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ON THE GROUP $\mathrm{Aut}_\Omega(X)$

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Abstract Felix and Murillo introduced the group $\mathrm{Aut}_\Omega(X)$ of self-maps f of X, which satisfy $\Omega f = 1_{\Omega X}$, and proved that the group is nilpotent with the order of nilpotency bounded by the Lusternik–Schnirelmann category of X. In this paper we construct a spectral sequence converging to the group $\mathrm{Aut}_\Omega(X)$ and derive several interesting consequences.

Keywords: self-homotopy equivalences; loop spaces; spectral sequences

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1. Introduction

Given a pointed CW-complex X let $\mathrm{Aut}(X)$ denote the set of homotopy classes of self-maps of X which are homotopy equivalences. When endowed with the operation induced by the composition of maps, $\mathrm{Aut}(X)$ becomes a group, called the *group of self-homotopy equivalences* of X. This group has been extensively studied (see the survey [**1**] or the recently published monograph [**11**]).

The purpose of this paper is to use spectral sequences to study the normal subgroup $\mathrm{Aut}_\Omega(X)$ of $\mathrm{Aut}(X)$, defined as the kernel of the homomorphism

$$\mathrm{Aut}(X) \to \mathrm{Aut}(\Omega X) f \mapsto \Omega f.$$

The homomorphisms induced by Ωf on the homotopy groups of ΩX are the same (after a shift in dimension) as those induced by f on the homotopy groups of X. It follows that $\mathrm{Aut}_\Omega(X)$ is a subgroup of the group $\mathrm{Aut}_\sharp(X)$, the kernel of the representation $\mathrm{Aut}(X) \to \mathrm{Aut}\,\pi_*(X)$. The group $\mathrm{Aut}_\Omega(X)$ was introduced by Felix and Murillo in [**5**], where they showed that $\mathrm{Aut}_\Omega(X)$ and $\mathrm{Aut}_\sharp(X)$ are generally different, that $\mathrm{Aut}_\Omega(X)$ is a nilpotent group, and that its order of nilpotency is bounded by the Lusternik–Schnirelmann category of X.

Our construction is modelled after the construction of a spectral sequence converging to the group $\mathrm{Aut}_\sharp(X)$, due to Didierjean in [**4**]. For later reference we recall that the initial term of Didierjean's spectral sequence is given by

$$E_1^{p,q} = H^{2p+q+1}(X^p; \pi_{p+1}X),$$

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where $p > 0$, $p+q \in \{-1,0,1\}$ and X^p is the pth Postnikov section of X. When X is either finite dimensional or a Postnikov piece, the spectral sequence converges to $\mathrm{Aut}_\sharp(X)$.

We will find that there is a spectral sequence converging to $\mathrm{Aut}_\Omega(X)$ whose initial term differs from those in the previously described one only in the terms $E_1^{p,-p}$, which are in general proper subgroups of the corresponding groups in the spectral sequence of Didierjean. The construction of the spectral sequence will be carried out in § 2. The usual methods for the construction of spectral sequences like exact couples are not appropriate, since they require the exactness of certain sequences, which is lacking in our case, due essentially to the fact that the exact homotopy sequence of a fibration is not exact in dimension 0. To overcome this difficulty some modifications are in order, and they are most easily described when the spectral sequence is constructed by means of a Cartan–Eilenberg system. The classical reference for Cartan–Eilenberg systems is [**3**], while the appropriate modifications can be found in [**2**], [**6**] and [**4**]. In § 3 we apply the spectral sequence to derive some general conditions when $\mathrm{Aut}_\Omega(X)$ is trivial or when it equals $\mathrm{Aut}_\sharp(X)$. Next we consider its relation to the localization: if P is a set of primes, and if X is simply connected, let $X \to X_P$ denote the localization with respect to P. In view of the theorems of Maruyama [**8**, **9**] one would expect that the natural homomorphism $\mathrm{Aut}_\Omega(X) \to \mathrm{Aut}_\Omega(X_P)$ P-localizes. This is indeed the case when X is a Postnikov piece, while for X a finite complex, some problems arise and the answer is still unsatisfactory. We close § 3 with some computational examples.

