caused by the effects of the Moon or the Sun appear in the results of PZT observations via four kinds of phenomena. In these terms, α refers to the right ascension of the stars observed, i.e. the local sidereal time of the observation, and L the mean longitude of the Moon or the Sun. These phenomena are as follows:

1) deflection of the plumb line,
2) deformation by the Earth tide,
3) forced polar motion by the external torque, (Oppolzer terms)
4) errors in adopted nutation coefficients.

Among these, 1) appears as a systematic variation in the adopted longitude or latitude, 2) and 3) appear as the longitude and latitude variations as a result of the forced polar motion, and 4) appears in the star positions to which the PZT observations are referred. These periodic terms in observed time and latitude, δt and $\delta\phi$, due to each phenomenon are tabulated in analytical forms in Table 1.

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Table 1. Relation among α, 2α, (α-2L), and (2α-2L) terms.

δt \ $\delta\phi$	$\frac{3}{2}\frac{G}{\omega^2}\left(\frac{M}{r^3}\right)_{\leftmoon}$		cos/sin α	sin/cos 2α	cos/sin $(\alpha-2L_{\leftmoon})$	cos/sin $(\alpha-2L_{\odot})$	sin/cos $(2\alpha-2L_{\leftmoon})$	sin/cos $(2\alpha-2L_{\odot})$
	$\frac{1000}{15}$ ms \ 1"	$\frac{C-A}{C}$	$(\cos i - \sin^2 i + \zeta)$		$\cos i$	ζ	$\cos i$	ζ
			$\times \sin\varepsilon \cos\varepsilon$	$\times \frac{\sin^2\varepsilon}{2}$	$\times \sin\varepsilon \frac{1+\cos\varepsilon}{2}$		$\times\left(\frac{1+\cos\varepsilon}{2}\right)^2$	
Deflection of plumb line	$\tan\phi$ \ $\cos 2\phi$	$(1+k-\ell)$ $\times \frac{C}{C-A} m$	-1	0	1	1	0	0
	1 \ $\sin\phi \cos\phi$		0	1	0	0	1	1
Deformation by earth tide	1	$k\left(\frac{m'}{3J_2}\right)$	0	0	$\frac{\mp 2 \frac{n_{\leftmoon}}{\omega}}{1-2\frac{A}{C}\frac{n_{\leftmoon}}{\omega}}$	$\frac{\mp 2 \frac{n_{\odot}}{\omega}}{1-2\frac{A}{C}\frac{n_{\odot}}{\omega}}$	0	0
Forced p.m. by ext. torque	$\tan\phi$ \ 1	1	∓ 1	0	$\frac{\pm 1}{1-2\frac{A}{C}\frac{n_{\leftmoon}}{\omega}}$	$\frac{\pm 1}{1-2\frac{A}{C}\frac{n_{\odot}}{\omega}}$	0	0
Errors in nuta. coef.	$\tan\phi$ \ 1	ξ	0	0	$\mp \frac{\omega}{2n_{\leftmoon}}$	$\mp \frac{\omega}{2n_{\odot}}$	0	0

The terms in δt and $\delta\phi$ are shown in a composite form in Table 1 to display the mutual relationship at a glance. Quantities devided by slanting lines and/or accompanied by double signs are to be read separately following the arranged order. That is, the left side of slanting lines and the upper sign in double signs refer to δt, and the right side and the lower sign to $\delta\phi$. For instance, the $(\alpha-2L)$ term in δt attributed to the deflection of plumb line must be read as follows:

$$\frac{3}{2}\,\frac{G}{\omega^2}\left(\frac{M}{r^3}\right)_{☾}(1+k-\ell)m\cdot\cos i\cdot\sin\varepsilon\;\frac{1+\cos\varepsilon}{2}\;\tan\phi\;\frac{1000}{15}^{\mathrm{ms}}\sin(\alpha-2L_{☾}).$$

The following notation is used in Table 1:

- i : inclination of the Moon's orbit to ecliptic,
- ε : obliquity of the ecliptic,
- M : mass of the heavenly body shown by the suffix,
- r : geocentric mean distance of the Moon or the Sun,
- G : constant of gravitation,
- ω : Earth's angular speed of rotation,
- n : mean motion of the Moon or the Sun,
- k : Love number,
- ℓ : Shida number,
- ϕ : latitude of the observing site,
- $m = \omega^2 a/g_e$,
- a : equatorial radius of the Earth,
- g_e : acceleration of gravity on the equator,
- $m' = \omega^2 a^3/(GM_{\oplus})$,
- $J_2 = (C-A)/(M_{\oplus}a^2)$,
- $\zeta = (M/r^3)_{\odot}/(M/r^3)_{☾}$,
- $1+\xi$: ratio of adopted to true values of nutation coefficients,
- A, A, C : principal moments of inertia of the Earth.

