*Proc. London Math. Soc.* (3) 92 (2006) 342–344 
doi:10.1112/S0024611505015613

# CORRIGENDUM

# HECKE ALGEBRAS OF CLASSICAL TYPE AND THEIR REPRESENTATION TYPE

(*Proc. London Math. Soc.* (3) 91 (2005) 355–413)

SUSUMU ARIKI

## 1. *Introduction*

In [**1**] and [**5**], a result by Rickard [**6**, Theorem 2] was used. Let $F$ be an algebraically closed field and $A$ be a self-injective finite-dimensional $F$-algebra. Then the result asserts that if there is an $A$-module with complexity greater than or equal to 3, then $A$ is wild. However, it has turned out recently that the proof has a gap, and specialists do not know how to correct it. I thank Dr K. Erdmann for this information.

Fortunately, the ways in which we used the result were not crucial, and we have different proofs for those places where the result was used. The aim of this corrigendum is to supply the arguments.

## 2. *The gap*

Let us explain what is wrong with the proof of [**6**, Theorem 2]. Crawley-Boevey's result [**3**, Theorem D] says that if there are infinitely many isomorphism classes of indecomposable $A$-modules $M$ with the same dimension such that $M$ and $\Omega^2\nu M$ are not isomorphic, then $A$ is wild. In [**6**, Lemma 1] it is stressed that when applying this result it is not necessary to assume that the $A$-modules $M$ are indecomposable, and Rickard made the following conclusion: if there are infinitely many isomorphism classes of $A$-modules $M$ of the same dimension with the property that the dimension of $\mathrm{Ext}^i_A(M, A/\operatorname{Rad} A)$ is not bounded as a function of $i$, then $A$ is wild. This is the statement of [**6**, Lemma 1].

Let $F$ be of characteristic 2 and let $A = FG$ be the group algebra of the Klein four group $G$. We know that $A$ is tame. Consider the family of 3-dimensional $A$-modules of the form $M = R_p \oplus F$ where $F$ is the trivial $A$-module and $R_p$ are the $A$-modules parametrized by $p \in P^1(F)$ as in [**2**, Proposition 7.1]. Then, because $\mathrm{Ext}^i_A(M, A/\operatorname{Rad} A) = \mathrm{Ext}^i_A(M, F)$ has $\mathrm{Ext}^i_A(F, F) = H^i(G, F)$ as a direct summand, and $H^\bullet(G, F) \simeq F[X, Y]$, the assumptions of [**6**, Lemma 1] are fulfilled. Thus, this gives a counterexample to the lemma.

As [**6**, Lemma 1] fails, we have to choose modules $L$ in the proof of [**6**, Theorem 2] in such a way that they are not only pairwise non-isomorphic but they are also indecomposable modules. Whether this is possible is now an open problem.

Received 31 July 2005.

2000 *Mathematics Subject Classification* 20C08, 20G05, 33D80.

## 3. *Replacements for the old arguments*

As the use of [**6**, Theorem 2] can also be avoided in [**5**], I explain the replacements in my paper only. The places where the result was used are

(1) 'Case 5b' in the proof of Theorem 42(2) [**1**, p. 392],

(2) 'Case $e = 2$' in § 4.4 [**1**, p. 399],

(3) Theorem 68(2) [**1**, p. 407].

Lemma 1. *The following hold.*

(1) *Let $A = FQ/I$ be a quiver with relations. If $Q$ contains two nodes* 1 *and* 2 *such that there is an arrow $1 \leftarrow 2$ and there are three loops on* 2*, then $A$ is wild.*

(2) *The quiver $F[X, Y, Z]/(X^2, Y^2, Z^2)$ is wild.*

For references see [**4**, I 10.8] for (1), and [**4**, I 10.10] for (2).