2. Construction of the spectral sequence

As we have already explained, the Cartan–Eilenberg method is best suited for our purposes. Recall that a *(non-abelian) Cartan–Eilenberg system* is given by the following data (for details, see [**4**]):

(i) abelian groups $H^{-1}(p,q)$ for $0 \leqslant p \leqslant q \leqslant \infty$,

(ii) groups $H^0(p,q)$ for $0 \leqslant p \leqslant q \leqslant \infty$,

(iii) pointed sets $H^1(p,p+1)$ with base points b_p for $0 \leqslant p < \infty$,

(iv) homomorphisms $\eta : H^n(p,q) \to H^n(p',q')$ for $n \in \{-1,0\}$, $p \geqslant p'$ and $q \geqslant q'$,

(v) homomorphisms $\delta : H^{-1}(p,q) \to H^0(q,r)$ for $p \leqslant q \leqslant r$,

(vi) actions of $H^0(p,q)$ on $H^1(q,q+1)$ which, through the action on the base-point, define functions

$$\delta : H^0(p,q) \to H^1(q,q+1) \quad \text{as } \delta : x \mapsto x \cdot b_q.$$

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These data satisfy some obvious commutativity and exactness conditions, most notably the requirement that the following sequences are exact:

(i) when $p \leqslant q \leqslant r$,

$$H^{-1}(q,r) \xrightarrow{\eta} H^{-1}(p,r) \xrightarrow{\eta} H^{-1}(p,q) \xrightarrow{\delta} H^0(q,r) \xrightarrow{\eta} H^0(p,r) \xrightarrow{\eta} H^0(p,q),$$

and

(ii) when $p \leqslant q$,

$$H^0(p,q+1) \xrightarrow{\eta} H^0(p,q) \xrightarrow{\delta} H^1(q,q+1).$$

The proof of the following theorem requires only a straightforward verification of the assertions (alternatively, follow [**2**]).

Theorem 2.1. *Every non-abelian Cartan–Eilenberg system defines a spectral sequence* (E_r, d_r) *of cohomological type whose initial term is given by*

$$E_1^{p,q} = H^{p+q}(p,p+1),$$

when $p+q \in \{-1,0,1\}$, *and* $E_1^{p,q} = 0$ *otherwise.*

If there exists some P *such that* $H^n(p,p+1)$ *is trivial when* $p > P$, *then this spectral sequence converges to the graded group* $H^0(0,\infty)$ *in the sense that the* E_∞ *term corresponds to the subquotients of the filtration*

$$F^pH^0(0,\infty) = \mathrm{Im}(H^0(p,\infty) \to H^0(0,\infty)) = \mathrm{Ker}(H^0(0,\infty) \to H^0(0,p)).$$

Finally, if $H^0(p,p+1)$ *and* $H^1(p,p+1)$ *are abelian groups, and if the function*

$$H^0(p,p+1) \to H^1(p+1,p+2), \quad x \mapsto x \cdot b_{p+1} - b_{p+1}$$

is a homomorphism, then the resulting spectral sequence consists of abelian groups and their homomorphisms.

We now apply the Cartan–Eilenberg method to construct a spectral sequence converging to $\mathrm{Aut}_\Omega(X)$. Let X be of the homotopy type of a simply connected CW-complex, and let

$$X^2 \leftarrow X^3 \leftarrow \cdots \leftarrow X^p \leftarrow \cdots \leftarrow X = \varprojlim X^p$$

be the Postnikov decomposition of X, where X^p is the homotopy fibre of the Postnikov invariant

$$\xi_p : X^{p-1} \to K(\pi_p X, p+1) \quad (p = 2,3,\dots).$$

The Postnikov decomposition of a space is natural in the sense that for $p \leqslant q$ there is a fibre map between function spaces

