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Dynamics of extrasolar systems at the 5/2 resonance: application to 47 UMa

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Abstract. A complete study is made of the 5/2 resonant motion of two planets revolving around a star, in the model of the general planar three body problem. Families of 5/2 resonant symmetric periodic orbits are computed numerically, for the masses of the extrasolar system 47 UMa. The phase of the two planets (alignment or antialignment of perihelia and position of each planet at perihelion or aphelion) plays an important role, and the change of the phase, other things being the same, may destabilize the system. Stable motion exists even in the case where the two planetary orbits intersect. A small value of the eccentricities, for the same phase, stabilizes the system. The above results are applied to the study of 47 UMa, which according to some observations is close to the 5/2 resonance.

Keywords. periodic orbits - resonances - extrasolar systems - 47 UMa

1. Introduction

The stability and the long term evolution of a planetary system is determined from the topology of its phase space. It is clear that the topology of the phase space of any dynamical system depends on the position and the stability properties of the periodic orbits, or equivalently, on the fixed points of the Poincaré map. In particular, the mean motion resonances in a planetary system correspond to periodic motion, in a rotating frame. This is the reason why the resonances play an important role in the study of the long term evolution of a planetary system, although the corresponding periodic orbits are a set of measure zero.

There are several planetary systems at different resonances, and we shall study in the present paper the 5/2 resonance. This resonance appears in our own planetary system, between Jupiter and Saturn. Also, according to Fisher *et al.* 2002, the extrasolar planetary system 47 UMa has a ratio of the planetary periods equal to $T_2/T_1 = 2.38$, which can be considered as close to the 5/2 resonance (but also close to the 7/3 resonance). A more recent analysis of the observational data for 47 UMa (Fisher *et al.* 2003) revise these values and give a new value for the planetary periods, $T_2/T_1 = 2.64$, which is close to the 8/3 resonance (but not far from the 5/2 resonance). Altough the planetary masses are also revised, the ratio m_1/m_2 is in all cases larger than unity.

There are several papers that study the dynamics of a planetary system at the 5/2 resonance and of the system 47 UMa in particular, by numerical simulations and analytic work (Ji *et al.* 2003, Barnes and Quinn, 2004, Gozdziewski, 2002, Laughlin *et al.* 2002, T.A.Michtchenko and S. Ferraz-Mello 2001, Beaugé *et al.* 2003). In the present study we make a global analysis of the 5/2 resonant motion of a planetary system. In this global analysis, we used for the planetary masses, the (minimum) masses given for the 47 UMa system by Fisher *et al.* (2002). Note that for these masses $m_1/m_2 \approx 0.30$, which is very close to the ratio of the masses of the Jupiter-Saturn system and consequently the dynamics is also applicable to our own planetary system.

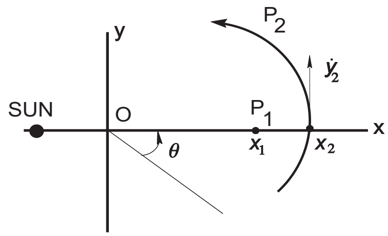

Figure 1. The rotating frame xOy

We find all the basic families of resonant periodic orbits at this resonance and in this way we have a complete knowledge of the regions of the phase space where stable motion exists. The orbits are periodic in a *non-uniformely* rotating frame, which means that the *relatative* configuration is repeated in space, and are symmetric with respect to the rotating x-axis, defined in the next section. The stability analysis of these families gives the regions of the phase space where a planetary system at the $5/2$ resonance can exist, and also the regions of the phase space where a planetary system could be trapped, if it had followed in the past a migration process. In addition, the motion close to a stable periodic orbit is the motion with small variation of the orbital elements, a condition which may play an important role in the appearance of life.

The general study of the $5/2$ resonance that we made is applied to the observed extra-solar planetary system 47 UMa, using the data given by Fisher *et al.* 2002. The stability of this system, for different initial phases, is compared with the exact resonant periodic motion at the $5/2$ resonance.

In all the following the central star will be called the *sun*, the inner planet will be called P_1 and the outer planet P_2.

2. The dynamical Model

The model we used in the study of periodic motion of the planetary system is the general three body problem, for planar motion.

