Judgment and Decision Making, Vol. 6, No. 3, April 2011, pp. 196–210

# Coming close to the ideal alternative: The concordant-ranks strategy

Neda Kerimi* Henry Montgomery† Dan Zakay‡

**Abstract**

We present the Concordant-Ranks (CR) strategy that decision makers use to quickly find an alternative that is proximate to an ideal alternative in a multi-attribute decision space. CR implies that decision makers prefer alternatives that exhibit concordant ranks between attribute values and attribute weights. We show that, in situations where the alternatives are equal in multi-attribute utility (MAU), minimization of the weighted Euclidean distance (WED) to an ideal alternative implies the choice of a CR alternative. In two experiments, participants chose among, as well as evaluated, alternatives that were constructed to be equal in MAU. In Experiment 1, four alternatives were designed in such a way that the choice of each alternative would be consistent with one particular choice strategy, one of which was the CR strategy. In Experiment 2, participants were presented with a CR alternative and a number of arbitrary alternatives. In both experiments, participants tended to choose the CR alternative. The CR alternative was on average evaluated as more attractive than other alternatives. In addition, measures of WED, between given alternatives and the ideal alternative, by and large agreed with the preference order for choices and attractiveness evaluations of the different types of alternatives. These findings indicate that both choices and attractiveness evaluations are guided by proximity of alternatives to an ideal alternative.

Keywords: multi-attribute decisions, concordant ranks, strategies, weighted Euclidian distance.

## 1 Introduction

The modern consumer society involves many decision situations in which one alternative needs to be chosen from a set of several fairly attractive alternatives. This is true for large investments such as the purchase of a car or a home, but also for everyday consumer choices such as deciding what to eat. In all these choice situations, there might be several alternatives that cannot easily be discarded. Indeed, a fundamental feature of market economies is to offer several alternatives that are attractive at least for some people some of the times, depending on their different tastes and monetary constraints. Thus, there are markets for attractive luxury cars, attractive budget cars, and so on. Usually, decision makers find a promising alternative that serves as a candidate for the final choice, which may be checked more or less thoroughly and differentiated from other alternatives before making the final choice (Brownstein, 2003; Montgomery, 1983, 1989; Svenson, 1992). They look through car ads, or browse a website for homes for sale, and may quickly find a promising alternative. This ability to make quick choices helps the consumer to solve life puzzles because lack of time is a fundamental feature of the modern consumer society (Jäckel & Wollscheid, 2007).

The present paper is concerned with the strategies people use in such situations. That is, how do people quickly find the most promising alternative out of several fairly attractive alternatives that are difficult to differentiate from each other with respect to their overall attractiveness? We suggest that in such situations individuals will choose the alternative that is close to their ideal alternative. This manifests itself by using a strategy that we call *the Concordant-Ranks strategy* (CR). According to this strategy, decision makers choose the alternative for which there is a concordance between the rank-order of the attribute values and importance weights of the attributes, provided that the overall attractiveness of the alternatives is approximately equal.

To study such choices and to test the validity of the CR strategy, we presented participants with individually tailored alternatives to choose from, each constructed to be approximately equal in attractiveness. Assuming that multi-attribute utility (MAU) (Humphreys & McFadden, 1980) could be used as an approximate indicator of alternative attractiveness, the alternatives were constructed such that they were equal in MAU based on attributes and attribute weights that participants generated themselves. The study focused on how participants quickly find the promising alternative (i.e., a tentative or prelimi-

*Department of Psychology, Stockholm University, Stockholm, Sweden, S-10691. Email: neda@psychology.su.se.

†Department of Psychology, Stockholm University, Stockholm, Sweden

‡Department of Psychology, Tel-aviv University, Tel-aviv, Israel



https://doi.org/10.1017/S1930297500001406 Published online by Cambridge University Press

nary choice,[1]) rather than on how they build up a complex argumentation structure to justify a final choice (Montgomery, 1983, 1989). Therefore, participants were presented with a large number of decision situations and had to quickly make a choice with a lack of time to build up a more complex argumentation structure.

## 1.1 The Concordant-Ranks Strategy

The following hypothetical example serves as an illustration of how the CR strategy might be used for finding an alternative that is proximate to the ideal alternative. Imagine a person—let us call her Jane—who wants to buy a mobile phone. Jane has found two phones: A and B that she finds quite attractive. The two phones have the same price. To make her choice Jane checks three attributes in order of importance: the design, battery time, and the quality of the camera (importance weights assumed to be 3, 2, 1, respectively). On a scale from 0 (minimally attractive) to 100 (maximally attractive), Jane assigns the following points to the phones on each attribute: Phone A: 65 (design), 60 (battery time), 50 (quality of camera); Phone B: 70 (design), 30 (battery time), 95 (quality of camera). Jane cannot immediately differentiate between the two phones with respect to over-all attractiveness. In fact, they have the same MAU (365). To make her choice Jane starts to look for how attractive the two phones are on specific attributes. She begins with the most important attribute, design, and finds that phone B is superior (70 vs. 65). She then continues to the next most important attribute: battery time. Here, phone B is quite unattractive (30) whereas A has a quite a good value (60). However, B has a very high value on the least important attribute, quality of camera (95), whereas A has a mediocre value on this attribute (50). Jane cannot still make up her mind. She does not want to base her decision on just one attribute because different attributes speak in favor of different alternatives.

To solve the problem Jane now looks for the alternative that overall is most similar to her ideal mobile phone (i.e., a phone that is 100 on each dimension). She then realizes that phone A, as opposed to B, has a nice pattern with concordance between the rank-order of the attribute values and the importance weights of the corresponding attributes. Because of this, she experiences that phone A is closest to the ideal alternative with attribute importance taken into account. More precisely, all *weighted* distances to the ideal alternative on each attribute (attribute weight x (100 – attribute value)) are relatively short for all attributes for this alternative. This follows from the fact that the rank of each attribute value agrees with the rank of the corresponding attribute weight for that alternative. In contrast, Alternative B deviates quite a lot from the ideal on the next most important attribute—battery time. Later, we will show that such a pattern is at hand in a weighted Euclidean space defined by those attributes that characterize the ideal alternative. That is, in this space an alternative with minimal WED to the ideal will exhibit concordance between attribute values and attribute weights provided that MAU is constant for all alternatives (Appendix A).

Consider now a case where Jane is presented with different phones on separate occasions, which makes it difficult to compare them with each other on single attributes. On each occasion Jane evaluates how attractive or unattractive the relevant phone is on a scale from 1 to 100. If it is true that a CR pattern is used as a proxy for minimization of the distance to the ideal, then one might expect the CR alternative to be more positively evaluated than the other alternatives, despite the fact that MAU is the same for all alternatives. On the other hand, if a CR pattern only is used as a tie-breaker, with no regard to the distance to the ideal, then there is no reason to expect more positive evaluations of single alternatives that are available one at the time. Thus, because we believe that distance to the ideal is a fundamental principle underlying both choices and evaluative ratings, we predict that CR alternatives will be preferred both in terms of choices and in terms of evaluations of single alternatives.

Zeleny (1976) hypothesized that alternatives that are close to some anchor (ideal) should be preferred. Building upon Zeleny's notion, Zakay and Dil (1984) found evidence that the preference for given alternatives depends on the alternative's distance to an ideal alternative. In another study, they found that this strategy had high predictive validity in actual choices (Zakay & Barak, 1984). In both studies, distance was calculated by summing the importance weights across those attributes where the attribute values of an offered alternative and the ideal alternative differed, thus not taking into account how much the attribute values of the offered alternatives differed from the attribute values of the ideal alternative.

