

ABSTRACTS OF AUSTRALASIAN PHD THESES

BANACH SPACES OF PSEUDOMEASURES ON COMPACT GROUPS WITH EMPHASIS ON HOMOGENEOUS SPACES

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Segal algebras and homogeneous Banach algebras, which are generalizations of the group algebra $L^1(G)$ of a compact, or locally compact abelian group G , have been studied in Reiter [4], [5], [6], Burnham [1], [2], [3]; and Wang [7].

In Chapter 3 of this thesis a generalized notion of homogeneous Banach space over a compact group is considered. No multiplication is assumed. If B denotes such a space then an operator $*$ may be defined from $M(G) \times B$ to B so that it corresponds to the convolution product in familiar cases. Then

$$\lim_{n \rightarrow \infty} \|k_n * b - b\|_B = 0$$

for any $b \in B$ and any approximate identity $(k_n)_{n \in \mathbb{N}}$ of $L^1(G)$. In fact this property is characteristic of homogeneous Banach spaces amongst those which are translation invariant. It is the basic tool used to study homogeneous Banach spaces.

Chapters 4 and 5 consider, in detail, the family B_n of homogeneous Banach spaces of pseudomeasures defined on a compact group. In Chapter 4 the elements B of B_n are characterized by means of norms defined on a family of trigonometric polynomials, and also by certain subspaces of

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$\underline{E}(\Sigma(G))$ which are identified with B^* . If B is also a subspace of $M(G)$ then it is a homogeneous convolution algebra; these algebras are the subject of Chapter 5.

A complete description of the homogeneous subalgebras of $L^1(G)$ is easily given. It is proved that the closed two-sided and closed left ideal theory of a homogeneous convolution Banach algebra B is precisely the same as that of one of the homogeneous subalgebras of $L^1(G)$. Moreover, an explicit representation of the ideals is given. One is also given for the $*$ -representations of any symmetric B - again the picture is similar to that of one of the symmetric homogeneous subalgebras of $L^1(G)$.

The seemingly disjoint second chapter considers a problem concerning weighted subspaces of some pointwise algebras of integrable functions defined on a compact abelian group G . This problem leads to the study of the tensor algebras $C(G)_{F_1} \hat{\otimes} C(G)_{F_2}$, where G is now a compact group and F_1, F_2 are subsets of $\Sigma(G)$. These algebras are homogeneous convolution Banach algebras in B_n .

References

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