The center of mass of the planetary system is considered as fixed in an inertial frame, and the study is made in a non-uniformely rotating frame of reference xOy, whose x-axis is the line *sun* - P_1, the origin O is the center of mass of these two bodies and the y-axis is perpendicular to the x-axis (Figure 1). In this rotating frame P_1 moves on the x-axis and P_2 in the xOy plane. The coordinates are the position x_1 of P_1, the position x_2, y_2 of P_2 and the angle θ between the x-axis and a fixed direction in the inertial frame. The coordinates x_1, x_2, y_2, define the position of the system in the rotating frame and the angle θ defines the orientation of the rotating frame, so these four coordinates determine the position of the system in the inertial frame. This is a system of four degrees of freedom, but it turns out that the angle θ is ignorable, and consequently the angular momentum integral $L = \partial\mathcal{L}/\partial\dot{\theta}$=constant, where $\mathcal{L}$ is the Lagrangian of the system. So the study is reduced to a system of three degrees of freedom, in the rotating frame only, and the angular momentum L is a fixed parameter (Hadjidemetriou 1975).

Symmetric periodic orbits exist in the above rotating frame, such that the planet P_2 starts perpendicularly from the x-axis ($y_2 = 0$, $\dot{x}_2 = 0$) and at that time $\dot{x}_1 = 0$, and after some time $t = T/2$ the planet P_2 crosses again the x-axis perpendicularly and at that time it is $\dot{x}_1 = 0$. This means that the non zero initial conditions of a symmetric periodic orbit, in the rotating frame, are x_{10}, x_{20}, $\dot{y}_{20}$. So, a family of symmetric periodic orbits is represented as a smooth curve in the three dimensional space $x_{10}\ x_{20}\ \dot{y}_{20}$. The

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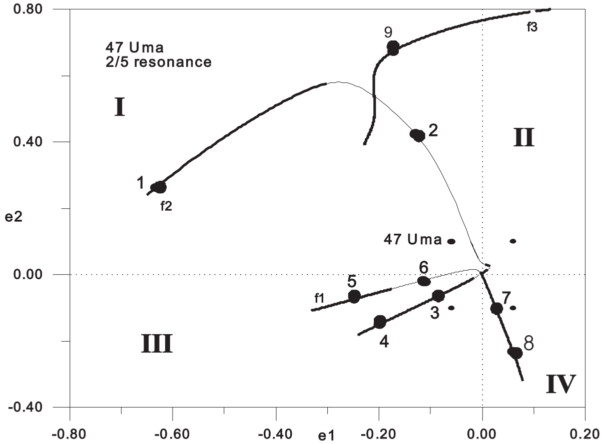

Figure 2. The three families of 5/2 resonant periodic orbits. The unstable sections are indicated by a thicker line. The sectors I-IV, defined by the sign of e_1 and e_2, correspond to the different phases given in Figure 3. The position of the 47 UMa is also shown, for all four possible phases.

symmetry implies that the perihelia of the two planets are either in the same direction or in opposite directions when the two planets and the sun are aligned. Viewed from the inertial frame, the line of apsides of the two planets precesses slowly, in such a way that $\Delta\omega$ is equal to 0 or 2π.

In order to avoid duplication of the results we fix the units of mass, length and time. This is achieved by taking the total mass of the system as the unit of mass, the gravitational constant equal to unity and also by keeping a fixed value of the angular momentum L for all the orbits of a family of periodic orbits. So, the normalizing conditions are $m_0 + m_1 + m_2 = 1$, $G = 1$, $L = constant$. In practice, we made the integration of the planetary system in the inertial frame (where the center of mass is fixed) and the reduction to three degrees of freedom, in the rotating frame, was made by a coordinate transformation. The method of integration was based on Taylor series expansion, and the accuracy was 10^{-14}.

3. The 5/2 resonance

The extrasolar planetary system 47 UMa is close to a resonant system, but not exactly resonant. The elements of this system that we used in the present study are (Fisher *et al.* 2002): $m_0 = 1.03\ M_{SUN}$, $m_1 \sin i = 2.54$ MJ, $m_2 \sin i = 0.76$ MJ, $a_1 = 2.09$ AU, $a_2 = 3.73$, $T_1 = 1089 \pm 3$ d, $T_2 = 2594 \pm 90$ d, $e_1 = 0.061 \pm 0.014$, $e_2 = 0.1 \pm 0.1$, $\omega_1 = 172^0$, $\omega_2 = 127^0$. Note that $\omega_1 - \omega_2 = 45^0$. For later updates see the web page maintained by Jean Schneider (*http://www.obspm.fr/encycl/catalog.html*). The planetary masses are multiplied by $\sin i$, where i is the inclination of the planetary orbit and are therefore the minimum masses. The observed values give a ratio of the planetary periods equal to $T_2/T_1 = 2.38$, which is close to the 5/2 resonance (but also close to the 7/3 resonance). In this paper we restrict the study to the computation of families of periodic orbits for the 5/2 resonance and for the minimum masses of the system 47 UMa. The study of the 7/3 and 8/3 resonances will be included in a forthcoming paper. In all the computations we used normalized values of the (minimum) masses, which are $m_0 = 0.996942$, $m_1 = 0.002354$, $m_2 = 0.000704$.