According to the CR strategy, weights of the underlying attributes are taken in consideration when trying to find an alternative that is proximate to an ideal alternative. More precisely, the decision maker has access to a subjectively defined decision space (Zakay & Barak, 1984) in which offered choice *alternatives* (e.g., a set of mobile phones) are characterized on a number of *attributes* (e.g., design, price, and battery time) in terms of *attractiveness values* (e.g., very attractive design, medium attractive price, and very unattractive battery time). Each at-

[1]Because the instructions were given in Swedish, the usage of the Swedish equivalent of "promising alternative" in the instructions ("lovande alternativ", 6150 Google hits) is more idiomatic (typically referring to a candidate for the final choice ) than "tentative choice" ("tentatitvt val", 1 Google hit.) and "preliminary choice " ("prelimin?rt val", 810 Google hits mostly referring to preliminary elections).

tractiveness value may correspond to a certain objectively given feature on the relevant attribute (e.g., a certain price in a given currency), but it is assumed that the decisions are based on the attractiveness values rather than on their corresponding objectively given features (Montgomery & Svenson, 1976). Furthermore, each attribute is associated with a subjectively defined importance *weight*, so that the attributes could be rank-ordered in line with their importance. Of particular importance for this study, we assume that the decision space includes an *ideal alternative*. This ideal alternative corresponds to a point in the space that is described by the optimal attractiveness values (i.e., maximally attractive values) on the attributes that build up the space (e.g., 100, 100, and 100). The ideal corresponds to a mobile phone with an optimal design, optimal price, and an optimal battery time (for other examples see Zakay & Barak, 1984). The decision maker's idea of the optimal value in terms of corresponding objectively given features depends on his or her knowledge of and previous experiences with the choice alternatives and may vary across different contexts. The ideal alternative may not actually exist, but is still an ideal to which the decision maker would like to come as close as possible.

Rubinstein and Zhou (1999) derived a mathematical model showing that, when the given alternatives are part of a Euclidean space, an individual will choose the alternative with the shortest Euclidean distance to a reference point within this space. However, using Euclidean distance requires the assumption that all decision makers share the same decision space. WED allows the dimensions in the decision space to differ for different individuals, in that individuals place different weights on each attribute in an alternative(De Leeuw & Pruszanky, 1978), as is also true for the MAU model (Humphreys & McFadden, 1980). Some studies have shown, however, that attribute weights do not change preference orders much (Dawes, 1979), while other studies, such as those testing lexicographic heuristics and Elimination by Aspects (EBA), have shown that decision makers use the relative importance of attributes as input for their decision (Dahlstrand & Montgomery, 1984; Tversky, 1972). In this study we tested both non-weighted Euclidean distance and WED as well as the lexicographic decision rule.

## 1.2 Other possible choice strategies when alternatives are approximately equal in overall value

When decision makers are faced with the task of making a quick choice between several alternatives that are close to equally attractive, at least two other possible strategies can guide them. They can either make a random choice, or they can base their choice on a simple and easily understood reason or heuristics.

The first strategy, making a random choice, is related to the flat maximum principle (von Winterfeldt & Edwards, 1986): when alternatives in the same choice set have approximately the same utility, choosing between them should be arbitrary. Several studies, however, have shown that even when alternatives are perceived as similar, decision makers will try to differentiate them (Mellers & Biagini, 1994; Slovic, 1975; Tversky, 1977), possibly because people want to justify their decisions in a clear-cut manner (Montgomery, 1983; Shafir, Simonson, & Tversky, 1993). For instance, the prominence effect (Slovic, 1975) implies that participants, when faced with alternatives with equated overall values, select the alternative that is superior on more important attributes.

According to the second strategy, individuals use simple and easily understood heuristics in order to make decisions (Gigerenzer, Todd, & the ABC Research Group, 1999; Tversky & Kahneman, 1974). Dahlstrand and Montgomery (1984) reported that the finding of a promising alternative was compatible with using non-compensatory heuristics that focus on positive attribute values, such as the lexicographic heuristic (choice of an alternative that is best on the most important attribute on which it differs from other alternatives: Gigerenzer & Goldstein, 1996; Gigerenzer et al., 1999) and the maximin heuristic (choice of an alternative with the best worst outcome: Coombs, Dawes, & Tversky, 1970; Dahlstrand & Montgomery, 1984). Heuristics can be used because they are efficient or frugal (Gigerenzer & Goldstein, 1996) or as a supplement to MAU when this principle is difficult to apply, or when it does not distinguish between the choice alternatives. On the other hand, results from other studies suggest that there is a more thorough type of information integration in choices and judgments, which may also involve an additive or a multiplicative type of information integration (e.g., Birnbaum & LaCroix, 2008; Glöckner & Betsch, 2008; Juslin, Jones, Olsson, & Winman, 2003; Lee & Cummins, 2004; Troutman & Shanteau, 1976; for an overview of early research, see Payne, Bettman, Johnson, & Luce, 1995). Here, Glöckner and Betsch's (2008) research is of particular relevance for the present study. These researchers showed that, when the information search is not restricted by the experimental procedure, quick choices might involve more thorough information integration (consistent with MAU). This is in contrast to earlier research where typically the experimental procedure has led to slow sequential choice processes, by which one piece of information is attended to at a time, (e.g., Payne et al., 1995; Rieskamp & Hoffrage, 1999), which in turn makes it natural for participants to use decision rules that rely on less thorough information integration.

## 1.3 Present study

In order to shed light on whether decision makers use the CR strategy in quick choices among a number of fairly attractive alternatives, we let participants choose among five alternatives individually tailored to have fairly high and close to equal MAUs. In Experiment 1, four of the alternatives were designed so that the choice of each alternative would be consistent with one particular choice strategy and not with strategies associated with the other alternatives. Thus, the choice made by the participants would indicate the strategy that they may have used. One alternative had the highest value on the most important attribute (lexicographic heuristic). One alternative had the best worst outcome (the maximin heuristic). The lexicographic and maximin heuristics were tested because these heuristics have been found to predict the selection of a promising alternative (Dahlstrand & Montgomery, 1984). One alternative had the shortest non-weighted Euclidean distance (ED) to the decision maker's ideal alternative. One alternative—the CR alternative—had the same rank-order within its attribute values as that of the importance weights, which also, as a rule (see Results), had the shortest WED to the ideal alternative. Note that the most attractive value on the most important attribute of the CR alternative was always lower than the corresponding value of the lexicographic alternative, which means that a choice of the CR alternative could not be interpreted as a choice according to the lexicographic rule. Finally, the fifth alternative consisted of attribute values that were arbitrary given the constraint that MAU for this alternative was the same as for the other alternatives in the set while it also was not compatible with the other strategies.

If MAU itself were the single basis for choosing between alternatives, the alternatives would be chosen randomly and hence have the same chance of being chosen. If participants use any of the other tested strategies, they would choose the alternative that was constructed to be preferable according to the strategy used (i.e., the CR alternative, the lexicographic alternative, the maximin alternative or the ED-alternative). However, as already mentioned, if participants choose the CR alternative, this does not necessarily indicate that concordant ranks are used as a proxy for finding an alternative with a minimal distance to the ideal, because it is conceivable that the only function of concordant ranks is to be used as a tie-breaker between equally attractive alternatives in situation where one alternative must be chosen. In addition, the CR alternative could be chosen as a result of sequential consideration of attributes, with the sequence affected by attribute importance (e.g., the elimination by aspects strategy: EBA, Gati, 1986; Payne et al., 1995; Svenson, 1979; Tversky, 1972; or the selection by aspects strategy: SBA, Barthélemy & Mullet, 1992). To check the relevance of these competing interpretations, participants in the present study were also asked to evaluate the attractiveness of single alternatives that were constructed in exactly the same way as the alternatives in the choice task. If this type of evaluation also favors the CR alternative, the competing interpretations will be less reasonable because they are confined to choices only.

