

## 2-D Radiative Transfer Simulations with Angle-Dependent Partial Frequency Redistribution

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**Abstract.** We demonstrate how the angle-dependent redistribution function can be incorporated into the 2-D transfer modelling of solar prominences. Some preliminary numerical simulations have been performed and we present their results by comparing the emergent hydrogen  $L\alpha$  line profiles computed with the angle-averaged and angle-dependent redistributions.

### 1. Introduction

Standard angle-averaged redistributions are currently used for Lyman lines in prominences and prominence-like structures, both in 1-D (Gouttebroze et al. 1993, Heinzel 1995), as well as in 2-D cases (Paletou 1995). Since 2-D transfer computations can explicitly account for strong anisotropies of the incident solar radiation, the angle-dependent approach seems to be more appropriate in such cases. In the present contribution we investigate this problem and demonstrate the differences between angle-averaged and more rigorous angle-dependent 2-D simulations for the case of prominences.

To achieve this, we have applied a 2-D code of Gorshkov (1996) to the problem of radiative transfer in solar prominences (Figure 1) for the case of a 4-level plus continuum HI atom. The main features of the code are: Multilevel Accelerated Lambda Iterations (MALI) scheme with Partial Frequency Redistribution (PRD) in resonance lines; modified long-characteristics method (timing is linearly proportional to the number of grid points) for 2-D solution of Radiative Transfer Equation (RTE); an ability to calculate angle- and height-dependent boundary conditions based on observational data.

### 2. Basic Formulae

#### 2.1. Angle-Dependent PRD

In our calculations we used the redistribution function in the form:

$$R(\nu', \nu, \Theta) = \gamma R_{II}(\nu', \nu, \Theta) + (1 - \gamma)\phi(\nu)\phi(\nu'),$$

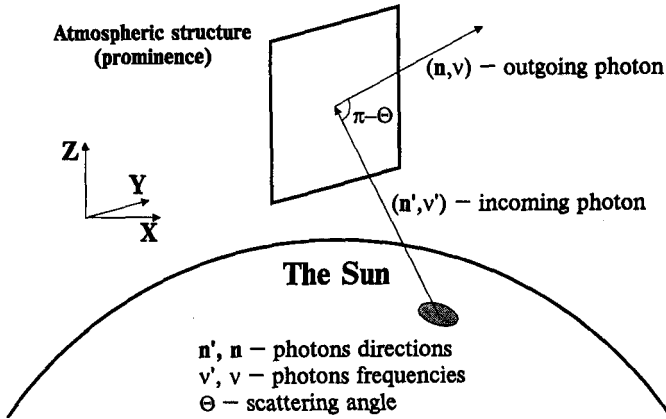


Figure 1. Photon scattering geometry.

where  $\gamma$  is the coherence parameter,  $\Theta$  is the scattering angle between directions of the incoming and the outgoing photons ( $\mathbf{n}'$  and  $\mathbf{n}$ , correspondingly), and  $R_{II}$  represents Hummer's (1962) function for the case of purely coherent scattering in the atom's frame.

Similarly to the angle-averaged case, PRD effects are taken into account by introducing the ratio of emission and absorption coefficients for a given line transition  $ij$  ( $i < j$ ):

$$\rho_{\nu n} \equiv \frac{\psi_{\nu n}}{\phi_{\nu}} = 1 + \gamma \frac{n_i}{n_j} \frac{B_{ij}}{P_j} (\bar{R}_{II}(\nu, n) - \bar{J}),$$

where  $n_i$  and  $n_j$  are the populations of atomic levels  $i$  and  $j$ ,  $B_{ij}$  is the Einstein coefficient for absorption,  $P_j$  represents a probability for an atom to leave the level  $j$ ;  $\bar{J} = (4\pi)^{-1} \int_0^\infty \int_0^{4\pi} I_{\nu' n'} \phi_{\nu'} d\nu' d\Omega'$  is the mean integrated intensity and  $\bar{R}_{II}$  stands for the scattering integral (see Hubený 1985). Contrary to the standard PRD,  $\rho$  now depends on the angle  $\Theta$  because  $\bar{R}_{II}$  has the following form:

$$\bar{R}_{II}(\nu, n) = (4\pi \phi_{\nu})^{-1} \int_0^\infty \int_0^{4\pi} R_{II}(\nu', \nu, \Theta) I_{\nu' n'} d\nu' d\Omega', \quad (*)$$

where the redistribution function is ( $x, x'$  are frequencies counted from the line center and expressed in Doppler units):

$$R_{II}(x', x, \Theta) = \begin{cases} \frac{g(\Theta)}{\pi \sin \Theta} \exp\{-[\frac{1}{2}(x - x')]^2 \csc^2 \frac{\Theta}{2}\} H[a \sec \frac{\Theta}{2}, \frac{1}{2}(x + x') \sec \frac{\Theta}{2}] & 0 < \Theta < \pi \\ \frac{1}{\sqrt{\pi}} H(a, x') \delta(x - x') & \Theta = 0 \\ \frac{a}{2\pi^{3/2}} \exp\{-(\frac{x-x'}{2})^2\} [( \frac{x+x'}{2} )^2 + a^2]^{-1} & \Theta = \pi \end{cases}$$

