Radiatively-Driven Stellar Winds

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1. Introduction

In this review, attention will be focussed exclusively on the winds of hot stars, concentrating for the most part on spectral types O and B. For these stars, a clear consensus has emerged that it is the gradient of selective radiation pressure - i.e., line-driving - that explains the high terminal velocities, typically ~ 2000 km s^{-1}, of their winds. Accordingly, few would now doubt that the supersonic zones of these winds present us with rather clean examples of line-driven flow, whose investigation therefore properly belongs under the heading "Radiation Hydrodynamics". Moreover, in marked contrast with other astronomical environments where line-driven winds may exist, the stellar case is geometrically and parametrically well defined and is thus by far the best natural example from which to learn about such flows.

Because of the nature of this volume, no attempt will be made to review the observational discoveries that have directly stimulated most of the theoretical work discussed herein. Nevertheless, it is worth emphasizing that another advantage stars offer the student of radiatively-driven flows is an abundance of superb data. With the phenomenon common even among the naked-eye stars, high signal-to-noise data is fairly readily obtained. Moreover, this area has been richly served by the remarkable advances over recent years in astronomical instrumentation, which have provided highly pertinent data all the way from X-ray to radio wavelengths. An important consequence of this relative data richness is that a number of "higher order" investigations can fruitfully be carried out, since there is still the definite prospect of a critical confrontation with observations. Among these are the growth of intrinsic instabilities to limiting amplitude and the perturbing effects of pulsation, rotation, and magnetic fields, as well as of companions.

A further restriction deriving from the nature of this volume is the exclusion of purely technical issues, especially the fascinating topic of numerical technique. This is nevertheless of crucial importance: The theoretical investigation of line-driven winds is now relatively mature and further advances are likely to require techniques that are not fundamentally built on such assumptions as monotonic flow, independently acting lines, and no continuum formation within the flow. These advances in technique are also required if purely theoretical wind models are to contribute input data to related topics in astrophysics, e.g., stellar evolution with mass loss.

Although somewhat peripheral to a discussion of theoretical problems and physical effects, some attention will be given to diagnostics. As is almost always true in astronomy, the observational data lack the information content for a reliable inversion to determine the run of physical conditions through these winds. Accordingly, theoretical models often cannot be tested by comparing with pre-existing deductions but must themselves be mapped onto the observational plane - and thought given to the uniqueness of any resulting success.

2. Basic Mechanisms

The basic acceleration mechanism is most readily understood by considering a radial flow in which a single ionic species is being driven away from a massless point con-

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tinuum source in consequence of the matter-radiation coupling provided by a single resonance transition at frequency ν_i. At the point where the velocity is v, the Doppler effect shifts continuum radiation of rest frequency $\nu_i(1+v/c)$ into resonant interaction with local ions, the average result of which per scattered continuum photon is the transfer of radial momentum $h\nu_i/c$ to each scattering ion. The velocity increment implied by this momentum transfer then brings the continuum at a somewhat higher frequency into resonance; the process therefore repeats and so the flow's acceleration continues. Accordingly, this mechanism naturally gives rise to the expectation that, whenever the total specific rate of momentum transfer due to lines exceeds a star's gravitational acceleration, a stationary wind solution will exist with a velocity law v(r) that is a monotonically increasing function of radius r.

This simple, qualitative description of line-driving requires a number of further comments:

1) The successive use of continuum radiation at higher and higher frequencies to accelerate matter was first invoked in the 1920's as a means of driving individual atoms away from stars [1,2]. In recent years, it has been recognized [3-5] that momentum coupling via Coulomb scatterings is usually strong enough to prevent significant drift velocities between the various scattering ions and the undriven matter - e.g., the protons. Accordingly, a one-fluid, hydrodynamic description of such flows may be used, with the radiative acceleration - i.e., the gradient of selective radiation pressure - appearing as a body-force term in the equation of motion. The resulting system of equations, though much studied, is still rich in research possibilities, especially with regard to time-dependent, non-linear effects. Nevertheless, we should not forget that at the lowest observed mass loss rates a breakdown of momentum coupling is indicated at only a modest distance above the star's surface. A multi-fluid treatment then becomes necessary, either to demonstrate that instabilities induced by the onset of differential streaming effectively maintain momentum coupling, or to follow the growth of streaming.

