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ORIGINAL ARTICLE

Theoretical Underpinnings of 'Land Abundance, Openness, and Industrialization'

How Openness Affects Output Elasticities in a 2 × 2 HOS Model with Product Differentiation

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Abstract
This paper develops a Heckscher–Ohlin model in which the effects on production structure of endowments and changes in trade policies depend in a continuous way on a country's degree of openness to trade. Its main contribution is to show semi-formally how the introduction of product differentiation causes the elasticities of output with respect both to factor endowments and to sectoral trade barriers to vary with import penetration (the share of foreign firms in home markets) and export orientation (the share of foreign markets in home-firm sales).

Keywords: Comparative advantage; differentiated products; openness to trade

1. The Supply Side of the 2 × 2 HOS Model

The purpose of this paper is to support the theoretical argument in Wood (2023) that the sectoral structure of a country's output is determined by the interaction of its factor endowments with its degree of openness to trade. This first section derives Wood's equation (1), which sets out the standard 2 × 2 Heckscher–Ohlin–Samuelson (HOS) model of a single country in a somewhat unfamiliar form. The two goods j are M (manufactures) and A (primary products), with the two factors i being N (land) and L (labour) and with primary production being land intensive. Both this and the next section largely reproduce results in Jones (1965) with minor changes in notation.

The supply side of the 2 × 2 HOS model is described by four equations in which the unit input coefficients a_{ij} link the outputs q_j with the factor inputs v_i, and the factor prices w_i with the goods prices p_j.

$$a_{NM}q_M + a_{NA}q_A = v_N \tag{1}$$

$$a_{LM}q_M + a_{LA}q_A = v_L \tag{2}$$

$$w_N a_{NM} + w_L a_{LM} = p_M \tag{3}$$

$$w_N a_{NA} + w_L a_{LA} = p_A \tag{4}$$

The shares of inputs employed in the production of the goods are written as $\lambda_{ij} = a_{ij}q_j/v_i$, with $\lambda_{NA} = 1 - \lambda_{NM}$ and $\lambda_{LA} = 1 - \lambda_{LM}$, and the shares of inputs in the cost of production of the two goods as $\theta_{ij} = w_i a_{ij}/p_j$, with $\theta_{LM} = 1 - \theta_{NM}$ and $\theta_{LA} = 1 - \theta_{NA}$.

Since primary production is land-intensive

$$a_{NM}/a_{LM} < a_{NA}/a_{LA} \tag{5}$$

and

$$a_{NM}/a_{NA} < a_{LM}/a_{LA} \tag{6}$$

and these inequalities are easily shown to imply

$$\lambda_{NM} < \lambda_{LM} \tag{7}$$

$$\theta_{NM} < \theta_{NA} \tag{8}$$

Taking logarithmic differentials of (1)–(4), where dx/x is written as $\hat{x}$, the envelope theorem (Shephard's lemma) implies that when factor prices change, changes in the cost-minimizing input coefficients satisfy

$$w_N \hat{a}_{Nj} + w_L \hat{a}_{Lj} = 0 \tag{9}$$

so differential changes in the price–cost equations satisfy

$$\hat{w}_N \theta_{Nj} + \hat{w}_L (1 - \theta_{Nj}) = \hat{p}_j \tag{10}$$

and

$$\hat{w}_N - \hat{w}_L = (\hat{p}_M - \hat{p}_A)/(\theta_{NM} - \theta_{NA}) \tag{11}$$

the Stolper–Samuelson theorem.

Differential changes in the full employment equations, with factor prices and therefore the input coefficients changing as well as factor supplies, give

$$\lambda_{iM}(\hat{a}_{iM} + \hat{q}_M) + (1 - \lambda_{iM})(\hat{a}_{iA} + \hat{q}_A) = \hat{v}_i \tag{12}$$

so

$$\hat{v}_N - \hat{v}_L = (\lambda_{NM} - \lambda_{LM})(\hat{q}_M - \hat{q}_A) + \lambda_{NM}\hat{a}_{NM} + (1 - \lambda_{NM})\hat{a}_{NA} - \lambda_{LM}\hat{a}_{LM} - (1 - \lambda_{LM})\hat{a}_{LA} \tag{13}$$