$$\mathrm{map}(X^q, X^q) \to \mathrm{map}(X^p, X^p),$$

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thus, by restriction, we obtain a fibre map

$$\mathrm{aut}_\Omega(X^q) \to \mathrm{aut}_\Omega(X^p).$$

Let F_p^q denote the fibre (over 1_{X^p}) of $\mathrm{aut}_\Omega(X^q) \to \mathrm{aut}_\Omega(X^p)$. For $p \leqslant q \leqslant r$ there is a fibration

$$F_q^r \hookrightarrow F_p^r \to F_p^q,$$

which allows the construction of a non-abelian Cartan–Eilenberg system: for $p \leqslant q$ and $n \in \{-1, 0\}$ let $H^n(p,q) := \pi_{-n}(F_p^q)$; the homomorphisms η are induced by restrictions, while the homomorphisms δ are determined by boundary homomorphisms in the homotopy long exact sequences of fibrations $F_q^r \hookrightarrow F_p^r \to F_p^q$ (note that the sequence starts at the level of classifying spaces, so we get homomorphisms at the π_0 level); for $p \geqslant 0$ let $H^1(p, p+1)$ be the set $[X^p, K(\pi_{p+1}X, p+2)]$ with ξ_{p+1} as base point; and, finally, let $H^0(p,q) = \pi_0(F_p^q)$ act on $H^1(q, q+1)$ by pre-composition.

All properties required for a Cartan–Eilenberg system, except the exactness of the sequence

$$H^0(p, q+1) \xrightarrow{\eta} H^0(p,q) \xrightarrow{\delta} H^1(q, q+1)\ ,$$

follow directly from the definitions. To verify the remaining condition observe that for an $f \in H^0(p,q)$, represented by a map $f : X^q \to X^q$ and satisfying $\Omega f = 1_{X^q}$, the condition $\delta(f) = \xi_{q+1} \circ f = \xi_{q+1}$ implies that there is a lifting $\bar{f} : X^{q+1} \to X^{q+1}$ for f, such that $\Omega \bar{f} = 1_{X^{q+1}}$.

Theorem 2.2. *Let X be a simply connected CW-complex. Then there exists a spectral sequence of cohomological type whose initial term is given as*

$$\begin{aligned} E_1^{p,-p-1} &= H^p(X; \pi_{p+1}X), \\ E_1^{p,-p} &= \mathrm{Ker}(\Omega : H^{p+1}(X; \pi_{p+1}X) \to H^p(\Omega X; \pi_{p+1}X)), \\ E_1^{p,-p+1} &= H^{p+2}(X^p; \pi_{p+1}X), \end{aligned}$$

where Ω is the cohomology suspension and $p > 0$.

If X is either finite dimensional or a Postnikov piece, then the spectral sequence converges to the group $\mathrm{Aut}_\Omega(X)$, i.e. the groups $E_\infty^{p,-p}$ are the subquotients of the filtration

$$F^p(\mathrm{Aut}_\Omega(X)) = \mathrm{Im}(\mathrm{Aut}_{\Omega,X^p}(X) \to \mathrm{Aut}_\Omega(X)),$$

where $\mathrm{Aut}_{\Omega,X^p}(X)$ is the subgroup of $\mathrm{Aut}_\Omega(X)$ consisting of self-equivalences which fix the Postnikov section X^p.