It should be noticed in this table that:

a) δt and $\delta\phi$ in each column of periodic terms are expressed solely by a sine or cosine, respectively, regardless of the phenomena,

b) 2α and $(2\alpha-2L)$ terms appear only via phenomenon 1),

c) the amplitude ratio of $(\alpha-2L)$ to $(2\alpha-2L)$ terms attributed to 1) is $2\tan\phi\sin\varepsilon/(1+\cos\varepsilon)$ for δt, and $4\cot 2\phi\sin\varepsilon/(1+\cos\varepsilon)$ for $\delta\phi$, regardless of Love number,

d) α and 2α terms must appear as a part of systematic errors in the star positions,

e) the amplitude of the periodic terms via 2) is negligiblly small as compared with those via 3) because of the existence of the factor of n/ω in 2),

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f) (α-2L) terms as obtained from PZT observations appear as an ensemble of the terms via 1) to 4). However, the one via 1) can be separated by use of the result of (2α-2L) terms as well as the amplitude relation mentioned in c).

g) $mC/(C-A)$ and $m'/(3J_2)$ in Table 1 are almost equal to unity.

Amplitudes of α, 2α, (α-2L), and (2α-2L) terms in Table 1 are evaluated for the site of the Tokyo PZT, and tabulated in Table 2, to two significant figures after the decimal point, the last figure of which is rounded.

Table 2. $\delta t/\delta\phi$ expected at the Tokyo PZT.

ms / 0".01	cos/sin α	sin/cos 2α	cos/sin (α-2L☾)	cos/sin (α-2L☉)	sin/cos (2α-2L☾)	sin/cos (2α-2L☉)
Defl. of plumb line	-0.53 / -0.36	+0.16 / +0.11	+0.38 / +0.25	+0.18 / +0.12	+1.28 / +0.91	+0.59 / +0.42
Deform. by earth tide	0	0	-0.00 / +0.00	-0.00 / +0.00	0	0
Forced p.m. by torque	-0.42 / +0.87	0	+0.32 / -0.67	+0.14 / -0.29	0	0
Errors in nut.coef.	0	0	+0.04 / -0.08	+0.25 / -0.53	0	0
Σ	-0.94 / +0.51	+0.16 / +0.11	+0.74 / -0.49	+0.56 / -0.70	+1.28 / +0.91	+0.59 / +0.42

Numerical values of the relevant constants adopted in this table are as follows:

$$m = 0.003\,4678, \qquad \phi = 35^\circ 40' 20''.707,$$
$$m' = 0.003\,4614, \qquad 1+k-\ell \equiv 1.20,$$
$$J_2 = 0.001\,0826, \qquad k \equiv 0.28,$$
$$\zeta = 0.45924, \qquad \xi \equiv -0.01.$$

Quantities in Table 2 divided by slanting lines are to be read separately for δt and $\delta\phi$, respectively, following the arranged order, as in the preceding table.

2. (α-2L$_☾$) and (2α-2L$_☾$) terms in δt as obtained from the PZT observations in Tokyo.

Observational data obtained by the Tokyo PZT during 15 years from 1962 through 1976 are analyzed here. The time data amounting to about 48,000 star observations made during the 15 years are reduced to the consistent system on the PZT star catalog, α_{75}, which has been used since 1975, allowing for all the past changes in the adopted longitude and the aberration constant etc. back to 1962. (UT0-UTC) for each star observation is converted to (UT1-UTC) by use of the results of the polar coordinates of the ILS and the IPMS, and then transferred to (UT1-TAI) by using the (UTC-TAI) data of the BIH.

In order to remove the seasonal and long term variations contained in the observed UT1, an observational equation composed of annual, semi-annual, biennial terms, and a cubic equation with respect to time is applied to the data of (UT1-TAI) by the method of least squares. This fitting process is repeated for every four-year interval of data, shifting by three-year steps. The residual, (O-A), over three years in the middle of each four-year interval are used for the subsequent calculation.