However, the relationship between factor inputs and factor prices depends on the elasticities of substitution

$$\hat{a}_{NM} - \hat{a}_{LM} = -\sigma_M(\hat{w}_N - \hat{w}_L) \tag{14}$$

$$\hat{a}_{NA} - \hat{a}_{LA} = -\sigma_A(\hat{w}_N - \hat{w}_L) \tag{15}$$

and the envelope theorem (9) implies

$$\theta_{NM}\hat{a}_{NM} + (1 - \theta_{NM})\hat{a}_{LM} = 0 \tag{16}$$

so

$$\hat{a}_{LM} = -\theta_{NM}(\hat{a}_{NM} - \hat{a}_{LM}) = \theta_{NM}\sigma_M(\hat{w}_N - \hat{w}_L) \tag{17}$$

$$\hat{a}_{NM} = -(1 - \theta_{NM})\sigma_M(\hat{w}_N - \hat{w}_L) \tag{18}$$

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and similarly

$$\hat{a}_{LA} = \theta_{NA}\sigma_A(\hat{w}_N - \hat{w}_L) \tag{19}$$

$$\hat{a}_{NA} = -(1-\theta_{NA})\sigma_A(\hat{w}_N - \hat{w}_L) \tag{20}$$

Hence, equation (13) becomes

$$\hat{q}_M - \hat{q}_A = \frac{\hat{v}_N - \hat{v}_L}{\lambda_{NM} - \lambda_{LM}} + \left[\frac{\lambda_{NM}\sigma_M + (1-\lambda_{NM})\sigma_A}{\lambda_{NM} - \lambda_{LM}} - (\theta_{NM}\sigma_M - \theta_{NA}\sigma_A)\right](\hat{w}_N - \hat{w}_L) \tag{21}$$

Substituting (11) into (21) gives

$$\begin{aligned}\hat{q}_M - \hat{q}_A = {} & \frac{\hat{v}_N - \hat{v}_L}{\lambda_{NM} - \lambda_{LM}} \\ & + \left[\frac{\lambda_{NM}\sigma_M + (1-\lambda_{NM})\sigma_A}{(\lambda_{NM} - \lambda_{LM})(\theta_{NM} - \theta_{NA})} - \frac{\theta_{NM}\sigma_M - \theta_{NA}\sigma_A}{\theta_{NM} - \theta_{NA}}\right](\hat{p}_M - \hat{p}_A)\end{aligned} \tag{22}$$

and when, for simplicity, the elasticity of substitution σ is the same in both sectors, this becomes

$$\hat{q}_M - \hat{q}_A = \frac{\hat{v}_N - \hat{v}_L}{\lambda_{NM} - \lambda_{LM}} + \sigma\left[\frac{1}{(\lambda_{NM} - \lambda_{LM})(\theta_{NM} - \theta_{NA})} - 1\right](\hat{p}_M - \hat{p}_A) \tag{23}$$

giving equation (1) in Wood (2023).

In this small open economy with homogeneous products, changes in domestic relative producer prices are determined by exogenous changes in foreign prices and in trade costs. In the event that goods prices are not just exogenous but fixed (so factor prices and input coefficients are fixed too), (22) and (23) become

$$\hat{q}_M - \hat{q}_A = (\hat{v}_N - \hat{v}_L)/(\lambda_{NM} - \lambda_{LM}) \tag{24}$$

the Rybczynski theorem, which shows that a variation in relative factor endowments generates a magnified change in relative outputs (just as the Stolper–Samuelson theorem shows that a variation in relative goods prices generates a magnified change in relative factor prices).

2. The Closed 2 × 2 Economy

The comparative static relationships derived in the previous section can be interpreted as describing a small open economy facing exogenous goods prices set in the world market. At the other extreme, consider a closed 2 × 2 economy in which relative goods prices adjust to equate supply and demand. Assume a constant elasticity demand function so that the relative demand for goods is a function of their relative prices with a constant elasticity γ

$$\hat{q}_M - \hat{q}_A = -\gamma(\hat{p}_M - \hat{p}_A) \tag{25}$$

Putting this together with the supply-side equation (22) gives the relationship between factor supplies and goods supplies in a closed 2-sector economy:

$$\hat{q}_M - \hat{q}_A = (\hat{v}_N - \hat{v}_L)\Big/\left[(\lambda_{NM} - \lambda_{LM}) + \frac{\Lambda}{\gamma(\theta_{NM} - \theta_{NA})}\right] \tag{26}$$

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where

$$\Lambda = (\lambda_{NM}\sigma_M + (1-\lambda_{NM})\sigma_A) - (\theta_{NM}\sigma_M - \theta_{NA}\sigma_A)(\lambda_{NM} - \lambda_{LM}) \tag{27}$$

and when $\sigma_M = \sigma_A = \sigma$, (26) becomes

$$\hat{q}_M - \hat{q}_A = (\hat{v}_N - \hat{v}_L) \Big/ \left[(\lambda_{NM} - \lambda_{LM}) + \frac{\sigma(1 - (\theta_{NM} - \theta_{NA})(\lambda_{NM} - \lambda_{LM}))}{\gamma(\theta_{NM} - \theta_{NA})} \right] \tag{28}$$

These equations for a closed economy will be a point of reference in the analysis in section 4 below of the effects of variation in openness to trade.