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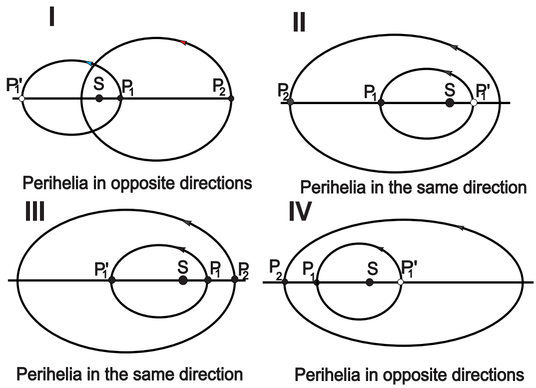

Figure 3. The four different configurations at $t = 0$ and $t = T/2$ (denoted by prime). The planet P_2 is at the same position after half a period, but the planet P_1 shifts from perihelion to aphelion (or vice versa). The panels I-IV correspond to the sectors of Figure 2 (for positive or negative eccentricities), as indicated.

The 5/2 resonance is of a different nature than the 2/1 resonance, studied by Psychoyos and Hadjidemetriou, 2004. In the present case there are two distinct resonant families, that bifurcate from the family of circular orbits from two points close to each other. One more family also exists, which does not bifurcate from the circular family.

3.1. *Periodic orbits*

In Figure 2 we present three families of 5/2 resonant periodic orbits, for the (normalized) values of the masses of the planetary system 47 UMa. To make the presentation clearer from the physical point of view, we present these families in the eccentricity space $e_1\ e_2$, instead of the initial condition space $x_{10}\ x_{20}\ \dot{y}_{20}$. To avoid artificial discontinuities in the presentation of the families, we use the notation $e_i > 0$ for position of the planet at aphelion and $e_i < 0$ for position at perihelion $(i = 1, 2)$. Note that for a symmetric periodic orbit the planets are either at perihelion or at aphelion at $t = 0$ and at $t = T/2$, where T is the period.

Along the families 1 and 2 (denoted by $f1$ and $f2$, respectively, in Figure 2), the eccentricities of the planets increase, starting from zero values, because these two families bifurcate from the circular family, where $e_1 = e_2 = 0$. For the family 3 (denoted by $f3$ in Figure 2), only the eccentricity e_1 of the first planet crosses the zero point, while e_2 stays at high values. We also computed the linear stability. The unstable sections of these families are indicated by a thicker line. In Figure 3 we give the four possible positions of a 5/2 resonant planetary system, corresponding to the four different configurations I-IV of Figure 2. It is simple geometry to see that there are eight different initial positions of the two planets at $t = 0$: The perihelia of the two planetary orbits can be aligned or antialigned and in each case the planets can be either at perihelion or at aphelion. These eight configurations are however equivalent in pairs, due to the fact that we are at a symmetric resonance, and consequently there are only four different configurations. Consider for example the case where the two planets are both at perihelia at $t = 0$ (cases III and IV in Figure 3). After half a period, at $t = T/2$, P_2 will be at perihelion but P_1 will be at aphelion.

3.2. *Stability*

In order to find the region of stability around a periodic orbit, we perturbed it and computed the Poincaré map on the surface of section $y_2 = 0$. We considered as perturbation

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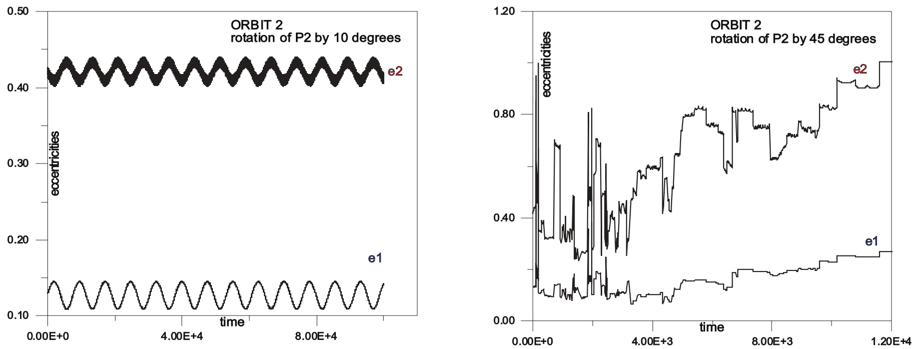

Figure 4. The evolution of the eccentricities of an orbit close to the stable periodic orbit 2 of the family 2, when the orbit of P_2 is rotated by 10^0 (panel a) and by 45^0 (panel b). The motion is bounded for a small perturbation, but for a larger perturbation the system is destabilized.