2) The acceleration mechanism is obviously fundamentally unchanged if a number of well separated lines act simultaneously, for then they contribute independently and cumulatively to the line-driving. Indeed, it was the realization [4,5] that numerous effective lines exist below the Lyman limit that first brought theory and observation into reasonable agreement for the hottest early-type stars. But now, if even closer agreement is to be achieved, account must be taken of violations of the assumption of independently acting lines. In fact, for realistic models [6], only a small minority of lines interact throughout the wind with pure continuum; the vast majority, having close violet neighbours, interact with a partially attenuated continuum and with a diffuse component comprising the continuum scattered by these neighbouring transitions. These multi-line effects [6-8] must now be treated accurately in all quantitative modelling of line-driven winds.

3) Implicit to the stated expectation of continued acceleration, resulting in stationary solutions with monotonic velocity laws, is the assumption that matter retains its ability to absorb momentum from the photospheric radiation field. This it clearly does if conditions of excitation and ionization change only slowly in the outflow. Excitation poses no problem in this regard since line-driving is dominated by resonance transitions [9,6] and each such momentum-transferring line-scattering is effectively instantaneously followed by de-excitation to the original (i.e., ground) state. Ionization also usually poses no problem: Although recombination time scales are not infinitesimal, ionization equilibrium in fact holds to a good approximation throughout the flow [3,5,10] - i.e., advection terms may be neglected - and, because the electron density and the energy density of ionizing radiation have similar radial dependences, predicted ionic abundance ratios are indeed slowly varying. However, exceptions may arise when a major constituent (i.e., H^+, He^+, or He^{++}) recombines within the wind, for then ionizing radiation sustaining (in some cases) or hitherto suppressing (in other cases) significant driving ions is greatly diminished. The resulting modulation of the driving force could conceivably exclude a stationary solution - e.g., by not accelerating matter to above the escape velocity - and thereby oblige the wind to undergo relaxation oscillations, though probably not with rigorous

preservation of spherical symmetry. A time-dependent treatment of line-driven winds is needed to investigate such possibilities.

4) The momentum gained by matter is above stated to be on average $h\nu/c$ in the radial direction per scattered continuum photon - i.e., exactly the photon's incident momentum. This is true if there is on average no change in an ion's radial momentum (a) as it decays back to the ground state subsequent to its scattering of a continuum photon, and (b) as it interacts with photons scattered by identical neighbouring ions. The first condition follows from the scattering phase function's fore-aft symmetry with respect to incident direction, and the second is a good approximation in laminar flow as a result of this same property of the phase function, for then the scattered radiation has negligible flux at ν_i. In fact, for a monotonic velocity law, this second condition is rigorously satisfied [11,12] in the narrow-line limit (Sobolev approximation). But in a real wind, two effects lead to an ion often finding itself in a scattered radiation field with non-negligible flux at ν_i, so that these ions' contribution to the total radiative acceleration g_R differs significantly from that obtained assuming pure extinction of the continuum. The first effect, already discussed, is a consequence of neighbouring transitions on the violet side of ν_i. The second is a consequence of local chaotic supersonic motions that make the flow multiply non-monotonic [13,14]. Accordingly, if one were to adopt a "mean field" approach in order to average over this local "turbulence", the corresponding effective phase function is asymmetric, so that some momentum transfer does then occur when an ion interacts with photons scattered by identical neighbouring ions. Note that these two effects cease to be independent and distinguishable when the amplitude of the chaotic motions exceeds the velocity separation of neighbouring transitions.

The above remarks relate to the radiative acceleration of an already existing outflow. Further concerns, therefore, are the flow's initiation and early acceleration, processes sometimes suspected of requiring an unconventional mechanism - i.e., one not included in standard treatments of line-driven winds.

The suspicion that line-driving cannot start what it finishes is prompted by the sharp drop in g_R for outflow velocities $\lesssim v_D$, the typical thermal velocity of the driving ions, an effect due to the shadowing of these ions by their counterparts in the essentially static photosphere. This consequence of photospheric line formation would thus seem to imply that some unconventional mechanism is responsible for the initial acceleration to $v \sim v_D$ and, as a corollary, that there is no wind when this mechanism is not operating. Thus the entire phenomenon of line-driven winds would then be fundamentally dependent on this mechanism, which would therefore completely determine the domain of mass loss as well as significantly influencing the winds' characteristics. Accordingly, even apart from possible diagnostics of physical conditions in the initial flow, the existence of an unconventional initiating mechanism should become evident when the predicted characteristics of winds are tested.

