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Minimal number of generators of some classes of groups

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The rank $d(G)$ of a group G is defined as the minimum of the cardinalities of the generating sets of G ; a generating set of cardinal $d(G)$ is called a minimal generating set of G .

Let G be a group with the presentation

$$G = \mathrm{gp}\left(a_1, a_2, \ldots, a_t; a_1^{p_1} = a_2^{p_2} = \ldots = a_t^{p_t} = a_1a_2 \ldots a_t = 1\right) ,$$

where each of the p_is occurring in the set of relations is a prime. It is shown that any permutation of the generators in the last relator $a_1a_2 \ldots a_t$ does not change the group. Let there be s different primes occurring in the presentation. It is shown that there is no loss of generality in assuming the first s primes, namely, $p_1, p_2, \ldots, p_s$, to be all distinct from one another.

Let $n = p_1p_2 \ldots p_s$ and $r_1, r_2, \ldots, r_s$ be the number of times each p_i , $i = 1, 2, \ldots, s$, occurs in the presentation. Again, it is shown that there is no loss of generality in assuming $r_1 \leq r_2 \leq \ldots \leq r_s$.

If now G^n is the group generated by all elements of G , each raised to the nth power, then with the above assumptions, it is proved that

(i) if $s = 1$, then $d(G/G^n) = t - 1$;

(ii) if $s \geq 2$ and $r_i \geq 2$ for some $1 \leq i < s$, then

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$$d(G/G^n) \geq r_s + 1 - \frac{2}{p_1 p_2 \cdots p_{s-1}} ;$$

(iii) if $s \geq 2$ and $r_i = 1$ for all $1 \leq i < s$, but $r_s \geq 2$, then

$$d(G/G^n) \geq \max\{1, r_s - 2\} + 1 - \frac{2}{p_s p_{s-2}} , \text{ where } p_0 = 1 .$$

In the second part of the thesis we show that if A is a finitely generated nilpotent group, B is a non-trivial finitely generated abelian group, and $A \text{ wr } B$ is their (standard, restricted) wreath product, then

$$d(A\text{wr}B) = \max\{1+d(A), d(A\times B)\}$$

(where $A \times B$ is the direct product). In fact, we construct minimal generating sets for all such wreath products. Some other wreath products are also considered.

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