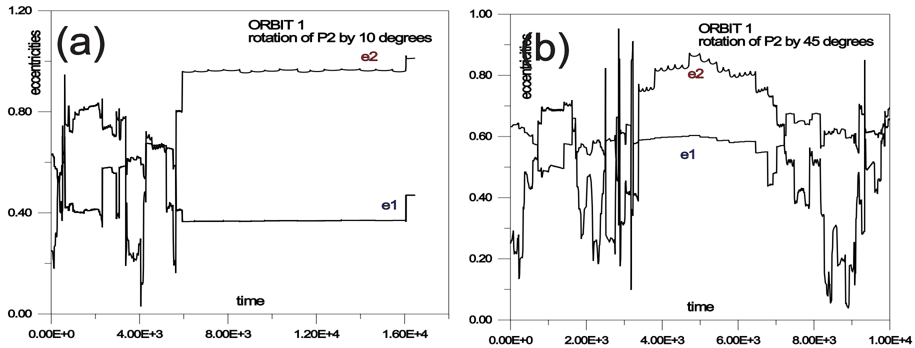

Figure 5. The evolution of the eccentricities of an orbit close to the unstable periodic orbit 1 of the family 2, when the orbit of P_2 is rotated by 10^0 (panel a) and by 45^0 (panel b). The motion is chaotic, implying that large values of the eccentricities destabilize the system.

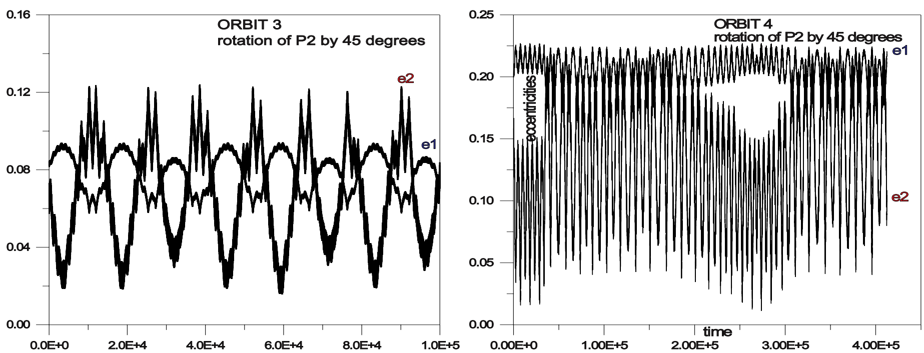

Figure 6. The evolution of the eccentricities of an orbit close to the unstable periodic orbits 3 (panel a) and 4 (panel b) of the family 2, when the orbit of P_2 is rotated by 45^0. The motion is bounded, but the amplitude of the variation is much larger for the orbit 4, which has larger eccentricities than the orbit 3.

the rotation of the orbit of P_2, at $t = 0$, by a certain angle, thus destroying the symmetry of the system. Some typical results are given bellow.

We considered first the stable periodic orbit 2 on the family 2 (Figure 4a), which corresponds to the phase where the perihelia are antialigned and P_1 is at perihelion and P_2 at aphelion (phase I). This orbit is linearly stable and a small perturbation, which in

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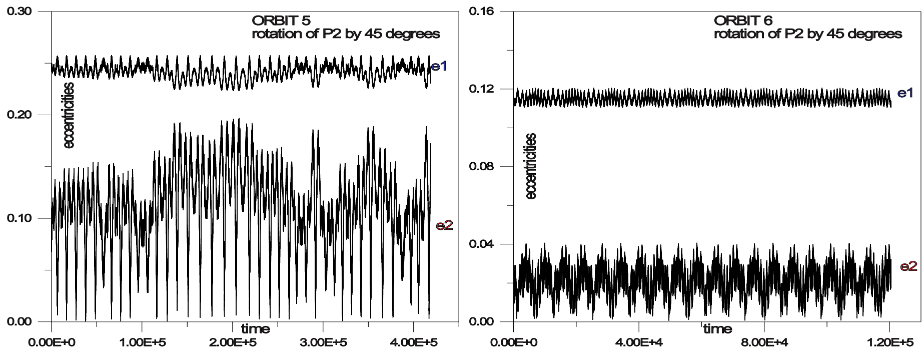