Although superficially plausible, the argument for the necessity of a distinct initiating mechanism is not in fact conclusive. Without adding unconventional effects or neglecting photospheric line formation, time-independent, line-driven wind solutions can be readily constructed [3,15,16]. In these solutions, acceleration to $v \gtrsim v_D$ is the result of an unbalanced gas pressure - i.e., an upwelling - a condition permanently maintained by the radiative expulsion of the overlying matter that might otherwise provide a confining weight. Moreover, again without adding unconventional effects or neglecting photospheric line formation, calculations [3,17] exist suggesting that these winds are the inevitable consequence of the non-existence of static reversing layers, a conclusion indicated by the finding that lines give $g_R > g$ in the high atmosphere, implying an outwardly directed net force that cannot be balanced.

A most interesting aspect of calculations of static and moving reversing layers is that the parameter domain wherein moving solutions exist greatly exceeds that wherein static solutions do not exist [3,17]. Thus there is an intermediate domain wherein a star has two options, remaining static or having a wind. Now, when the most detailed calculations [17] are compared with observational data, stars are indeed found to lose mass when no static reversing layer exists but, most significantly, show little or no

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evidence of mass loss when both options are open. These findings coupled with the absence of any surviving successful prediction deriving from the hypothesis of a distinct initiating mechanism strongly suggest that no such mechanism plays more than a minor part in initiating line-driven winds.

The view presented above is that stars have line-driven winds in consequence of the non-existence of hydrostatic equilibria for their reversing layers. As already noted, this suggestion derives from calculations [3,17] giving $g_R > g$ in the high atmosphere where low electron density allows new ions to appear which, not having counterparts in the photosphere, do not have their contributions to g_R much reduced by shadowing (i.e., line formation). But the published investigations of this effect are not self-consistent macroscopically, since the contribution of lines to g_R is computed only after the atmosphere's stratification is prescribed. Somewhat surprisingly, hydrostatic equilibria can in fact be found when self-consistency is demanded, at least for line formation by coherent scattering in a Schuster-Schwarzschild atmosphere. These equilibria have the form of a normal atmosphere surmounted by a low density, "zero-gravity" halo whose extent is such as to build up the column density of the "new" ions to the point that line formation reduces g_R to just below g. But now these solutions are not self-consistent microscopically, since the halo densities are so low that momentum sharing breaks down. Accordingly, a deeper explanation for the occurrence of line-driven winds would seem to involve the viability rather than the strict existence of static reversing layers. Nevertheless, the existing determination [17] of the domain of unavoidable mass loss is likely to remain approximately valid since the criterion $g_R > g$ at small optical depths is a good indicator of a non-viable static reversing layer.

3. The Standard Model

Given that an unconventional mechanism for initiating line-driven winds is neither necessary theoretically nor indicated observationally, a standard model for such a wind rather naturally defines itself as being the spherically symmetric, time-independent flow whose mass-loss rate Φ, velocity law $v(r)$, temperature stratification $T(r)$, as well as the complete radiation field $I_\nu(r,\mu)$ are determined with mutual consistency from the relevant equations and boundary conditions. Specifically, this definition implies that the only forces acting are those due to gravity and to the gradients of the gas and radiation pressures, and that $T(r)$ is determined by energy exchange with the radiation field and by adiabatic expansion. In particular, therefore, mechanical and thermal effects due to rotation, magnetic fields, pulsations, and instabilities are neglected.

With the standard model thus defined, we know without actually obtaining solutions that they would be unstable, and also that they would fail to explain several observed phenomena. Among these are: superionization [3,18,19], X-ray emission [20-23], flat-bottomed absorption troughs [24,13,14], narrow absorption components [25-27], and non-thermal radio emission [28,29]. Given these problems, one might well ask: Why compute standard models so defined? Some answers to this question are:

1) To investigate the nature, location and importance of omitted effects by quantifying the failures of the standard model, especially its predictions of Φ and v_∞.

The suggestion that standard models should indeed be computed is thus not tantamount to the discounting of additional effects, rather it stems in part from the desire to advance their discussion beyond mere verbal speculations or simple, ad hoc modelling. Needless to say, to be informative, the failures must be established with calculations not compromised by inadequate approximations or incomplete physical data. When thus used, standard models play a rôle analogous to that of spherical stellar models in studies of cluster H-R diagrams, where failures are interpreted as evidence of mixing, mass loss, etc. Many other examples could be cited of the unquestioned usefulness of standard models that omit known effects.

A specific example of the diagnostic use of the standard model concerns the hot corona - cool wind model [30,31], which is still of current interest [32]. In this model, line-driving serves only to accelerate a flow whose mass loss rate is already determined by non-radiative effects in the coronal zone. If so, the standard model must be expected to fail in predicting both Φ and v_∞. However, if used to investigate only the cool, supersonic flow, the standard model would then be expected to confirm that line-driving indeed accounts for the momentum flux of the terminal flow. Such a combination of failure and success would identify the base of the flow as the location of the standard model's deficiency and thereby provide support for the hot corona - cool wind model. Of course, pulsations and turbulence, both of which may have kinetic energy densities comparable to that of the mean flow at the sonic point, would be plausible alternatives.