Experiment 2 was performed to check if a CR alternative is preferable in an additional context than the one tested in Experiment 1. Similar to Experiment 1, one alternative was constructed as a CR alternative, but in contrast, the other four alternatives consisted of arbitrary attribute values.

# 2 Mathematical proof of compatibility of Concordant-Ranks Strategy with minimizing distance to an ideal alternative

In Appendix A, we present a mathematical proof concerning the relationship between the CR strategy and WED to an ideal alternative for a number of alternatives that have the same MAU. The proof assumes that the decision maker has an idea of what is maximally good on all the important attributes, and that decision makers try to choose an alternative that is proximate to this ideal. Given this assumption, the proof clarifies the relationship between MAU and WED in a situation where both MAU and WED values are calculated from ratings of attribute values and importance weights of the attributes used in a given decision problem. We show first that the squares of the rated weights that are used for calculating MAU could replace the weights in the equation for WED. We then show that when WED is thus defined, minimization of WED, in a case where MAU is constant for all alternatives, implies that the rank order of the attribute values agrees with the rated importance weights. This implies that searching for an alternative that has such a relation between attribute values and attribute weights is compatible with, although not necessarily equivalent to, finding an alternative that is proximate to an ideal alternative. This also shows that alternatives that are equally attractive in terms of MAU are not necessarily equally attractive to the decision maker although the alternatives reasonably still are fairly equal in attractiveness.

## 2.1 Discussion

It should be noted that our proof—that minimization of WED implies concordant ranks when MAU is kept constant—does not imply the reverse relationship—that

https://doi.org/10.1017/S1930297500001406 Published online by Cambridge University Press

a given CR alternative has lower WED than other alternatives with the same MAU. (Note the CR is based on ranking, and distances can changes without changing the ranking.) The latter relationship, that CR signals minimal WED, is of particular importance in the present study because because we assume that decision makers may use CR as a proxy for finding an alternative with minimal WED. We have checked that it is indeed possible to construct cases where WED is lower for another alternative than a particular CR alternative in a situation where all alternatives have the same MAU. Thus, it is an empirical question to find out how often CR implies lowest WED in given choice situations. We will report on this relationship in connection with presenting the results of the experimental studies.

# 3 Experiment 1: Concordant-ranks versus other choice strategies

In this experiment the aim was to find out if participants would choose an alternative according to the CR strategy rather than other possible choice strategies tested in this study: lexicographic, maximin, Euclidean, or some arbitrary strategy. In addition, we examined whether the CR alternative was evaluated as more attractive than the other alternatives. To increase the validity of the results, the alternatives were individually tailored by using attributes that the participants themselves provided. Two different procedures were used to identify attributes: inferring attributes from thinking aloud about an imagined decision or direct stating relevant attributes. This precaution is in line with previous research showing that different procedures for getting data on decision processes (e.g., think aloud data or retrospective reports) may also influence the decision process more or less strongly (Russo, Johnson, & Stephens, 1989).

## 3.1 Method

### 3.1.1 Participants

In Experiment 1, 61 (42 women) participants from Stockholm University participated in the experiment. Participants were offered either course credits or a movie ticket for their participation. Participants completed the experiment individually.

### 3.1.2 Procedure

The following steps were conducted in Experiment 1:

*1. Presentation of experiment and think aloud task.* The participants were divided into two groups who followed different procedures for identifying their most important attributes in the given decision situation. Think-aloud participants (N = 45) were told that the experiment consisted of two tasks, a think-aloud task and a second task, which would be conducted by means of a computer. When participants gave consent to continue, they received an exercise in the think-aloud method. After the exercise, participants were randomly assigned to one of two decision-making tasks (house choice or car choice) with identical instructions. The same decision making task was used throughout in later phases of the experiment. Participants were told to think aloud while imagining several alternatives for the given task (e.g., several cars or houses), and to eventually choose one of the imagined alternatives. Afterwards, participants were asked some control questions such as whether they had previously encountered similar decision-making tasks. Data from the control questions are not reported in this study because they did not affect the results. Direct stating participants (N = 16), instead of thinking aloud, were told to provide a list of the most important attributes for the given decision-making task.

*2. Identification of attributes.* To make it possible for the experimenter to have time to identify participants' important attributes from think-aloud data on the spot, think-aloud participants completed a questionnaire on decision-making styles (GDMS, Scott & Bruce, 1995). Data from the GDMS questionnaires are not reported in this study because they did not affect the results. Meanwhile, the attributes of the ideal alternative were identified by rerunning think-aloud recordings. When necessary, the attributes were relabeled so that they could be used for comparative attractiveness evaluations across alternatives. This was also done for direct-stating participants. For instance, if the subject had said that it was important for him or her to live centrally, this attribute was relabeled as an attribute called location. Once relabeled, the attributes were entered into the computer application by the experimenter. In the forthcoming steps the procedure was identical for think-aloud participants and direct-rating participants.

*3. Review and ranking of attributes.* The computer application presented the attributes to the participants at the same time and in order of identification. Participants could add or remove attributes if they felt that the attributes were not representative. After reviewing the attributes, participants clicked "Continue" to go to the next screen, on which the attributes that had previously been identified were presented. Participants then rated the importance of each attribute by distributing 100 points across the attributes, which is in line with the Point Allocation (PA) Method (Shoemaker & Waid, 1982). This method has been found to result in importance weights that are more equal to each other than is true for other methods for generating attribute weights. However the predictive accuracy was the same for this method as for other more complicated weight elicitation methods, involving pairwise comparison of alternatives or attributes (Shoemaker & Waid, 1982). Participants were informed that the same points could not be given to two different attributes and that a rating of zero could not be given to any of the attributes. If this occurred, the participants were reminded of the constraints and were asked to re-enter their answers. In addition, participants could not continue to the next screen until the sum of the given values equaled 100.

*4. Presentation and choice of alternative.* After receiving and ranking the attributes, participants were then presented with instructions for the successive pages. They were informed that ten sets of alternatives, each consisting of five unique alternatives, would be presented and that their task was to choose, as quickly as possible, the most promising alternative. This was intended to be equivalent to finding the best alternative within the limits of making a quick decision. After the instructions, participants were shown an example of how a set of alternatives might appear. Each alternative attribute was presented as a pie diagram, with each diagram representing the value for that attribute. For example, an alternative consisting of five attributes would have five pie diagrams. A full pie diagram represented the best possible value for that attribute; a half pie diagram represented a half-good value, and so forth, with no numbers given. While viewing the example, participants could ask questions in order to understand how the alternatives were going to be presented, after which they could continue to the next screen.

The attributes of each alternative were presented in the order they had been identified, which, in most cases, differed from participants' importance weighting of them. The pie diagrams within an alternative had the same color, but differed in color for different alternatives. The order of the alternatives in a set was random. For instance, the lexicographic alternative could at one point be on the leftmost side and at another point on the rightmost side.

The choice alternatives were constructed based on the attributes and the attribute weightings the participants had given earlier in the experiment. Each alternative was constructed according to one and only one of the five tested strategies. For example, the lexicographic alternative was not eligible according to the maximin heuristic, shortest Euclidean distance or shortest WED. Based on the attributes and attribute weightings given by the participants, a computer application calculated the MAU of a participant's ideal alternative. This was assumed to consist of maximally positive attribute values (i.e., 100 on a 0–100 scale). Afterwards, the computer application adjusted each generated alternative's MAU to have a value equal to 80% of the ideal alternative's MAU. The attribute values of each alternative were presented on graphical scales (in the shape of pie diagrams with no numbers given), under the assumption that we could use these scales as a proxy for people's subjective representation of attractiveness levels (Svenson, 1979).