Figure 7. The evolution of the eccentricities of an orbit close to the unstable periodic orbit 5 (panel a) and the stable periodic orbit 6 (panel b) of the family 1, when the orbit of P_2 is rotated by 45^0. The motion is bounded, but the amplitude of the variation is much larger for the unstable orbit 5 which has larger eccentricities than the orbit 6.

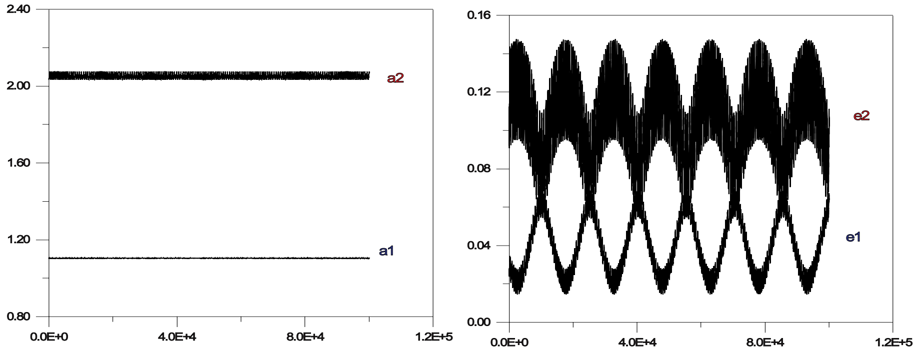

Figure 8. The evolution of the semimajor axes (panel a) and of the eccentricities (panel b), of an orbit close to the unstable periodic orbit 7 of the family 1, when the orbit of P_2 is rotated by 10^0. The motion is bounded.

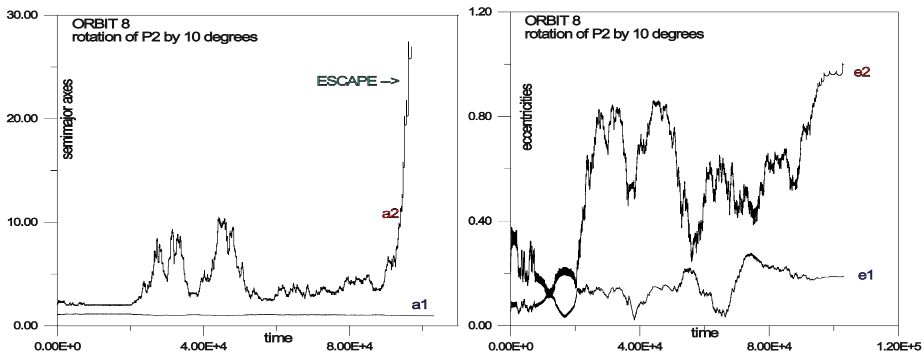

Figure 9. The evolution of the semimajor axes (panel a) and of the eccentricities (panel b), of an orbit close to the unstable periodic orbit 8 of the family 1, when the orbit of P_2 is rotated by 10^0. The motion is chaotic.

this case is the rotation of P_2 by 10^0, gives bounded motion. We remark that the two planetary orbits intersect, and still the motion is stable. A larger perturbation however (rotation by 45^0), destabilizes the system (Figure 4b). Next, we consider the unstable orbit 1 of the same family 2, which has the same phase but larger eccentricities than the orbit 2. A rotation of the orbit of P_2 by 10^0 and also by 45^0 results to instability

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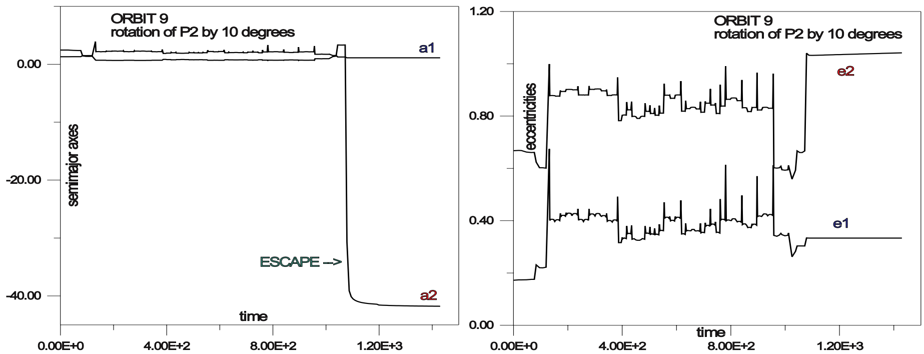

Figure 10. The evolution of the semimajor axes (panel a) and of the eccentricities (panel b), of an orbit close to the unstable periodic orbit 9 of the family 3, when the orbit of P_2 is rotated by 10^0. The motion is chaotic.