3. The 2 × 2 Model with Differentiated Products

Now suppose that within each of the two sectors goods are differentiated, and that consumers have preferences between varieties. For simplicity, assume that the home country produces only one variety of each good which it sells in the home market and in foreign markets. Let each market be described by a standard two-level CES demand system where aggregate consumption of each good in each market is described by

$$Q_{kj} = \left(b_{kj}^{\frac{1}{\beta_{kj}}} q_{kj}^{\frac{\beta_{kj}-1}{\beta_{kj}}} + (1-b_{kj})^{\frac{1}{\beta_{kj}}} z_{kj}^{\frac{\beta_{kj}-1}{\beta_{kj}}} \right)^{\frac{\beta_{kj}}{\beta_{kj}-1}} \tag{29}$$

where $k = H, F$, the home and foreign markets, $j = M, A$, and z_{kj} is an index of sales of foreign varieties of j in k. Aggregate market consumption is described by

$$Q_k = \left(B_k^{\frac{1}{\gamma_k}} Q_{kM}^{\frac{\gamma_k-1}{\gamma_k}} + (1-B_k)^{\frac{1}{\gamma_k}} Q_{kA}^{\frac{\gamma_k-1}{\gamma_k}} \right)^{\frac{\gamma_k}{\gamma_k-1}} \tag{30}$$

where for both sectors $\beta_{kj} > \gamma_k$.

Corresponding to these quantity indices are two sets of price indices: at the product level

$$P_{kj} = (b_{kj} p_{kj}^{1-\beta_{kj}} + (1-b_{kj})\pi_{kj}^{1-\beta_{kj}})^{\frac{1}{1-\beta_{kj}}} \tag{31}$$

where π_{kj} is a price index of sales of foreign varieties of j in k; and at the market level

$$P_k = (B_k P_{kM}^{1-\gamma_k} + (1-B_k) P_{kA}^{1-\gamma_k})^{\frac{1}{1-\gamma_k}} \tag{32}$$

The constant elasticity demand functions are:

$$\frac{q_{kj}}{Q_{kj}} = b_{kj} \left(\frac{p_{kj}}{P_{kj}} \right)^{-\beta_{kj}} \tag{33}$$

$$\frac{z_{kj}}{Q_{kj}} = (1-b_{kj}) \left(\frac{\pi_{kj}}{P_{kj}} \right)^{-\beta_{kj}} \tag{34}$$

$$\frac{Q_{kM}}{Q_k} = B_k \left(\frac{P_{kM}}{P_k} \right)^{-\gamma_k} \tag{35}$$

$$\frac{Q_{kA}}{Q_k} = (1 - B_k)\left(\frac{P_{kA}}{P_k}\right)^{-\gamma_k} \tag{36}$$

which imply that the share of expenditure in market k on the home variety of good j is

$$sd_{kj} = \frac{p_{kj}q_{kj}}{P_{kj}Q_{kj}} = b_{kj}\left(\frac{p_{kj}}{P_{kj}}\right)^{(1-\beta_{kj})} \tag{37}$$

while the shares of the two goods in total expenditure in market k are

$$s_{kM} = \frac{P_{kM}Q_{kM}}{P_kQ_k} = B_k\left(\frac{P_{kM}}{P_k}\right)^{(1-\gamma_k)} \tag{38}$$

$$s_{kA} = \frac{P_{kA}Q_{kA}}{P_kQ_k} = (1 - B_k)\left(\frac{P_{kA}}{P_k}\right)^{(1-\gamma_k)} \tag{39}$$

When $\gamma_k = 1$, (30) and (32) are replaced by Cobb–Douglas functions but (33)–(39) are unchanged.

A property of price indices like (31) and (32) is that the proportional effect of the individual price on the price index is the market share of the individual good, so

$$\hat{P}_{kj} = sd_{kj}\hat{p}_{kj} + (1 - sd_{kj})\hat{\pi}_{kj} \tag{40}$$

$$\hat{P}_k = s_{kM}\hat{P}_{kM} + s_{kA}\hat{P}_{kA} \tag{41}$$

The demand functions (33), (35), and (36) then imply that

$$\hat{q}_{kj} = -[\beta_{kj}(1 - sd_{kj}) + \gamma_k sd_{kj}]\hat{p}_{kj} + \gamma_k\hat{P}_k + \hat{Q}_k \tag{42}$$

4. The Effects of Openness on Output Elasticities

Drawing on the previous sections, this section provides formal support for a key proposition in the theory section of Wood (2023): that greater openness increases the elasticity of a country's manufactured–primary output ratio with respect to its land–labour ratio. It also shows that greater openness has an ambiguous effect on the relationship between the manufactured–primary output ratio and relative sectoral trade costs.