Proof. Let us begin with the identification of the E_1 term. Denote by $\mathrm{aut}_{\sharp X^p}(X^{p+1})$ the space of self-maps over X^p of X^{p+1}, which induce identity automorphisms on homotopy groups. By the remark following Proposition 3.1 of [**4**] and the proposition itself, the map

$$\Phi : \mathrm{map}(X^p, K(\pi_{p+1}X, p+1)) \to \mathrm{aut}_{\sharp X^p}(X^{p+1}),$$

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which to an $\alpha : X^p \to K(\pi_{p+1}X, p+1)$ assigns the composition

$$X^{p+1} \xrightarrow{(1,\alpha\circ \mathrm{pr})} X^{p+1} \times K(\pi_{p+1}X, p+1) \xrightarrow{\mu} X^{p+1} ,$$

is a homotopy equivalence. It follows that $E_1^{p,-p-1}$, the fundamental group of (the identity component of) $\mathrm{aut}_{\sharp X^p}(X^{p+1})$, is isomorphic to $H^p(X^p; \pi_{p+1}X)$, which is in turn isomorphic to $H^p(X; \pi_{p+1}X)$.

Since the Postnikov sections of ΩX are the spaces ΩX^p, and since clearly α represents an element of $\pi_0(F_p^{p+1})$ if and only if $\Omega\alpha \simeq 0$, as a map from ΩX^p to $K(\pi_{p+1}X, p)$, we obtain

$$E_1^{p,-p} = \pi_0(F_p^{p+1}) = \mathrm{Ker}(\Omega : H^{p+1}(X^p; \pi_{p+1}X) \to H^p(\Omega X^p; \pi_{p+1}X)).$$

From the cohomology sequence of the pair (X^p, X) we see that $H^{p+1}(X^p; \pi_{p+1}X)$ is isomorphic to the kernel of the connecting homomorphism

$$\delta : H^{p+1}(X; \pi_{p+1}X) \to H^{p+2}(X^p, X; \pi_{p+1}X).$$

Moreover, $H^{p+2}(X^p, X; \pi_{p+1}X) \cong \mathrm{Hom}(H_{p+2}(X^p, X), \pi_{p+1}X)$, so, by the Hurewicz Theorem, $H_{p+2}(X^p, X) \cong \pi_{p+2}(X^p, X) \cong \pi_{p+1}X$, so $H^{p+1}(X^p; \pi_{p+1}X)$ can be described as the kernel of the homomorphism

$$H^{p+1}(X; \pi_{p+1}X) = [X, K(\pi_{p+1}X, p+1)] \to \mathrm{Hom}(\pi_{p+1}X, \pi_{p+1}X),$$

which to an $\alpha : X \to K(\pi_{p+1}X, p+1)$ assigns the induced homomorphism in π_{p+1}. Similarly, by considering the pair $(\Omega X^p, \Omega X)$, the group $H^{p+1}(\Omega X^p; \pi_{p+1}X)$ can be identified with the kernel of the analogous homomorphism

$$[\Omega X, K(\pi_{p+1}X, p)] \to \mathrm{Hom}(\pi_{p+1}X, \pi_{p+1}X).$$

Since an $\alpha : X \to K(\pi_{p+1}X, p+1)$, which satisfies $\Omega\alpha = 0$, induces a trivial homomorphism in homotopy, it represents an element of $H^{p+1}(X^p, \pi_{p+1}X)$, and hence an element of $\mathrm{Ker}(\Omega : H^{p+1}(X^p; \pi_{p+1}X) \to H^p(\Omega X^p; \pi_{p+1}X))$.

The convergence is proved as in Theorem 2.3. of [**4**]. □

3. Applications

We begin with two corollaries, which follow from elementary properties of the cohomology suspension.

Corollary 3.1. *If X is a co-H-space, then* $\mathrm{Aut}_\Omega(X)$ *is trivial.*

Proof. It is well known that in co-H-spaces the cohomology suspension is injective, hence all terms $E_1^{p,-p}$ are trivial. □

On the opposite extreme we have the following corollary.

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Corollary 3.2. *If X is a rational space, then* $\mathrm{Aut}_\Omega(X) = \mathrm{Aut}_\sharp(X)$.