The above-mentioned test of line-driving in supersonic flow has in fact already been carried out successfully for ζ Puppis [6]. It remains to be seen, however, whether a complete standard model for this well-observed star gives the correct Φ and v_∞.

2) The standard model's predictions might well be rather accurate, in which case empirical mass loss formulae used in following the evolution of massive stars could be replaced by Φ's derived from first principles.

Some reasons for expecting usable predictions are:

a) The fair degree of success already achieved [17,33,34] with the CAK model [15] together with the reasonable expectation that remaining residuals are largely due to compromising approximations and not to a neglected unconventional mechanism.

b) To the extent that they are understood, the effects responsible for the standard model's immediate observational failures are not of great structural consequence. For example, the ions giving evidence of superionization are only trace constituents and so contribute negligibly to g_R [34]. Also the asymmetric effective phase function implied by flat-bottomed absorption troughs has only a moderate impact on computed Φ's [6].

c) When time averages are taken of equations that purport to describe the finite amplitude state reached by these unstable winds [35], the resulting corrections to the equations of the standard model are $\sim \eta = \frac{1}{2}$ U/V, where U and V are the propagation and star frame speeds, respectively, of radiatively-driven shocks. Typical values are believed to be V $\sim$ 2500 km s^{-1} and U $\sim$ 500 km s^{-1}, so that $\eta \sim 0.1$, suggesting that a solution of the time-averaged equations will differ little from the corresponding standard model. Note that η measures the local inefficiency of radiative driving [35] - i.e., the fraction of work done by radiative forces lost by radiative cooling at shock fronts.

The effects most likely to frustrate hopes of usable predictions are those that would be introduced by large amplitude turbulence in the transonic flow. If this turns out to be so, the computation of Φ's by applying the standard model to the supersonic flow with v_∞ imposed from observation [6] would still be accurate, and so the requirements of stellar evolution investigators could still be met.

3) To provide zero-order models for studies of the instabilities of line-driven winds.

Of itself, however, this is not a compelling reason: neither the existence of instabilities nor their growth rates appear to depend delicately on a wind's structure.

The standard model here defined has strictly laminar flow. However, as described earlier, an interesting variant is obtained by averaging over local supersonic "turbulence". Because several of the immediate failures of the standard model are probably consequences of this turbulence, this modified standard model should further improve the predictive power of stellar wind calculations.

4. The CAK Model

Because computing standard models as defined above is a formidable undertaking, earlier investigators have sensibly adopted simplifying approximations. These are here identified and discussed in the case of the well-known CAK model [15], with the intent of suggesting that its usefulness is by now exhausted, at least for the prediction of wind parameters. Future investigators should therefore not content themselves with minor, cosmetic improvements of this model.

The major departures of the CAK model from the ideal standard model are the following:

1) A core-halo structure is assumed - i.e., continuum formation is neglected in the wind as are dynamical effects in the photosphere.

This is well justified for O and B stars, for which the model was intended. But for W-R stars, the modelling of whose winds is a major technical challenge, continuum formation remains significant out to quite high velocities and so must be taken into account in calculating g_R and in determining the structure of the transonic flow.

2) The emergent photospheric radiation field is not affected by the wind and so can be taken from a standard model atmosphere.

Although continuum emission from the winds of O and B stars can reasonably be neglected, line emission cannot, especially for O stars in the EUV [6]. The back-warming or blanketing effect of this back-radiation from the wind changes the emergent radiation from the photosphere with both dynamical [13,6] and diagnostic [36,37] consequences and should be incorporated into future models. Needless to say, for the W-R stars this effect is of major importance for the temperature stratification of the continuum-forming layers which, as noted above, include much of the accelerating supersonic flow. No reliable analyses of W-R spectra can be performed until models taking this effect into account become available.

3) Throughout the wind, each line interacts only with the star's diluted photospheric radiation - i.e., multi-line transfer effects are neglected.