Let us start with 'Case 5b'. We want to show that $\mathcal{H}_2(q, Q) \otimes_F \mathcal{H}_2^A(q)$ is wild. In Case 2b we proved that $\mathcal{H}_2(q, Q)$ is Morita equivalent to $FQ/I$ where $Q$ has the adjacency matrix $\left(\begin{smallmatrix}1&1\\1&1\end{smallmatrix}\right)$. We also know that $\mathcal{H}_2^A(q) \simeq F[X]/(X^2)$. Let $S_1$ and $S_2$ be simple $\mathcal{H}_2(q, Q)$-modules, and $S$ be the simple $\mathcal{H}_2^A(q)$-module. Then $\mathcal{H}_2(q, Q) \otimes_F \mathcal{H}_2^A(q)$ has two simple modules $S_1 \otimes S$ and $S_2 \otimes S$. Let $P_i$ be the projective cover of $S_i \otimes S$, for $i = 1, 2$. Then $\operatorname{Rad} P_1/\operatorname{Rad}^2 P_1 = 2S_1 \otimes S \oplus S_2 \otimes S$ and $\operatorname{Rad} P_2/\operatorname{Rad}^2 P_2 = S_1 \otimes S \oplus 2S_2 \otimes S$. Thus the Gabriel quiver of $\mathcal{H}_2(q, Q) \otimes_F \mathcal{H}_2^A(q)$ satisfies the assumptions of Lemma 1(1). The result follows.

In 'Case $e = 2$' of § 4.4, we have to show that $\mathcal{H}_2^A(q) \otimes \mathcal{H}_2^A(q) \otimes \mathcal{H}_2^A(q)$ is wild. As $\mathcal{H}_2^A(q) \simeq F[X]/(X^2)$, this follows from Lemma 1(2).

Finally, we consider Theorem 68(2). The only place we have to consider is the proof of the claim that divisibility by $(x - q)^3$ implies wildness. Before proving this, we recall the possibility for $\mathcal{H}_2(q, Q)$ when it is tame. In Case 1b, it is the Kronecker algebra $F[X, Y](X^2, Y^2)$. In Case 2b there are two simple modules $S_1$ and $S_2$ such that the projective covers both satisfy $\operatorname{Rad} P/\operatorname{Rad}^2 P = S_1 \oplus S_2$. In Case 4b, we have two simple modules $S_1$ and $S_2$ such that the projective covers satisfy $\operatorname{Rad} P_1/\operatorname{Rad}^2 P_1 = S_2$ and $\operatorname{Rad} P_2/\operatorname{Rad}^2 P_2 = S_1 \oplus S_2$ respectively. As is stated in the proof, $\mathcal{H}_4^A(q)$ is Morita equivalent to Case 4b and $\mathcal{H}_5^A(q)$ has a block algebra which has the same Gabriel quiver as Case 2b. Note that $\mathcal{H}_n^D(q)$ cannot be tame. Therefore, if $\mathcal{H}_n^X(q)$, for some $n$ and $X$, is tame then we may assume either that $\mathcal{H}_n^X(q)$ is $F[X, Y]/(X^2, Y^2)$ or that $\mathcal{H}_n^X(q)$ has two simple modules $S_1$ and $S_2$ such that the projective covers satisfy $[\operatorname{Rad} P_1/\operatorname{Rad}^2 P_1 : S_2] = 1$ and $\operatorname{Rad} P_2/\operatorname{Rad}^2 P_2 = S_1 \oplus S_2$.

Now assume that $(x - q)^3$ divides $P_W(x)$. If there are distinct $i$, $j$ and $k$ such that $x - q$ divides $P_{W_i}(x)$, $P_{W_j}(x)$ and $P_{W_k}(x)$, then Lemma 1(2) implies that $\mathcal{H}_W(q)$ is wild. If $(x - q)^2$ divides $P_{W_i}(x)$ but $(x - q)^3$ does not divide $P_{W_i}(x)$ and $x - q$ divides $P_{W_j}(x)$, for $j \neq i$, then $\mathcal{H}_{W_i}(q)$ is tame and the Gabriel quiver of $\mathcal{H}_W(q)$ satisfies the assumptions of Lemma 1(1) or (2). Thus $\mathcal{H}_W(q)$ is wild as desired.

## 4. *Lemma* 30

I also use this occasion to give a different argument for Lemma 30 [**1**, p. 366]. This avoids the use of Theorem 28, under the additional assumption that the length of the cycle is even.

https://doi.org/10.1112/S0024611505015613 Published online by Cambridge University Press

Let the cycle length be even. Then we may choose the orientation of the subquiver so that every vertex is either a sink or a source. Thus the result follows from Theorem 6(3).

The additional assumption is harmless because the only place where Lemma 30 is used is in the proof of Lemma 67, in which the length is 4.

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*Susumu Ariki*
*Research Institute for Mathematical Sciences*
*Kyoto University*
*Kyoto 606–8502*
*Japan*

ariki@kurims.kyoto-u.ac.jp