Table 1: Means, standard deviations (*SD*) for the attractiveness evaluations and mean WEDa values of each of the five alternative types and percent of alternative type with the lowest WED in Experiment 1.

| | Think-aloud (*n*=45) | | Direct-stating (*n*=16) | |
|---|---|---|---|---|
| Chosen alternative | Frequency | % | Frequency | % |
| Lexicographic | 77 | 17.1 | 24 | 15.0 |
| Maximin | 80 | 17.8 | 36 | 22.5 |
| CR | 187 | 41.5 | 65 | 40.6 |
| Euclidean | 59 | 13.1 | 18 | 11.3 |
| CR | 47 | 10.4 | 17 | 10.6 |

The choice of an alternative was indicated by clicking on the radio button that was associated with that alternative. Participants could not change their choice answer once it had been made. The mean choice time was 30.43 seconds (*SD* = 14.37), which we interpret as corresponding to fairly quick choices.

After completing the ten choice tasks, participants in the think-aloud group were presented with a new set of five alternatives and were asked to think aloud while quickly choosing the most promising alternative. These data were collected for exploratory purposes for planning future research.

*5. Attractiveness evaluations.* After alternatives were chosen, alternatives from two previously shown sets were randomly chosen and presented to the participants one by one. Participants were required to assign each alternative a rating indicating its perceived attractiveness on a scale from 0 (not attractive at all) to 100 (most attractive).

We assumed that there was a negligible risk that participants had memories of their previous choices that influenced their evaluations because these choices involved a large number of fairly complex alternatives (totally fifty choice alternatives, each of which was described on three to seven attributes).

## 3.2 Results and discussion

### 3.2.1 Choices

Table 1 shows the choice frequencies of the different alternative types for each of the two attribute identification groups: think-aloud and direct-testing.

As indicated by t-test, there was no significant difference between the choice tasks, car or house, therefore the data from the two choice tasks were merged. This was also the case for the two attribute identification groups (think-aloud and direct-stating). Therefore, the two groups were collapsed in subsequent data analyses. One-way repeated measures ANOVA, with the five alternative types as the within-participants factor, resulted in a strong significant effect, $F\ (4, 240) = 23.07, p < .001$. Post hoc analysis using Fisher's LSD showed that the CR alternative was chosen significantly more often than the other alternatives ($p < .001$). In addition, the lexicographic alternative was chosen significantly more often than the arbitrary alternative ($p < .05$), and the maximin alternative was chosen significantly more often than the lexicographic ($p < .001$), and the arbitrary alternative ($p < .01$).

Note that the PA Method that we used for getting attribute weights from participants could result in more equal weights than is true for other weight elicitation methods, such as methods based on pairwise comparisons of attributes (Shoemaker & Waid, 1982). This means that MAU computed with alternative methods would have resulted in unequal MAU estimates of the choice alternatives in the present study. This in turn invites the possibility that the choice frequencies found for different alternatives in the present study, especially the predominance of CR-choices, resulted from other MAU levels, rather from usage of particular decision rules other than MAU. To check for this possibility, the MAU-weights were "stretched out" by linear transformation using the formula $\mathrm{W}i^* = C(Wi - M) + M$, where $\mathrm{W}i^*$ is the "new" weight estimate for attribute $i$ to be used in the calculations of MAU; $Wi$ is the original "old" weight estimate for attribute $i$ (i.e., the weights that the participant provided using the PA method), $M$ is the mean of all the attribute weights in the choice set, and $C$ is a constant ($>$). We tried various values of $C$ and found that the distribution of predicted choice alternatives was the same in the interval $C > 1.5 < 5$. We chose $C = 2$ for the final computations, which were conducted for all the 61 participants in Experiment 1. This linear transformation is justified by the fact that the weights based on the different methods reported in the Shoemaker and Waid (1982) study were linearly related to each other. A theoretical rationale for the linear transformations based on range-frequency theory (e.g., Wedell & Parducci, 1988) is described in Appendix C.

The linear transformation resulted in some negative MAU-weights (although the sums of negative weights were only 1% of the sum of positive weights). For this reason, we calculated two sets of MAU-vales, one where weights were allowed to be negative, and one where the negative weights were assumed to be zero. It was found that the resulting new MAU-values indeed tended to favor the CR-alternative (CR and highest "new" MAU coinciding in 52% of the cases). However, choices of the CR-alternative were significantly more common (42% of the choices) than choices alternative with highest new MAU-value, both when negative weights were allowed, ($t\ (60) = 3.77, p < .001$, choice percentage = 32%.) and when negative weights were set to zero, ($t\ (60) = 4.14, p < .001$, choice percentage = 31%.). In sum, the predominance of CR-choices seen in Table 1 does not seem to have resulted from an equal weighting bias.

The variation in choice frequencies for each of the five alternative types has two possible sources. One could result from variation *within* individuals, assuming that the responses in the ten choice situations come from a common population of responses. This variability could be calculated from the binomial distribution $Pr = f\ (k; n,$ p), where $Pr$ is the probability of receiving exactly $k$ choices of one of the alternative types among the ($n$=10) choice situations, given an expected probability $p$ of choice of the relevant alternative (across all samples). Another source could result from variation *between* individuals, reflecting that individuals have different propensities to prefer specific types of alternatives. To examine the relative importance of these two sources of variation, an $F$-test was conducted for each alternative type. The data used for this test were responses from the participants. The numerator in the $F$-ratio was the variance of the empirically obtained distribution of choices of the alternatives ($df$ = 60). The denominator was the variance of the binomial distribution for the same alternatives ($df$ = $\infty$), with $Pr$ estimated from the mean proportion of alternative choices across individuals. Thus, to the extent that the variance of the latter distribution underestimates the empirical distribution across individuals, implying $F > 1$, the more the variation between individuals in their propensity to prefer a certain strategy will account for the distribution across individuals.

Table 2 shows significant $F$-ratios for each of the four alternatives that were constructed in line with a specific choice rule, whereas the $F$-ratio for the arbitrarily constructed alternative was non-significant. This suggests that individuals differed in their propensities to choose a specific type of alternative with an exception for the arbitrary alternative. This finding is especially clear for the CR alternative, in which the obtained variance is more than three times greater than the variance that would be expected if the variability were completely due to intra-individual factors.

Table 3 shows descriptive data illustrating the relatively high inter-subject consistency. Each cell shows the number of participants who had the relevant row choice

Table 2: Variances of the empirical and binomial distribution, and the F-value and its probability ($p$-value) based on the ratio of the variances in Experiment 1.

| Chosen alternative | Empirical variance | Binomial variance | $F$ | $p$ |
|---|---|---|---|---|
| Lexicographic | 2.33 | 1.45 | 1.61 | 0.01 |
| Maximin | 2.12 | 1.53 | 1.39 | 0.05 |
| CR | 8.12 | 2.41 | 3.38 | 0.01 |
| Euclidean | 1.40 | 1.01 | 1.38 | 0.05 |
| Arbitrary | 1.20 | 0.97 | 1.24 | 0.1 |

Table 3: Choice frequency of a particular alternative in Experiment 1.

| Choice frequency | Lexicographic | Maximin | CR | Euclidean | Arbitrary |
|---|---|---|---|---|---|
| 10 | 0 | 0 | 1 | 0 | 0 |
| 9 | 0 | 0 | 3 | 0 | 0 |
| 8 | 0 | 0 | 5 | 0 | 0 |
| 7 | 0 | 1 | 5 | 1 | 0 |
| 6 | 1 | 0 | 6 | 1 | 0 |
| 5 | 1 | 2 | 8 | 1 | 0 |
| 4 | 6 | 4 | 9 | 0 | 3 |
| 3 | 10 | 13 | 4 | 8 | 3 |
| 2 | 12 | 12 | 4 | 12 | 11 |
| 1 | 12 | 20 | 8 | 11 | 21 |

frequency (from zero to maximum of ten choices of that strategy) of the relevant column alternative. The CR alternative was chosen in almost half of the trials by 29 (47.5%) participants while only 8 participants (13.1%) chose one of the other alternatives in at least half of the trials.