In order that a line should nowhere interact with photons scattered by its nearest neighbour, it must be separated from that neighbour by $\Delta\nu > 2\nu v_\infty/c$. Now, if the spectrum were covered with strong lines having just this minimum separation, half the photons emitted by the star would scatter in the wind and so Φ would be half the single-scattering limit - i.e., $\Phi = \frac{1}{2}L/cv_\infty$. Thus, whenever Φ exceeds this critical value, multi-line transfer must play some rôle. In reality, this value is barely exceeded at the highest mass loss rates found for O and B stars, a result suggesting perhaps that multi-line effects may indeed be neglected. However, the extreme clumpiness of the line distribution ensures that these effects are in fact important even for mass loss rates far below this critical value [6]. Accordingly, multi-line transfer must be included in future quantitative modelling of O and B star winds. Moreover, models of W-R winds that ignore these effects must inevitably be hopelessly inadequate.

4) The narrow-line limit (Sobolev approximation) is used in computing each line's contribution to the radiative acceleration g_R.

With regard to a specific transition, an ion in the wind finds itself able to interact radiatively with another identical ion provided their velocity of approach or separation is $\lesssim v_D$. Now, because v_D in these cool winds is small compared to the typical wind speed, the distance within which this interaction is possible is small compared to the characteristic dimension of the wind. Sensibly taking advantage of the existence of a small parameter ($v_D/v_\infty \sim 10^{-3}$), we commonly take the narrow-line limit ($v_D/v_\infty \to 0$), in which case the region of interaction becomes vanishingly small. This limit, when combined with assumptions 1) - 3), yields a simple formula [11,12] for a single line's contribution to g_R that is basic to the CAK formulation.

For the standard model with laminar flow, the Sobolev approximation retains its supreme usefulness. Phenomena such as wind-blanketing, continuum formation in the wind, and multi-line transfer in no way forbid its use, and indeed it greatly facilitates their incorporation into the standard model [6]. However, for a modified standard model that somehow accounts for local velocity fluctuations, the validity of this approximation must be reconsidered. For example, Hamman [38,39] of necessity avoided its use when exploring the implications of high microturbulence in these outflows. On the other hand, the highly supersonic velocity fluctuations implied by the derived microturbulent velocities ($\sigma \sim 150$ km s^{-1}) suggest shocks, and their effect in line formation can be investigated with a piece-wise application of the Sobolev approximation - i.e., between consecutive shocks [13,14].

5) The mass-loss rate Φ is determined by the requirement of regularity at a critical point in the suprsonic flow - i.e., down-wind from the sonic point.

Assuming isothermal flow for purposes of illustration, we can write the equation of motion for steady, spherical flow as

$$(v^2 - a^2)\,\frac{1}{v}\frac{dv}{dr} = -\frac{2a^2}{r} + g_R - g \quad , \tag{1}$$

where a is the Newtonian speed of sound. We then see that the right-hand side must vanish at the sonic point ($v = a$) if the velocity derivative is to remain finite. This constraint, which is additional to a complete set of boundary conditions, in general cannot be satisfied with arbitrary Φ, which is therefore an eigenvalue. With the assumption of line formation by pure extinction, this eigenvalue can be determined with shooting integrations starting at the photosphere [3]. However, when re-emission following line absorption is allowed for or when continuum formation within the wind is included, the subsonic structure is affected by radiation received from beyond the sonic point and so an iterative solution procedure must be devised.

In the CAK model, the contribution of lines to g_R is computed in the narrow-line limit; and the implications of this for equation (1) are fully accepted. Now, in this limit, a pencil of radiation completes its interaction with a particular transition within an infinitesimal distance, and so the attenuation suffered depends only on the physical conditions and the velocity derivative at the point of resonance. The resulting dependence of g_R on dv/dr [11,12] then alters the solution topology of equation (1). At the sonic point, the right-hand side can now be made to vanish by suitable choice of dv/dr, so that this singular point no longer determines Φ. However, a new critical point appears downwind in the supersonic flow where, because of g_R's dependence on dv/dr, equation (1) cannot in general be solved for dv/dr. The eigenvalue Φ is then determined by demanding regularity at this critical point [15].

However, the CAK critical point is an artefact of the narrow-line limit, in that it has no counterpart when this limit is not taken [40]. The velocity "derivative" appearing in the formula for g_R should clearly be regarded as an average gradient in an interval of half-width $\sim v_D$ and so should not be accorded the same status as the derivative on the left hand side of equation (1). In fact, if a simple averaging algorithm is used to determine this "derivative", numerical integrations can be carried through the CAK critical point without difficulty.

