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A counter-example to a conjecture by Deakin

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Michael A.B. Deakin (*Bull. Austral. Math. Soc.* 17 (1977), 161-175) states (on p. 173) a conjecture, which, if true, would lead to simpler proofs of a number of known theorems. This conjecture is, however, shown to be false by means of a counter-example.

Given a finitely determined real-valued $C^\infty\left(R^n\right)$-germ f and germs $g_i(x) = O\left(|x|^2\right)$, $i = 1, \ldots, n$, the conjecture ([1], p. 173) asserts the existence of a diffeomorphism germ

$$\varphi = x + O\left(|x|^2\right) ,$$

such that

$$f + \sum_1^n g_j D_j f = f \circ \varphi .$$

But such a diffeomorphism does not exist in general as shown by

$$f = x^2 y + y^5 , \quad g_1 = y^2 , \quad g_2 = 0 .$$

Namely, the germ f is 5-determined ([2], p. 292). Let φ be given by

$$\varphi(x, y) = (x, y) + \left(p_1 + z_1, z_2\right)$$

with $z_1 = O\left(\left(x^2+y^2\right)^{3/2}\right)$, $z_2 = O\left(x^2+y^2\right)$, and p_1 homogeneous of degree two. Then modulo terms of degree greater than 5,

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$$x^2y + y^5 + y^2 \cdot 2xy - (x+p_1+z_1)^2(y+z_2) - (y+z_2)^5$$
$$= \left\{2xy^3-2xyp_1-x^2z_2\right\} - \left\{p_1^2y+2xp_1z_2+2xyz_1\right\} .$$

By the conjecture this is identically zero for some p_1, z_1, z_2 ; and so the term $2xy^3$ implies that

$$p_1 = y^2 + ax^2 + bxy ,$$

the fifth order term becoming

$$-y^5 + x \cdot p_2$$

for some homogeneous polynomial p_2 . Here $-y^5$ cannot be cancelled for any p_1, z_1, z_2 .

References

[1] Michael A.B. Deakin, "New proofs of some theorems on infinitely differentiable functions", *Bull. Austral. Math. Soc.* **17** (1977), 161-175.

[2] Christopher Zeeman, "The classification of elementary catastrophes of codimension ≤ 5 ", (notes written and revised by David Trotman), *Structural stability, the theory of catastrophes, and applications in the sciences*, 263-327 (Proc. Conf. Battelle Seattle Research Center 1975; Lecture Notes in Mathematics, **525**. Springer-Verlag, Berlin, Heidelberg, New York, 1976).

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