### 3.2.2 Attractiveness evaluations

Table 4 shows means and standard deviations for the attractiveness evaluations, where participants had to evaluate how attractive each alternative was perceived.

The data were subjected to a 2x5 mixed model ANOVA with attribute identification procedure as the between-participants factor and type of alternative as the within-participants factor. Unexpectedly, the evaluations were lower for the think aloud procedure participants ($M$ = 56.89) than for direct stating participants ($M$ = 65.29), $F$ (1, 60) = 6.89, $p < .05$. It may be speculated that this effect follows from the more thorough analysis of the decision situation than the think aloud task. However, the effect does not interfere with the aim of the present experiment, which does not concern the overall attractiveness level across different types of alternatives, but rather the relative attractiveness level for different types of alternatives. Indeed, type of alternative yielded a strongly significant effect, $F$ (4, 240) = 8.88, $p < .001$, but there was no significant interaction between attribute identification procedure and type of alternative, which indicates that the effect of alternative type is stable across the two identification procedures. Post hoc analyses conducted across all 61 participants using LSD (Fisher's) criterion indicated that the CR alternatives were evaluated as significantly more attractive than all the other alternatives ($p < .05$). In addition, the maximin alternatives were evaluated as significantly more attractive than the Euclidean and arbitrary alternatives ($p < .05$).

Table 4 also shows the average WED value for each type of alternative computed according to Eq. 1 in Appendix A. A 2x5 mixed model ANOVA with attribute identification procedure as between-participants factor and type of alternative as within-participants factor showed no effect of the attribute identification procedure. There were strongly significant differences between the WED values for the various types of alternatives, $F$ (4, 240) = 138.065, $p < .001$. Post hoc analyses with Fisher's LSD showed that all alternative types differed significantly ($p < .05$) from one another with an exception for the lexicographic versus the maximin alternative. The CR alternative had the lowest WED followed by the maximin, lexicographic, arbitrary, and Euclidean alternative in that order. The rightmost column of Table 4 gives the overall percentage of each type of alternative that had the lowest WED to the ideal alternative in each choice situation. It can be seen that the CR alternative also had lowest WED in a large majority (85.6%) of the choice situations.

The results of this experiment show that concordance between the rank order of the attribute values and attribute weights of an alternative seems to play an important role when decision makers are finding the most promising alternative. In addition, the CR alternative was evaluated as more attractive. However, it is conceivable that the constraints imposed on the construction of alternatives were of importance for the results. That is, having the same MAU, while trying to differentiate between the alternatives according to different choice strategies, could lead to the alternatives being constructed in such a way that might promote choosing the CR alternative. To control for this possibility, Experiment 2 was conducted.

Table 4: Means, standard deviations (SD) for the attractiveness evaluations and mean WEDa values of each of the five alternative types and percent of alternative type with the lowest WED in Experiment 1.

| | Think-aloud (*n*=45) | | | Direct-stating (*n*=16) | | | All participants (*n*=61) |
|---|---|---|---|---|---|---|---|
| Type of alternative | Mean | *SD* | Mean WED | Mean | *SD* | Mean WED | % lowest WED |
| Lexicographic | 56.0 | 17.0 | 1052 | 64.9 | 17.7 | 1026 | 8.0 |
| Maximin | 59.3 | 17.1 | 1024 | 66.8 | 18.2 | 1024 | 3.3 |
| CR | 65.0 | 12.4 | 949 | 72.3 | 18.2 | 934 | 85.6 |
| Euclidean | 50.4 | 18.7 | 1121 | 60.9 | 21.4 | 1134 | 0.2 |
| Arbitrary | 53.8 | 14.9 | 1065 | 61.6 | 18.9 | 1058 | 3.1 |

# 4 Experiment 2: Concordant-Ranks versus arbitrary strategy

In this experiment, we relaxed the constraints under which the alternatives were constructed, to check if a CR alternative would still be preferred to other alternatives with the same MAU. More specifically, one alternative was in line with the CR strategy while the other alternatives consisted of arbitrary attribute values. In addition, the alternatives were constructed to be less similar to each other although still having the same MAU. We checked that the choice according to the CR alternative would be at hand also in this experiment.

## 4.1 Method

### 4.1.1 Participants

A total of 31 undergraduate students (21 women) from Stockholm University participated. The subject recruiting procedure was the same as in Experiment 1 and 2.

### 4.1.2 Construction of choice alternatives.

One out of the five alternatives in a choice set was compatible with the CR strategy. The remaining four alternatives were forced to be incompatible with the CR strategy and consisted of arbitrary attribute values as long as the overall MAU was the same for all the alternatives in a set. In addition, each alternative in a choice set had to differ from the other four alternatives by 10 points (on the 0 to 100 scale used for the attribute values) on at least one attribute. This was done because the computer otherwise generated alternatives that would be difficult to differentiate from the CR alternative. It should be noted that the arbitrary alternatives in this experiment were different from the ones in Experiment 1 in that by chance they could be compatible with the lexicographic, maximin or Euclidean choice strategies.

### 4.1.3 Procedure

The procedure in Experiment 2 was the same as in Experiment 1. The mean time to make a choice was 26.00 seconds (*SD* = 10.48).

## 4.2 Results and discussion

### 4.2.1 Choices

Conducting a one-sample t-test on the difference between the actual percentage of choices of the CR alternative and the expected percentage of choices (20%) computed for each subject across the ten choice sets again showed that the CR alternative was significantly more chosen (29.4%) as compared to the arbitrary alternatives on average (17.7%), $t$ (30) = 2.346, $p < .05$.

### 4.2.2 Attractiveness evaluations

A paired sample t-test on the attractiveness values of CR alternative and the arbitrary alternatives showed that the CR alternative was rated as significantly more attractive (70.23) than the arbitrary alternatives (65.15), $t$ (30) = 2.349, $p < .05$. It may be concluded that choices in line with the CR strategy again were the most common type of choices among alternatives that were equal in MAU. Also, in terms of attractiveness evaluation, the CR alternative was evaluated as more attractive than the arbitrary alternatives.

# 5 General discussion

The aim of this study was to investigate strategies decision makers may use when making quick choices and attractiveness evaluations where the alternatives are approximately equal in overall value. At least three decision strategies are possible: using the CR strategy, random choice, and use of simple decision heuristics. In two experiments, comprising a total of 92 participants, it was found consistently that the most frequently chosen alternative was the CR alternative. Using alternative weights in the MAU assessments indicated that it indeed was the availability of a CR pattern that explains this result rather than a higher true MAU value of the CR alternative. Thus, the predominance of the CR strategy seems to be quite robust by not requiring strict equality of true MAU values for the alternatives at hand.