In an apparent refutation of the above criticism, the physical significance of the CAK critical point seemed to be established by Abbott's [41] demonstration that the propagation characteristics of radiative-acoustic waves is such that this critical point is the last point capable of communication with every part of the flow. This analysis was, however, carried out using the Sobolev approximation and so effectively only wavelengths long compared to the Sobolev length $L = v_D/(dv/dr)$ were considered. When generalized to include shorter wavelengths, the backward propagation of information in the supersonic flow no longer occurs, at least in the case of line formation by pure extinction [42]. This finding undermines the physical significance claimed for the CAK critical point and tends to support the primary rôle of the sonic point in determining Φ. Owocki and Rybicki caution, however, that somewhat different propagation characteristics may be found when they treat line formation by scattering.

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If it should indeed turn out that radiative-acoustic waves in the scattering case can carry information from the supersonic flow back through the sonic point, this will just be a means additional to that already provided by the direct irradiation of the subsonic flow by the wind - i.e., the wind-blanketing effect discussed earlier. The existence of this back-flow of information does not invalidate the rôle of the sonic point singularity in determining Φ for the steady solution but it does raise the tantalizing prospect of delayed feedback instabilities.

5. Instabilities

Stability analyses of line-driven winds have been prompted by the plausible hope that non-radiative effects due to instabilities entering the non-linear regime will explain several of the immediate observational failures of the purely laminar standard model. From the substantial literature [3,43-51] on local wind instabilities arising from fluctuations in the driving term g_R, it emerges that the most rapidly growing instability is that due to short length-scale velocity perturbations. The physical effect can be readily understood by subjecting an optically thin blob to a positive velocity increment. Driving ions in the unperturbed blob see a photospheric radiation field that is partly attenuated by matter somewhat closer to the star. But this attenuation or shadowing is decreased in consequence of the extra Doppler shift given by the velocity increment; the resulting fluctuation δg_R is therefore positive and the perturbation amplifies. For line formation by pure extinction and on the assumption of non-interacting driving lines, the growth rate [46,49] of this instability allows ~ 100 e-foldings to occur in a typical wind time-scale. Accordingly, non-linear effects would seem to have ample opportunity of manifesting themselves.

The above picture is somewhat modified when the analysis is generalized to include scattered line radiation [50,42,51]. Because the unperturbed solution is everywhere expanding, the diffuse radiation field comprising photons scattered by a transition of rest frequency ν_i is seen by a co-moving blob to start abruptly (in the narrow-line limit) at ν_i and to extend to the red. If we therefore now subject this blob to a velocity increment it acquires the ability to scatter some line photons emitted from its direction of motion but loses that ability with respect to its wake. Accordingly, there is a retarding effect - line drag - that opposes the growth of instability. Quantitatively, this line-drag term essentially eliminates instability at the beginning of the flow but because of sphericity becomes much less effective at higher velocities. Thus, although the conclusion that line-driven winds are highly unstable thereby survives, the strong reduction of growth rate in the region of initial high acceleration raises the prospect that non-linear effects in winds derive from the advecting out of fluctuations due to atmospheric instabilities rather than from the intrinsic instabilities discussed here.

It should perhaps be emphasized that this line-drag term emerges from an analysis that, apart from the Doppler effect, neglects terms of order v/c. This term is therefore intrinsically stronger than the Thomson photon drag so important for perturbations in the pre-recombination Universe and should be the dominant drag term whenever the spectrum of the diffuse radiation has discontinuities.

Delayed feedback instabilities that might result from the mutual radiative interaction of a wind and its underlying photosphere will of course not be revealed by the local analyses hitherto carried out.

6. Finite Amplitude State

Given the existence of rapidly growing instabilities, our interest naturally turns to the finite-amplitude state reached when non-linear terms halt further growth. Ultimately, a full understanding of this state can only come from following the growth of instabilities numerically. Such an approach is in fact the subject of a brief report by Wolf [52], who claims that line-driven instabilities give rise to strong, X-ray emitting shocks. A direct attack on this problem is also planned by Castor, Owocki, and Rybicki.

Another, though decidedly less satisfactory approach is the phenomenological one familiar to astronomers in the context of the mixing-length theory of turbulent convection. For unstable, line-driven winds, this approach [23,35] allows the important finite-amplitude effects to be tentatively identified and their observational implications to be crudely evaluated. Among the phenomena thus investigated are radiatively-driven shocks, shadowing in multiply non-monotonic flow in regard to both the maintenance and the destruction of shocks, and the inefficiency of radiative driving due to dissipation by shocks.

An early speculation [3] concerning dissipation by shocks assumed to result from wind instabilities was that the ambient temperature might thereby be raised to $\sim 2\times10^5$ K, thus creating anomalous ions by collisional ionization [3,4]. But following the discovery [20] of X-ray emission, it was recognized [23] that at the densities prevailing in O stars' winds shock-heated gas cooled too quickly to heat the ambient gas. More plausible, therefore, is a model in which the X-ray emitting, shock-heated gas occurs in thin sheets, with radiative driving of the cooled gas emerging from these sheets maintaining the shocks against radiative losses.