Assume that the home country is small relative to foreign markets so the prices of foreign firms are given and price indices in foreign markets are also given, and that the home country sells at the same price in both home and foreign markets.

The supply response with the same elasticity of substitution σ in both sectors is given by (23). On the demand side, assume that all the βs are equal and both the γs are equal, so from (42) we have

$$\hat{q}_{HM} - \hat{q}_{HA} = -[\beta(1 - sd_{HM}) + \gamma sd_{HM}]\hat{p}_M + [\beta(1 - sd_{HA}) + \gamma sd_{HA}]\hat{p}_A \tag{43}$$

Even with the βs equal, this does not give $\hat{q}_{HM} - \hat{q}_{HA}$ as an exact function of $\hat{p}_M - \hat{p}_A$ unless the home-market share of home producers is the same in both sectors.

However, from (43) we can write

$$\begin{aligned}\hat{q}_{HM} - \hat{q}_{HA} = &-[\beta - (\beta - \gamma)sd_{HM}](\hat{p}_M - \hat{p}_A) \\ &+ [(\beta - \gamma)(sd_{HM} - sd_{HA})]\hat{p}_A\end{aligned} \tag{44}$$

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and

$$\hat{q}_{HM} - \hat{q}_{HA} = -[\beta - (\beta - \gamma)sd_{HA}](\hat{p}_M - \hat{p}_A) + [(\beta - \gamma)(sd_{HM} - sd_{HA})]\hat{p}_M \quad (45)$$

If $\hat{p}_M$ and $\hat{p}_A$ have opposite signs (relative to fixed foreign prices) the final right-hand side term is positive in one of the above equations and negative in the other. In the present context, it is reasonable to assume that domestic prices will move in opposite directions relative to world prices as a result of a change in factor endowments or in trade policy.

It follows that there is some number sd_H between sd_{HM} and sd_{HA} such that

$$\hat{q}_{HM} - \hat{q}_{HA} = -\epsilon_H(\hat{p}_M - \hat{p}_A) \quad (46)$$

where

$$\epsilon_H = \beta(1 - sd_H) + \gamma sd_H \quad (47)$$

In the foreign market, the relationship corresponding to (43) is

$$\hat{q}_{FM} - \hat{q}_{FA} = -[\beta(1 - sd_{FM}) + \gamma sd_{FM}]\hat{p}_M + [\beta(1 - sd_{FA}) + \gamma sd_{FA}]\hat{p}_A \quad (48)$$

If the share of home firms in foreign markets is small, then q_{FM}/q_{FA} is approximately a constant elasticity function of p_M/p_A with elasticity β. More precisely, using the same argument as for (46) there is a small sd_F such that

$$\hat{q}_{FM} - \hat{q}_{FA} = -\epsilon_F(\hat{p}_M - \hat{p}_A) \quad (49)$$

where

$$\epsilon_F = \beta(1 - sd_F) + \gamma sd_F \quad (50)$$

The elasticity of relative demand in each market will thus be a market-specific weighted average of the elasticities of substitution between varieties and between goods.

The combined home-market and foreign-market changes in demand for home-firm output in the individual sectors are

$$\hat{q}_M = \hat{q}_{HM}ss_{HM} + \hat{q}_{FM}(1 - ss_{HM}) \quad (51)$$

$$\hat{q}_A = \hat{q}_{HA}ss_{HA} + \hat{q}_{FA}(1 - ss_{HA}) \quad (52)$$

where the ss_{Hj} are the shares of home-firm supply of j sold in the home market. If the two ss_{Hj} were equal, their common value could be used to calculate the elasticity of relative demand for the home varieties of the two goods as a weighted average of the two market-specific elasticities in (47) and (50), and clearly that elasticity would be higher the larger the share of home-firm supply sold in the foreign market.