The runner-up chosen alternative in Experiment 1 was the maximin alternative, and the third most chosen alternative was the lexicographic alternative. These results imply that heuristic-based strategies seem to be preferred by some individuals (Dahlstrand & Montgomery, 1984). A reason that the heuristic-based alternatives were not the overall most chosen alternatives may be found in the information search process. Both the lexicographic and maximin heuristics consider a small portion of the information and stop looking as soon as this information search is satisfied. However, empirical findings show that in many situations, including situations where quick choices are required, decision makers search through more information (for examples see Glöckner & Betsch, 2008; Lee & Cummins, 2004; Troutman, & Shanteau, 1976). The CR strategy takes into account all given information, although on a rank order level. No computations are needed except checking the rank order of attribute values and importance weights.

Choosing the CR alternative may overlap with using an EBA (or SBA) decision strategy. To check for this possibility, each subject evaluated the attractiveness of ten randomly selected alternatives. If the participants used an EBA strategy, then judging each alternative independently would not give consistently higher ratings for an alternative because it would be difficult for them to search for information across the alternatives as is required by EBA. Conversely, if a CR strategy was used, then the alternative selected by the CR strategy should be judged as more attractive because it is closer to the ideal than the other alternatives, as was indeed found in both Experiment 1 and 2. Moreover, the fact that the CR alternative tended to be preferred both in choices and in attractiveness evaluations support the idea that the CR strategy is used as a proxy for finding the minimum distance to an ideal, independently of the type of judgment, rather than just being used as a tie-breaker.

The present results may be related to research on the prominence effect (Slovic, 1975; Tversky, Sattah, & Slovic, 1988) that decision makers will choose an alternative (e.g., an apartment) that is better on the more important attribute (e.g., price), but worse on less important attribute (e.g., location), although the two alternatives have been matched to be equal in attractiveness. Interestingly, in a series of studies, Montgomery, Selart, and their collaborators found that the prominence effect is also obtained not only for choices but also for attractiveness evaluations (Montgomery, Selart, Gärling, & Lindberg, 1994; Selart, Montgomery, Romanus, & Gärling, 1994; Selart, Gärling, & Montgomery, 1998), which implies that more important attributes loom larger in both choices and attractiveness evaluations as compared to matching tasks. This pattern of results parallels the present findings that more important attributes loom larger in choice and attractiveness evaluations than was expected from MAU estimates. To the extent that alternatives that have been matched as being equally attractive corresponds to equal MAU, these paralleling results mean that there might be a common explanation for the present findings and the prominence effect. That is, the prominence effect could result from the fact that choices and attractiveness evaluations, but not the performance in matching tasks, are guided by closeness to an ideal alternative.

Support for the possibility that WED to an ideal alternative may explain results in Experiment 1 comes from the fact that WED was related to type of alternative in the same way for choices and attractiveness evaluations, with one minor exception (choice of arbitrary vs. Euclidean in Experiment 1). According to all three types of data (i.e., choices, evaluations and WED) the CR strategy was most preferable followed by maximin, lexicographic, arbitrary and Euclidean, in that order. These results are compatible with weighted Euclidean distance to an ideal being a general principle guiding the evaluation of alternatives that are about equally attractive, with CR as the most distinct manifestation of WED to the ideal.

The present study concerned relatively quick choices. However, it is possible that the results also shed light on more prolonged decision processes. As already mentioned, a great number of studies have shown that people tend to make a preliminary decision quite early in the decision process and that they tend to stick to this choice in their final decision, although this decision may be justified in terms of arguments that emerge after the preliminary decision (Montgomery, 1983; 1989).

Finally, a note of caution should be made with regard to the significance of the present results. The data do not clearly discern MAU from WED as choice principles, because MAU was kept constant in both experiments. It cannot be excluded that participants (mistakenly) used concordant ranks as a proxy for evaluating MAU rather than WED. It would be of interest to design choice experiments in which choice alternatives are tailored in such a way that MAU and WED predict different choices independently of whether the rank order of attribute values and importance weights agree or not for a given alternative. Moreover, additional research is needed to test whether concordant ranks are used as proxy for minimization of WED to an ideal. Possibly, the CR strategy is used with no regard to WED or, conversely, that minimization of WED does not rely on CR but on some other unknown judgment strategy. In addition, it may be argued that the procedural difference in the think-aloud and direct-stating groups could have led to different reasoning when choosing. However, results indicated that there were no differences between the groups in terms of preferred choice strategy and therefore it is plausible to assume that, even though the attributes were derived differently in the two groups, the choices were still consistent with the CR strategy.

Although other possibilities of interpreting the data remain, arguments can be raised in support of our approach—and against the alternative interpretations. First, the assumption that the predominance of choices of the CR alternative reflects minimizing the WED to an ideal has a clear theoretical rationale, as shown by the mathematical proof in Appendix A. The validity of this theoretical rationale is strengthened by the fact that the CR alternatives in a large majority of the choice situations had shorter WED to the ideal than was true for the other choice alternatives. The lack of a corresponding theoretical rationale in other strategies gives an advantage to the CR strategy. Second, there is the striking fact that not only choices but also attractiveness evaluations favored the CR alternative. The alternative interpretations discussed earlier do not explain why also the attractiveness evaluations favored the CR alternative. Third, in contrast to the alternative interpretations, the results of the present study suggest an overriding principle—minimization of WED to an ideal—as a basis for the preference order between *all* the different types of alternatives investigated in the present study.

In summary, the results from the present two experiments support the possibility that choices could be based on the concordance of the rank order of the attribute values in an alternative and the corresponding importance weights. In addition, such an alternative will also have the shortest WED to an ideal alternative, provided that MAU is kept constant across alternatives. Especially striking is that attractiveness evaluations of single alternatives also showed a tendency to favor the CR alternative, which speaks against explanations based on comparisons between alternatives. Moreover, the notion that proximity to an ideal is of importance for choice and attractiveness evaluations is further underpinned by the high degree of agreement between the WED measures and preference order for choices and attractiveness evaluations of different types of alternatives. However, additional research is needed to clarify the generality of the findings of the present study.

# References

Barthlemy, J. P., & Mullet, E. (1992). A model of selection by aspects. *Acta Psychologica*, *79*, 1–19.

Birnbaum, M. H., & LaCroix, A. R. (2008). Dimension integration: Testing models without trade-offs. *Organizational Behavior and Human Decision Processes*, *105*, 122–133.

Brownstein, A. L. (2003). Biased predecision processing. *Psychological Bulletin*, *129*, 545–568.

Coombs, C. H., Dawes, R. M., & Tversky, A. (1984). *Mathematical psychology* (Coombs, Ed.). Englewood Cliffs, NJ: Prentice Hall.

Dahlstrand, U., & Montgomery, H. (1984). Information search and evaluation processes in decision making: A computer based process tracing study. *Acta Psychologica*, *56*, 113–123.

Dawes, R. M. (1979). The robust beauty of improper linear models in decision making. *American Psychologist*, *34*, 571–582.

De Leeuw, J., & Pruzansky, S. (1978). A new computational method to fit the weighted euclidean distance model. *Psychometrika*, *43*, 479–490.

Gati, I. (1986). Making career decisions: A sequential elimination approach. *Journal of Counseling Psychology*, *33*, 408–417.

Gigerenzer, G., & Goldstein, D. (1996). Reasoning the fast and frugal way: Models of bounded rationality. *Psychological Review*, *103*, 650–669.

Gigerenzer, G., Todd, P. M., & ABC Research Group the. (1999). *Simple heuristics that make us smart*. New York: Oxford University Press.