If a shock's strength is indeed being maintained, the flow seen by an observer travelling with the shock will be quasi-stationary; the equations describing rigorously steady flow relative to the shock should therefore allow essential aspects of shock maintenance to be identified. For an isothermal shock (i.e., infinitesimal cooling zone), the relevant equation of motion for plane-parallel flow is

$$(w^2 - a^2)\,\frac{1}{w}\frac{dw}{dx} = g_s + g - g_R \quad , \tag{2}$$

where g_s is the shock's acceleration in the star's frame, and w is the velocity relative to the shock front. Here both w and x are taken to be positive in the direction back towards the star.

With the equation written in this way, the post-shock flow of a line-driven shock is seen to be closely analogous to the outflow of a line-driven wind - cf., equn. (1). Thus, again the right-hand side must vanish at the sonic point ($w = a$) if a singularity in the transonic flow is to be avoided. However, in contrast with the laminar wind, here g_R must dominate ($> g_s+g$) in the subsonic flow ($w < a$) and viceversa in the supersonic flow. Accordingly, radiative driving can maintain a radiating shock if it provides a confining force in that part of the shock's wake that is still in acoustic communication with the shock front. In effect, radiative driving then provides the shock with an in situ piston whose rate of working makes up for the radiative losses occurring in the cooling zone.

Shadowing in the non-monotonic flow created by shocks allows line driving to provide the pattern of force needed to maintain the shocks. Thus, $g_R > g_s + g$ can be satisfied for $w < a$ if this immediate post-shock zone is for the most part directly irradiated by the star, whereas $g_R < g_s + g$ for $w > a$ can be satisfied if this more distant zone is shadowed by faster outflowing material nearer the star. It is also evident that a shock's decay must be expected when its subsonic wake is shadowed - i.e., when its in situ piston is removed. This is likely to occur when a shock is slowed by running into a density enhancement left by a previously decayed shock or when a following shock experiences exceptional acceleration.

In the calculation of g_R for a post-shock flow, radiation scattered back from ahead of the shock gives a line-drag term opposing the in situ piston supplied by the direct photospheric radiation. A quantitative analysis using the transfer theory for scattering complexes created by non-monotonic flow [13] reveals that the piston is neutralized at the base of the wind but, as in the instability analysis, sphericity diminishes the fractional contribution of line drag, thereby allowing shocks to be maintained out in the wind.

Because the instability's growth rate is high at short wavelengths, a high spatial frequency of shocks is a natural consequence. This then provokes the conjecture that, in the finite-amplitude state, the first scatterings of all photons that interact with

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wind matter occur in immediate post-shock wakes ($w < a$). If this is indeed approximately so, then matter is being driven away from hot stars basically in consequence of the velocity increments inflicted by passing shocks, with line-driving demoted to the task of maintaining the shocks.

7. Diagnostics of the Finite Amplitude State

The importance of securely established failures of physical models is well illustrated by the development of a diagnostic relating to the local kinematic character of line-driven winds. Thus, with purely laminar flow, no combination of velocity law and radial distribution of scattering ions can reproduce the broad black absorption troughs of strong P Cygni profiles [24]. Moreover, although departures from laminar flow in the form of microturbulence improve the computed profiles [38,39], the observed extent of blackness is still not predicted [13]. However, if the flow is assumed to be multiply non-monotonic, as implied by the expectation of a high spatial frequency of shocks, the resulting multiple points of resonance inhibit the continued forward propagation of a photon, giving rise to an effectively back-scattering medium and therefore black absorption troughs [13,14]. Accordingly, with the secure exclusion of alternatives, extended black absorption troughs would seem to be a reliable diagnostic of multiply non-monotonic flow. Nevertheless, accurate fits to entire profiles are not achieved with the simple non-monotonic models hitherto considered [13,14]. Further improvement requires the inclusion of the stochastic element of the microturbulent model [38,39].

Additional information about the finite-amplitude state is contained in the data on narrow absorption components [25-27]. In the context of the multiple-shock model stressed here, the "shell" producing these components can perhaps be interpreted as comprising numerous density enhancements left behind by decayed shocks. Such enhancements might get locked to the observed velocity as a result of shadowing at higher velocities and radiative driving by P Cygni emission components at lower velocities. Other interpretations [25-27] remain viable, however.