Proof. For dimensional reasons all elements in $H^{p+1}(X^p; \pi_{p+1}X)$ are products, and are therefore annihilated by the cohomology suspension. Hence, the spectral sequences for $\mathrm{Aut}_\Omega(X)$ and for $\mathrm{Aut}_\sharp(X)$ coincide. □

Let P be a set of primes, and let $X \to X_P$ denote the localization with respect to P. Since the localization commutes with the loop space construction, it is natural to ask if the induced homomorphism $\mathrm{Aut}_\Omega(X) \to \mathrm{Aut}_\Omega(X_P)$ is a localization of nilpotent groups. This is indeed true when X is a Postnikov piece but unfortunately we have not been able to prove the analogous result for X a finite-dimensional complex. The difficulty is of the same kind as in the case of the localization of the group of self-equivalences which induce identity on all homotopy groups. Maruyama [**8**] proved that when X is a finite-dimensional complex, then $\mathrm{Aut}_{\sharp n}(X) \to \mathrm{Aut}_{\sharp n}(X_P)$ is the P-localization, where $\mathrm{Aut}_{\sharp n}(X)$ denotes the group of self-equivalences which induce identity of the first n homotopy groups. However, it is still unknown if $\mathrm{Aut}_\sharp(X) \to \mathrm{Aut}_\sharp(X_P)$ also localizes.

Theorem 3.3. *Let X be a Postnikov piece. Then for any set of primes P the natural homomorphism* $\mathrm{Aut}_\Omega(X) \to \mathrm{Aut}_\Omega(X_P)$ *is the P-localization.*

Proof. The proof is by comparison of spectral sequences. Indeed, the localization $X \to X_P$ induces the localization between the corresponding spectral sequences, which in turn implies that $\mathrm{Aut}_\Omega(X) \to \mathrm{Aut}_\Omega(X_P)$ is a P-localization. □

If X is an n-dimensional CW-complex, then the natural homomorphism $\mathrm{Aut}(X) \to \mathrm{Aut}(X^N)$ is bijective for $N \geqslant n$. Let $\mathrm{Aut}_{\Omega,N}(X)$ denote the subgroup of $\mathrm{Aut}(X)$ consisting of classes represented by maps $f : X \to X$, such that $f^N \in \mathrm{Aut}_\Omega(X^N)$. Clearly, $\mathrm{Aut}_{\Omega,N}(X) \cong \mathrm{Aut}_\Omega(X^N)$, so we get the following result analogous to Theorem 0.1 of [**8**].

Corollary 3.4. *If X is an n-dimensional CW-complex, then for any set of primes P and for $N \geqslant \dim(X)$ the natural homomorphism* $\mathrm{Aut}_{\Omega,N}(X) \to \mathrm{Aut}_{\Omega,N}(X_P)$ *is a P-localization.*

A similar restriction is required for the following result, which compares the groups $\mathrm{Aut}_\sharp(X)$ and $\mathrm{Aut}_\Omega(X)$.

Corollary 3.5. *If the natural homomorphism* $\mathrm{Aut}_\Omega(X) \to \mathrm{Aut}_\Omega(X_{(0)})$ *is a rationalization, then* $\mathrm{Aut}_\sharp(X)/\mathrm{Aut}_\Omega(X)$ *is a finite group.*

Proof. By the universal property, the group $(\mathrm{Aut}_\sharp(X))_{(0)}$ can be identified with a subgroup of $\mathrm{Aut}_\sharp(X_{(0)})$. It follows that the group $(\mathrm{Aut}_\sharp(X)/\mathrm{Aut}_\Omega(X))_{(0)}$ is smaller than $\mathrm{Aut}_\sharp(X_{(0)})/\mathrm{Aut}_\Omega(X_{(0)})$, which is trivial, by Corollary 3.2. □

We now give some simple computational examples.

Theorem 3.6. $\mathrm{Aut}_\Omega(S^m \times S^n) \cong \mathrm{Aut}_\sharp(S^n \times S^m)$.