Glöckner, A., & Betsch, T. (2008). Multiple-reason decision making based on automatic processing. *Journal of Experimental Psychology: Learning, Memory, and Cognition*, *34*, 1055–1075.

Humpreys p, & MacFadden, W. (1980). Experiences with maud: Aiding decision structuring versus bootstrapping the decision maker. *Acta Psychologica*, *45*, 51–69.

Jäckel, M., & Wollscheid, S. (2007). Time is money and money needs time? a secondary analysis of time-budget data in germany. *Journal of Leisure Research*, *39*, 86–108.

Juslin, P., Jones, S., Olsson, H., & Winman, A. (2003). Cue abstraction and exemplar memory in categorization. *Journal of Experimental Psychology: Learning, Memory, & Cognition*, *29*, 924–941.

Lee, M. D., & Cummins, T. D. R. (2004). Evidence accumulation in decision making: Unifying the take the best and the rational models. *Psychonomic Bulletin and Review*, *11*, 343–352.

Mellers, B. A., & Biagini, K. (1994). Similarity and choice. *Psychological Review*, *101*, 505–518.

Montgomery, H. (1983). Decision rules and the search for a dominance structure: Towards a process model of decision making. In P. Humphreys, O. Svenson, & A. Vari (Eds.), *Analyzing and aiding decision processes* (pp. 471–483). Amsterdam: North-Holland.

Montgomery, H. (1989). From cognition to action: The search for dominance in decision making. In H. Montgomery & 0. Svenson (Eds.), *Process and structure in human decision making* (pp. 23–49). Chichester, UK: John Wiley & Sons.

Montgomery, H., Selart, M., Gärling, T., & Lindberg, E. (1994). The judgment-choice discrepancy: Noncompatibility or restructuring? *Journal of Behavioral Decision Making*, *7*, 145–155.

Montgomery, H., & Svenson, O. (1976). On decision rules and information processing strategies in multiattribute decision making. *Scandinavian Journal of Psychology*, *17*, 283–291.

Payne, J. W., Bettman, J. R., Johnson, E. J., & Luce, M. F. (1995). An information processing perspective on choice. In J. R. Busemeyer, R. Hastie, & D. Medlin (Eds.), *Decision Making from the perspective of cognitive psychology* (pp. 137–175). San Diego, CA: Academic Press.

Rieskamp, J., & Hoffrage, U. (1999). When do people use simple heuristics and how can we tell. In G. Gigerenzer, P. M. Todd, & the ABC Research Group. *Simple heuristics that make us smart* (p. 141–167). New York: Oxford University Press.

Rubinstein, A., & Zhou, L. (1999). Choice problems with a ?reference? point. *Mathematical Social Sciences*, *37*, 205–209.

Russo, J. E., Johnson, E. J., & Stephens, D. L. (1989). The validity of verbal protocols. *Memory and Cognition*, *17*, 759–769.

Scott, S. G., & Bruce, R. A. (1995). Decision making style: The development and assessment of a new measure. *Educational and Psychological Measurement*, *55*, 818–831.

Selart, M., Gärling, T., & Montgomery, H. (1998). Compatibility and use of information processing strategies. *Journal of Behavioral Decision Making*, *11*, 59–72.

Selart, M., Montgomery, H., Romanus, J., & Gärling, T. (1994). Violation of procedure invariance in preference measurement: Cognitive explanations. *European Journal of Cognitive Psychology*, *6*, 417–436.

Shafir, E., Simonson, I., & Tversky, A. (1993). Reason-based choice. *Cognition*, *49*, 11–36.

Shoemaker, P. H., & Waid, C. C. (1982). An experimental comparison of different approaches to determining weights in additive utility models. *Management Science*, *28*, 185–196.

Slovic, P. (1975). Choice between equally valued alternatives. *Journal of Experimental Psychology: Human Perception and Performance*, *1*, 280–287.

Svenson, O. (1979). Process descriptions of decision making. *Organizational Behavior and Human Performance*, *23*, 86–112.

Svenson, O. (1992). Differentiation and consolidation theory of human decision-making: A frame of reference for the study of pre-and post decision processes. *Acta Psychologica*, *80*, 143- 168.

Troutman, C. M., & Shanteau, J. (1976). Do consumers evaluate products by adding or averaging attribute information? *Journal of Consumer Research*, *3*, 101–106.

Tversky, A. (1972). Elimination by aspects: A theory of choice. *Psychological Review*, *79*, 281–299.

Tversky, A. (1977). Features of similarity. *Psychological Review*, *84*, 327–352.

Tversky, A., & Kahneman, D. (1974). Judgment under uncertainty: Heuristics and biases. *Science*, *185*, 1124–1131.

Tversky, A., Sattah, S., & Slovic, P. (1988). Contingent weighting in judgment and choice. *Psychological Review*, *95*, 371–384.

Von Winterfeldt, D., & Edwards, W. (1986). *Decision analysis and behavioral research* (D. von Winterfeldt & W. Edwards, Eds.). New York: Cambridge University Press.

Wedell, D. H., & Parducci, A. (1988). The category effect in social judgment: Experimental ratings of happines. *Journal of Personality and Social Psychology*, *55*, 341–356.

Zakay, D., & Barak, A. (1984). Meaning and career decsion making. *Journal of Vocational Behavior*, *24*, 1–14.

Zakay, D., & Dil, D. (1984). Degree of post-decisional confidence as a function of the distances of the offered alternatives from an ?ideal? alternative. *Archiv fr Psychologie*, *136*, 293–300.

Zeleny, M. (1976). The attribute-dynamic attitude model. *Management Sciences*, *23*, 12–25.

# Appendix

## A Proof that minimized weighted Euclidean distance implies agreement between rank-order of weights and attribute-values given a constant MAU

We will now prove that an alternative having minimized weighted Euclidean distance to an ideal alternative will also have the same rank-order within the attribute values and importance weights as in the ideal alternative, given a constant MAU. The shortest non-weighted Euclidean distance, $d$, is calculated by the Pythagorean Theorem:

$$d = \sqrt{\sum_{i=1}^{n}(o - a_i)^2}.$$

where $a_i$ is the attractiveness value on attribute $i$, and $o$ is the attractiveness value on any attribute of the ideal alternative ($o$ is assumed to be constant for all attributes). The weighted Euclidean distance used in his paper, is based on squared weights (de Leuw & Pruszanky, 1978) That is, the non-squared difference $(o - a_i)$ is multiplied by $a$ weight $w_i$. It is then assumed that the weights can be seen as included in the distance implying that $w_i(o-a_i)$ is equivalent to the distance $w_i a_i - w_i o$. Squaring this expression yields $w^2(o - a_i)^2$. Thus, the shortest weighted Euclidean Distance (WED) between an alternative $j$, (e.g., the WED alternative) and the ideal alternative is calculated by including the squared weight of each attribute:

$$\mathrm{WED}_j = \sqrt{\sum_{i=1}^{n} w_i^2(o - a_i)^2}. \qquad (1)$$

where $w_i$ is the importance weight of attribute $i$.
For simplification assume the following notation:

$$I_i = w_i o.$$

$$X_i = w_i a_i.$$

We show the theorem for three attributes, a generalization is straightforward.
We rewrite $\mathrm{WED}_j$:

$$\mathrm{WED}^2 = (I_1 - X_1)^2 + (I_2 - X_2)^2 + (I_3 - X_3)^2 =$$
$$I_1^2 + X_1^2 - 2I_1X_1 + ... + I_3^2 + X_3^2 - 2I_3X_3. \qquad (2)$$