Here  $H(a, x)$  is the Voigt function and  $a$  the damping parameter. Examples of the function  $R_{II}(x', x, \Theta)$  are shown in Figure 2. Since the function varies very sharply with  $\Theta$  and  $x'$ , we used appropriate  $\Theta$ - and  $x'$ -dependent frequency quadratures for the evaluation of the scattering integral (\*).

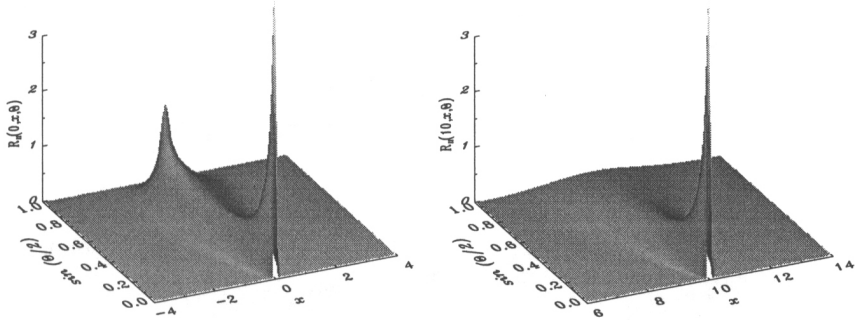


Figure 2. Function  $R_{II}(x', x, \Theta)$  for  $x' = 0$  (left) and  $x' = 10$  (right).

### 2.2. Changes in the Formal Solution of RTE

Since we need to know specific intensities  $I_\nu$  to calculate the integral (\*), the following changes in a standard Feautrier scheme of the formal solution of RTE are necessary:

$$\begin{aligned} d^2u/d\tau^2 &= u - \bar{S} + d\Delta S/d\tau, \\ d^2v/d\tau^2 &= v - \Delta S + d\bar{S}/d\tau \end{aligned}$$

Here  $u = (I_\nu^+ + I_\nu^-)/2$  and  $v = (I_\nu^+ - I_\nu^-)/2$  are Feautrier variables;  $\bar{S} = (S_\nu^+ + S_\nu^-)/2$  and  $\Delta S = (S_\nu^+ - S_\nu^-)/2$  represent averaged sum and difference of source functions in positive (+) and negative (-) directions. The scale of optical depths  $\tau$  is calculated in a positive direction. To solve these transfer equations, we introduced corresponding changes in an improved Feautrier method of Rybicki and Hummer (1991).

### 3. Results and Conclusion

Using the above-described approach, we have computed 2-D transfer for the prominence model having the following parameters: dimensions  $\Delta Z = 2000$  km,  $\Delta Y = 2000$  km, low boundary at the height  $H = 10,000$  km above the solar surface, temperature  $T = 8000$  K, gas pressure  $P_{gas} = 0.05$  dyn/cm<sup>2</sup> turbulent velocity  $V_{turb} = 5$  km/s. The angle-dependent incident radiation field was used similarly as in Gorshkov (1996), where other details of our numerical procedure are described. As a result of preliminary simulations, we present a comparison of emergent  $L\alpha$  line profiles (taken in the center of the slab) for the cases of angle-averaged and angle-dependent PRD (see Figure 3). The main effect seen here is a lowering of the intensity in the line core. As a next step in

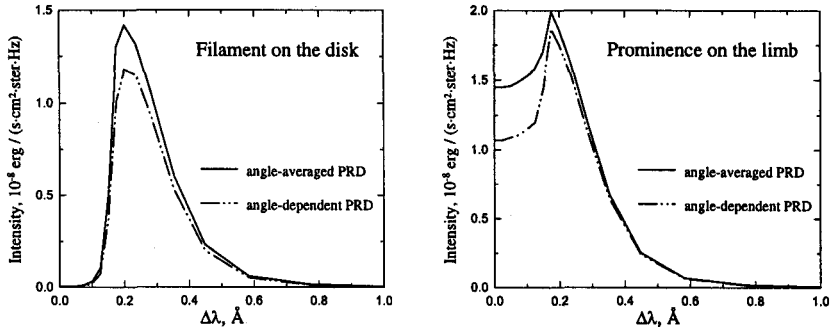


Figure 3. Calculated emergent profiles for the  $L\alpha$  line as seen on the disk (left) and on the limb (right).

this work we intend to demonstrate the influence of the angle-dependent PRD on spectral diagnostics of prominence plasmas.

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