Further decisive contributions from X-ray astronomy with regard to shocks and coronal zones can be anticipated. Higher quality spectral information will allow stronger statements concerning the radial distribution of X-ray emitting gas in the winds of single O stars [20-23]. Further studies of hot stars with neutron stars orbiting in their winds should provide important data on the scales and amplitudes of wind inhomogeneities [53]. Moreover, future absorption studies of such systems should test the claim [54] that a source of soft X-rays distributed throughout the cool winds is demanded by the data.

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M. Ibanez: Have you taken into account dust effects in some of your models?

L. Lucy: No. For hot stars, dust condensation occurs far downstream and so is of no consequence for the acceleration zone near the star.

R. Opher: Anne Underhill published a paper (Ap.J. (Letters) 268, L127) on observational data of a series of OB stars in which stars of higher luminosity had smaller terminal velocities, indicating that the winds were not radiatively driven. Can you comment on this?

L. Lucy: As I explained in my talk, Abbott and I devised a critical test of line-driving as the explanation of high terminal velocities and found that it confirms the operation of this mechanism for the best-observed object, ζ Puppis. This test could usefully be repeated for Underhill's stars if their basic parameters are well determined.

Deviations from correlations expected for pure line-driving are most probably due to perturbing effects in the neighbourhood of the sonic point rather than to an additional acceleration mechanism in the high speed outflow.

V. Icke: It's probably greedy to ask for more, but I'd like to know (a) if these models can be extended to the relativistic case $v/c \simeq 1$, so that they'd be applicable in galactic nuclei with a power-law UV spectrum, and (b) what scope there is for radiation-driven winds in stars where the main source of opacity is dust?

L. Lucy: Line-driving to relativistic velocities assuming a power-law continuum is the subject of several papers attempting thus to explain narrow absorption lines in quasar spectra.

Dust-driven wind models for cool stars, especially the carbon stars, have been constructed. My own current interest in this area concerns the use of such models as boundary conditions for evolutionary models on the asymptotic-giant branch. Interesting work remains to be done on instabilities and dust-driven shocks seem plausible.

J. Krolik: Leon having raised the subject, I'd like to describe my work with John Raymond, showing where we agree or disagree with him. We calculated the actual ionization and thermal structure behind the kind of shocks that have been posited to exist in stellar winds. For reasons of conceptual simplicity, we assumed that no material left the shock once it had been swept up, in contrast to Leon's picture in which mass flux continuity is imposed everywhere. We found that it is by no means a good assumption to suppose, as Leon does, that all shocks are isothermal. In fact, the dividing line between shocks that do or don't cool is right in the middle of the expected range. Letting gas flow out the backs of shocks only reduces the importance of cooling.

From these ionization and thermal structure results, we evaluated the opacity of the post-shock material, and hence the force on it from scattering an O-star continuum. Much of the force is due to the opacity of hot gas, so that the time-averaged radiation force on a large number of shocks and inter-shock regions is far from identical to the force predicted by CAK-like models. For this reason, and the presence of shocks themselves, the averaged velocity law may bear little resemblance to the CAK result. Raymond and I are beginning work on a hydrodynamics simulation in which we will study these effects.

As a final note, I'd like to suggest that the physics we have been discussing in the context of the O-star winds may be equally applicable to other situations in which there are strong thermal UV continua, such as planetary nebulae or cataclysmic variables.

M. Shull: Some years ago, Wolf-Rayet star winds had more momentum, $\dot{M}V$ / L/c, than could be accounted for in single-scattering. Could you review the current status of

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the observations, theories, in the light of the new models and the effects:

1) Multi-scattering
2) Backwarming of stellar photosphere
3) Continuum formation in wind

L. Lucy: Models incorporating all the effects you list are not yet to hand. My expectation, however, is that they will combine to yield successful models for W-R winds. This success will come partly from the enhanced mass loss allowed by multi-scattering and partly from a revision of W-R parameters. Note that the low effective temperatures and therefore low luminosities deduced from UV continua have been derived using standard model atmospheres, thus ignoring the degradation of the spectral energy distribution that occurs within these dense winds.

G. Fisher: Is there a characteristic length scale between shocks? What is it?

L. Lucy: In the phenomenological model I described, the condition that the post-shock flow receives sufficient direct radiation to maintain the shock determines the length scale between shocks.

A.R. Taylor: For what one might take to be reasonable values for the magnetic fields in winds of hot stars, can one account for the observed non-thermal radio emission in terms of your picture via shock acceleration of electrons?

L. Lucy: According to R.L. White (Ap.J. 289, 698), the answer is yes.