Because $o$ is constant, $\mathrm{WED}^2$ can be computed as a function of the variables $X_1, X_2$, and $X_3$:

$$F(X_1, X_2, X_3) = (I_1^2 + I_2^2 + I_3^2) + (X_1^2 + X_2^2 + X_3^2)$$
$$-2(I_1X_1 + I_2X_2 + I_3X_3).$$

We assume a constant MAU, $k$, which implies:

$$k = X_1 + X_2 + X_3. \qquad (3)$$

We next prove that the alternative with minimized Euclidean distance and constant MAU must have the same order on its attributes as the importance weight of the ideal alternative. The proof makes use of Lagrange multipliers, whereby we minimize the squared WED, given by the function $F$, under the constraint of a constant MAU:

$$L(X_1, X_2, X_3, \lambda) = F(X_1, X_2, X_3) - \lambda(X_1 + X_2 + X_3 - k).$$

As customary, we differentiate $L$ with respect to each of the variables and equate with 0 to get the critical values:

$$\frac{\partial L}{\partial X_1} = 2X_1 - 2I_1 - \lambda = 0. \qquad (4)$$

$$\frac{\partial L}{\partial X_2} = 2X_2 - 2I_2 - \lambda = 0. \qquad (5)$$

$$\frac{\partial L}{\partial X_3} = 2X_3 - 2I_3 - \lambda = 0. \qquad (6)$$

$$\frac{\partial L}{\partial \lambda} = (X_1 + X_2 + X_3 - k) = 0. \qquad (7)$$

A rearrangement of the above gives:

$$X_1 = \frac{\lambda}{2} + I_1. \qquad (8)$$

$$X_2 = \frac{\lambda}{2} + I_2. \qquad (9)$$

$$X_3 = \frac{\lambda}{2} + I_3. \qquad (10)$$

$$X_1 + X_2 + X_3 = k. \qquad (11)$$

The sum of the Equations 8, 9 and 10 is:

$$\frac{\lambda}{2} + I_1 + \frac{\lambda}{2} + I_2 + \frac{\lambda}{2} + I_3 = X_1 + X_2 + X_3 = k$$

$$\Rightarrow \frac{\lambda}{2} = \frac{k}{3} - \frac{1}{3}(I_1 + I_2 + I_3). \qquad (12)$$

According to Equations 8, 9, and 10:

$$I_1 - X_1 = \frac{\lambda}{2}, I_2 - X_3 = -\frac{\lambda}{2}, I_3 - X_3 = \frac{\lambda}{2}. \qquad (13)$$

According to Equations 12 and 13:

$$I_1 - X_1 = I_2 - X_2 = I_3 - X_3 = \frac{1}{3}(I_1 + I_2 + I_3) - \frac{k}{3}.$$

We now substitute $I_i$ and $X_i$ to solve the minimized attributes $a_i$:

$$I_i - X_i = w_i o - w_i a_i = \frac{(w_1 + w_2 + w_3)o - k}{3}$$

$$\Rightarrow a_i = o - \frac{(w_1 + w_2 + w_3)o - k}{3w_i}. \tag{14}$$

Note that $o$ and the numerator in equation 14 are the same for every attribute $a_i$. Hence, the weight $w_i$ in the denominator determines the attribute value for $a_i$. This shows that the higher the weight, the higher the attribute value. Given unique attribute values, this implies the attribute values are in the rank-order.

Q.E.D

The above proof can be extended by the following corollary, which shows shorthand for calculating the weighted minimum Euclidean distance for the given MAU($k$). Substituting $I_i - X_i$ with equation 13 gives the minimum WED(WED$_{min}$)

$$\begin{aligned}
\text{WED}^2_{min} &= \left(\frac{(w_1 + w_2 + w_3)o - k}{3}\right)^2 + \\
&\left(\frac{(w_1 + w_2 + w_3)o - k}{3}\right)^2 + \\
&\left(\frac{(w_1 + w_2 + w_3)o - k}{3}\right)^2 \\
&= 3\left(\frac{(w_1 + w_2 + w_3)o - k}{3}\right)^2 \\
\text{WED}_{min} &= \frac{1}{\sqrt{3}}(w_1 + w_2 + w_3)o - k.
\end{aligned}$$

## B Calculation of constant MAU for each alternative

In order to apply MAU values that were 80% of the ideal alternative, we first calculated MAU for the ideal alternative, and then we reduced MAU for the generated alternatives with 20%. The MAU for the ideal alternative (MAU$_{opt}$) was calculated by:

$$\text{MAU}_{opt} = \left(\sum_{i=1}^{n} o w_i\right).$$

where $o$ is the ideal value that is considered to always be 100 and $w_i$ are the attribute weights given by the participants. This means that MAU$_{opt}$ will always be 10000 (because the weights always sum up to 100 for any number of attributes). In order to get the desired MAU(MAU$_D$), which was 80% of MAU$_{opt}$ (hence is 8000), on each generated alternative, we used the following function:

$$\text{MAU}_D = 0.8 \times \text{MAU}_{opt}.$$

We let the computer generate random numbers $_R$i with mean=80 (80% of the ideal value 100) and SD=10. Our generated alternative has MAU:

$$\text{MAU}_R = \sum_{i=1}^{n} R_i w_i.$$

MAU$_R$ will most likely not be equal to MAU$_D$, we therefore modify $R_i$ such that MAU$_R$ $= MAU_D$. We do this compensation by multiplying each $R_i$ by a constant $c$, to get the desired MAU. The constant is:

$$c = \frac{\text{MAU}_D}{\text{MAU}_R}.$$

So we will now have attribute values $a_i$, where

$$a_i = R_i c = R_i \frac{\text{MAU}_D}{\text{MAU}_R}.$$

We repeat generating random numbers to ensure that $\leq a_i \leq 100$. We verify that our alternative will have MAU equal to MAU$_D$:

$$\text{MAU}_D = c \times \text{MAU}_R = \sum_{i=1}^{n} a_i w_i = c \sum_{i=1}^{n} R_i w_i.$$

Because the computer generated alternatives with MAU value equal to 8000, the attribute values could sometimes be in decimals. These values were rounded to the closest whole number implying a maximal deviation of $0.5/8000 = 0.0625\%$.

## C Theoretical basis, based on range frequency theory, for linear transformation of the attribute weights estimated in the present study

A theoretical basis for the linear transformations calculated on the attribute weights in the present study can be found in range frequency theory (e.g., Wedell & Parducci, 1988):

$$J = wR + (1 - w)F.$$

where $J$ is the observed judgment for a given stimulus $S$ in a given context, $w$ is a weight ($< 1$), $R$ is the range value of the stimuli (= the position of $S$ on a continuum going from $S_{min}$ to $S_{max}$ in the included context) and $F$ is the frequency value for $S$ (= the rank of $S$ in the total number of contextual stimuli). In our data $J$ will be the empirical weight estimates, $R$ will be the weights that actually were used by the participants in choice and judgment, and $F$ = the frequency the value of the weight.

Assuming that the weight estimates are anchored in equal weights (Weber & Borcherding, 1993) implies that the $F$ can be assumed to be equal to the mean of the weights given by the participants for a given set of alternatives (if the ranks are expressed in terms of percentages). Solving for $R$, letting $R = W{*}_i$ and $J = W_i$ ($i$ = attribute $i$) and letting $F = M$ (of the weights), yields:

$$W{*}_i = \frac{(W_i - (1 - w))M}{w}.$$

This equation can be reformulated to the equation that we used for the linear transformations of the weight estimates used in our "old" MAU data: $W{*}_i = C(W_i - M) + M$, if we let $C = \frac{1}{w}$.

https://doi.org/10.1017/S1930297500001406 Published online by Cambridge University Press