(Figure 5). We note that the orbit 1 has the same phase as the orbit 2, which implies that small values of the eccentricities are essential, in the 5/2 resonance, for the stability of the system.

The third orbit that we considered is orbit 3, on family 2, which corresponds to the phase where the perihelia are aligned and both planets are at perihelia (phase III). This orbit is linearly unstable, but a rotation of P_2 by 45^0 gives bounded motion (Figure 6a). The fourth orbit on same family that we consider is the unstable orbit 4, which has larger eccentricities than the orbit 3. A rotation of the orbit of P_2 by 10^0 in this latter case results to bounded motion, but now the variation of the eccentricities is much larger, compared to the orbit 3 on this family, which has the same phase. Again we see that a small value of the eccentricities is necessary for the stability of the system.

We repeated the same procedure as for the orbits 1-4 of the family 2, to the orbits 5-8 on the family 1. The orbits 5 and 6 have the same phase (phase III). The motion in their neighborhood is bounded, when the system is perturbed by rotating the orbit of P_2 by 45^0, but the variation of the eccentricities is much larger in the unstable orbit 5 (Figure 7a) than in the stable orbit 6 (Figure 7b).

The orbits 7 and 8 are linearly unstable, and correspond to the same phase (phase IV), but close to the orbit 7, which has small planetary eccentricities, the motion is bounded, as shown if Figure 8. On the contrary, the same perturbation to the orbit 8 results to destabilization of the system, (Figure 9). Again we note that a small value of the eccentricities is necessary for a stable planetary system at the 5/2 resonance.

All orbits of the family 3 are unstable and a small perturbation results to the destabilization of the system. A typical example is shown in Figure 10 for the orbit 9.

We remark that in all bounded orbits of the system, the amplitude of the variation of the semimajor axes is small. This is in agreement with Barnes and Quinn (2004), even in the cases where the variation of the eccentricirties is quite large.

3.3. *The system 47 UMa*

In Figure 2 we show the position of the system 47 UMa for all the four different phases, which are symmetric with respect to the origin and also symmetric with respect to the e_1 and e_2 axes. We note that these positions are close to periodic motion which has a stable region in its vicinity. In order to study the stability of these systems, we computed the evolution of these four different configurations, by the Poincaré map on a surface of section, by rotating the orbit of P_2 by 45^0 (because in the observed system it is $\omega_1 - \omega_2 = 45^0$). In all cases the numerical computations showed that we have a

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well defined bounded motion. Note that for these small eccentricities we have bounded motion both for alignment and for antialignment of the apsides. This is in agreement with Ji *et al.* (2003).

4. Discussion

From all the above results we come to the conclusion that small planetary eccentricities are necessary for stable motion at the 5/2 resonance. We remark that stable motion exists even in the case where the planetary orbits intersect (orbit 2), and even in the case where the system is linearly unstable (orbits 3, 7), provided that the phase is III (alignment of perihelia and position of both planets at perihelion) or IV (antialignment of perihelia and position of both planets at perihelia). We also note that the system 47 UMa lies close to the regions in the e_1 e_2 space of Figure 2, where we have bounded motion. This system would be unstable if the eccentricities were higher (see also Laughlin *et al.* 2002, Barnes and Quinn, 2004), contrary to the 2/1 resonance, where a large value of the eccentricities stabilizes the system (Psychoyos and Hadjidemetriou, 2004), because, in this latter case, the minimum distance between the planets is increased when the eccentricities increase (for the same phase). We also remark that the symmetry of the orbit is a stabilizing factor, and the system is destabilized if the symmetry is destroyed, other parameters being the same, since a rotation of the orbit of P_2 results to instability in some cases.

Since the ratio of the masses of the Jupiter-Saturm system is very close to the ratio we used for 47 UMa, the same remarks for the stability apply to our own solar system.

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