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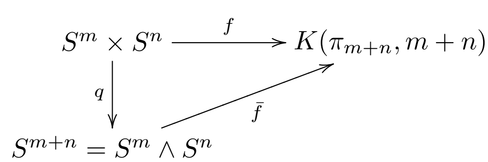
Proof. It is sufficient to show that the $E_1^{p,-p}$ terms of the spectral sequence for $\mathrm{Aut}_\Omega(S^m \times S^n)$ coincide with the corresponding terms of the spectral sequence for $\mathrm{Aut}_\sharp(S^m \times S^n)$. In the second case, a straightforward computation shows that the only non-trivial term is

$$E_1^{m+n-1,-m-n+1} = \pi_{m+n}(S^m) \oplus \pi_{m+n}(S^n),$$

so we only need to show that

$$\Omega : H^{m+n}(S^m \times S^n, \pi_{m+n}) \to H^{m+n-1}(\Omega S^m \times \Omega S^n, \pi_{m+n})$$

(where $\pi_{m+n} = \pi_{m+n}(S^m \times S^n)$) is trivial. In order to do so, observe that every map $f : S^m \times S^n \to K(\pi_{m+n}, m+n)$ can be factored up to homotopy as



and that $\Omega q = 0$, since q represents a product in $H^{m+n}(S^m \times S^n)$. It follows that

$$E_1^{m+n-1,-m-n+1} = H^{m+n}(S^m \times S^n; \pi_{m+n}) = \pi_{m+n}(S^m \times S^n).$$

□

The groups $\mathrm{Aut}_\sharp(S^m \times S^n)$ are computed in Proposition 5.5 of [**10**], so we obtain

$$\mathrm{Aut}_\Omega(S^m \times S^n) \cong \mathrm{Coker}([\imath_m, -]) \oplus \mathrm{Coker}([\imath_n, -]),$$

where $[\imath_m, -] : \pi_{n+1}(S^m) \to \pi_{n+m}(S^m)$ and $[\imath_n, -] : \pi_{m+1}(S^n) \to \pi_{m+n}(S^n)$ are Whitehead products with fundamental classes of S^m and S^n, respectively. By standard computation,

$$\mathrm{Aut}_\Omega(S^3 \times S^n) \cong \pi_{n+3}(S^3) \oplus \pi_{n+3}(S^n) \quad \text{for } n > 4$$

and

$$\mathrm{Aut}_\Omega(S^3 \times S^4) \cong \pi_7(S^3) \oplus \pi_8(S^5).$$

Finally, putting together the above theorem with the previous results on localization yields the following corollary.

Corollary 3.7. *Assume that X is a simply connected space, which is p-equivalent, with respect to some prime p, to a product of two spheres. Then $(\mathrm{Aut}_\Omega(X))_{(p)} = (\mathrm{Aut}_\sharp(X))_{(p)}$.*

Proof. First observe that $\mathrm{Aut}_\Omega(S^m \times S^n)$ is finite, hence there exists an N such that $\mathrm{Aut}_\Omega(X_{(p)}) = \mathrm{Aut}_\Omega((X^N)_{(p)})$. Then apply Theorems 3.3 and 3.6. □

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Note that the group $\mathrm{Aut}_\sharp$ has been computed for many spaces satisfying the assumptions of the last corollary (see, for example, [**7**]).

More complicated examples can be treated by combining the localization with some special techniques for the computation of Aut_Ω for products or wedges of simpler spaces. Details will appear elsewhere.

We conclude the paper with two open problems. Felix and Murillo [**5**] showed that $\mathrm{Aut}_\Omega(X) \neq \mathrm{Aut}_\sharp(X)$ in general, but their example is an infinite-dimensional space. It is still unknown if there exists a finite complex X such that $\mathrm{Aut}_\Omega(X) \neq \mathrm{Aut}_\sharp(X)$. Moreover, in view of Corollary 3.1 we have the following related question: is $\mathrm{Aut}_\sharp(X)$ trivial for every finite co-H-space X?

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