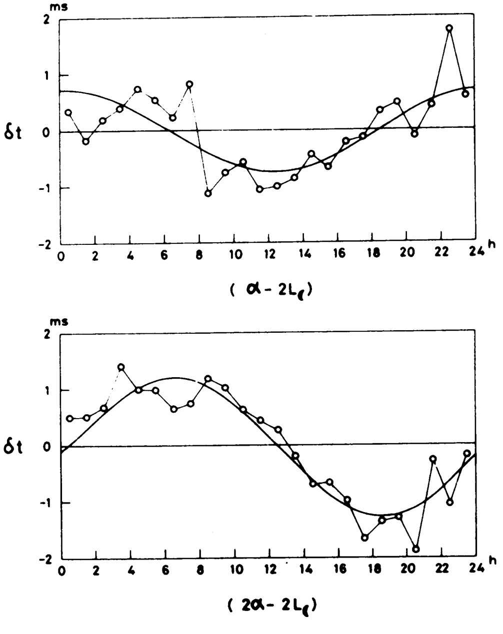

Fig. 1. Results of (α-2L$_☾$) and (2α-2L$_☾$) terms.

The data of (O-A) for each star observation are classified with

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respect to the phase of $(\alpha - 2L_☾)$ or $(2\alpha - 2L_☾)$ into one-hour intervals such as 0h - 1h, 1h - 2h,......, 23h - 0h. The hourly mean values of (O-A) in phase are illustrated in Figure 1 for $(\alpha - 2L_☾)$ and $(2\alpha - 2L_☾)$ terms in δt. Each small circle in these graphs is the mean value of about 2,000 star observations. The results shown in Figure 1 are given analytically as follows:

$$\delta t = \underset{\pm .99}{\overset{\text{ms}}{0.725}} \cos(\alpha - 2L_☾) + \underset{\pm .98}{\overset{\text{ms}}{0.071}} \sin(\alpha - 2L_☾), \tag{1}$$

$$\delta t = \underset{\pm .75}{1.213} \sin(2\alpha - 2L_☾) - \underset{\pm .76}{0.169} \cos(2\alpha - 2L_☾). \tag{2}$$

The amplitude of the cosine term of $(\alpha - 2L_☾)$ in (1) and that of the sine term of $(2\alpha - 2L_☾)$ in (2) are in rather good agreement with the corresponding values in Table 2. By comparing the result of 1.213 ms obtained for the $\sin(2\alpha - 2L_☾)$ term in (2) with the corresponding value in Table 2, the value of $(1+k-\ell)$ can be estimated as 1.14. The amplitude of 1.213 ms for the $\sin(2\alpha - 2L_☾)$ term in (2) also gives the amplitude of the $\cos(\alpha - 2L_☾)$ term due to the deflection of plumb line as 0.361 ms, by applying the ratio of $2 \tan\phi \sin\varepsilon/(1+\cos\varepsilon) = 0.2978$. After removing 0.361 ms just obtained and 0.322 ms due to 3) in Table 2 from the ensemble result of 0.725 ms for $\cos(\alpha - 2L_☾)$ term in (1), 0.042 ms is obtained as the residual which corresponds to the term ascribable to the errors in nutation coefficients for $2L_☾$. Thus the results, $\Delta\varepsilon$ and $\sin\varepsilon \cdot \Delta\Psi$, are obtained as follows:

	$\Delta\varepsilon$	$\sin\varepsilon \cdot \Delta\Psi$
Tokyo	$0''.0893 \pm 0''.0022$,	$-0''.0818 \pm 0''.0022$.

McCarthy (1976) obtained similar results from the data of PZT observations made at Washington, Richmond, and Herstmonceux, assuming the value of $(1+k-\ell)$ equal to 1.13. These results are shown here for comparison:

	$\Delta\varepsilon$		$\sin\varepsilon \cdot \Delta\Psi$	
Washington	$0''.0907$	$\pm 0''.0016$,	$-0''.0832$	$\pm 0''.0015$,
Richmond	977	08,	897	07,
Herstmonceux	889	17,	816	16.

The present results are rather close to those obtained from Herstmonceux data by McCarthy.

Reference

McCarthy, D.D.: 1976, Astron. J. 81, p. 482.