However, when the shares of home-firm supply in the foreign market differ between sectors, the four individual elasticities of demand derived from (40–42) have to be aggregated to give

$$\hat{q}_M - \hat{q}_A = -\epsilon_M \hat{p}_M + \epsilon_A \hat{p}_A + (ss_{HM} - ss_{HA})\hat{Q}_H \quad (53)$$

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where

$$\epsilon_M = [\beta(1 - sd_{HM}) + \gamma(1 - s_{HM})\, sd_{HM}]ss_{HM} + \beta(1 - ss_{HM}) + \gamma\, s_{HM} sd_{HM} ss_{HA}$$

$$\epsilon_A = [\beta(1 - sd_{HA}) + \gamma(1 - s_{HA})\, sd_{HA}]ss_{HA} + \beta(1 - ss_{HA}) + \gamma\, s_{HA} sd_{HA} ss_{HM}$$

The last term in (53) is an income effect in the home economy that results from the adjustment in the relative goods price to an exogenous change in factor supplies or in trade policy. If $ss_{HM} - ss_{HA}$ is positive, a rise in p_M/p_A is a deterioration in the home country's terms of trade, so $\hat{Q}_H$ will be negative, while if $ss_{HM} - ss_{HA}$ is negative $\hat{Q}_H$ will be positive. Thus the income effect will somewhat reinforce the direct price effects. However, the income effect is related not to the openness of the economy but only to the sectoral difference in openness, and so does not change the conclusion that the elasticity of relative demand is higher the more open is the economy.

We therefore have

$$\hat{q}_M - \hat{q}_A = -\epsilon\,(\hat{p}_M - \hat{p}_A) \tag{54}$$

where the aggregate elasticity ϵ is not a fixed parameter, and depends on home-firm shares in home-market demand and home-market shares in home-firm supply. It will be closer to γ if the home country is relatively closed so that home firms have high shares of their home market (and therefore less elastic demand) and exports to higher-elasticity foreign markets are a low share of sales. The more open the home country, therefore, the closer will ϵ be to the higher β elasticity. In short, competition is stronger in a more open economy, first because home firms face stronger competition in their home markets, and secondly because home firms sell more of their output in more competitive foreign markets.

This imprecisely defined elasticity of relative demand was derived from the demand functions (33), (35), and (36). Rotunno and Wood (2020) work with an exact elasticity of substitution, using a result derived by Sato (1967) for a two-level CES production function. Sato's more precise result, however, depends on assumptions that are unsatisfactory in the context of assessing the effects of changes in trade policy or in factor endowments, and which the present approach avoids, as explained in the Appendix at the end of this paper.

Although this paper addresses two-sector models, it is worth noting that the openness–moderated relationship in (54) applies to any pair of goods in a many-good HOS model with product differentiation, and that much of the analysis of the rest of this section will extend to many-good models.

4.1 Openness-Conditioned Output-Endowment Elasticity

The effect of factor endowment changes on home output with differentiated products is similar in kind but different in degree to that in a closed economy:

$$\hat{q}_M - \hat{q}_A = (\hat{v}_N - \hat{v}_L)\Big/\left[(\lambda_{NM} - \lambda_{LM}) + \frac{\sigma(1 - (\theta_{NM} - \theta_{NA})(\lambda_{NM} - \lambda_{LM}))}{\epsilon(\theta_{NM} - \theta_{NA})}\right] \tag{55}$$

in which the γ of (28) is replaced by ϵ because (25) is replaced by (54). In an almost-closed economy, ϵ will be close to γ, especially if there is little domestic competition, and the elasticity described by (55) will be close to the closed-economy elasticity in (28). In a very open economy, ϵ will be close to the much higher elasticity β and the effects of endowment changes on output changes will be larger. With a very low degree of product differentiation, product varieties are almost perfect substitutes, ϵ approaches infinity, and (55) approaches the Rybczynski result (24) for the small open economy with homogeneous products.

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The relationship between equation (55) and the HOS and closed economy elasticities in (24) and (28) has an intuitive explanation. An expansion of, say, labour supply in the home country would cause, at given prices, a Rybczynski magnified expansion of manufacturing output and a contraction of primary output. In a small open HOS economy without product differentiation and in the absence of trade policy changes, these output changes would be fully reflected in trade flows and would not generate any price adjustments. In a closed economy, the change in the output mix causes the relative price of manufactures (and therefore of labour) to fall sufficiently to restore equilibrium, with the relevant elasticity being the inter-sectoral substitution elasticity. In an open 2×2 economy with product differentiation, intra-sectoral substitution, with a much higher elasticity, comes into play. The reduced price of manufactures has a bigger effect, so the initial relative increase in output of manufactures is absorbed with smaller changes in goods (and factor) prices, the moderation of the initial Rybczynski effect is smaller, and the output elasticity is greater.

4.2 Openness-Conditioned Output-Trade-Cost Elasticity

Even with no change in factor endowments, sectoral outputs can be changed by trade policy. The supply response in (23) with $\hat{v}_L = \hat{v}_N = 0$ is:

$$\hat{q}_M - \hat{q}_A = \sigma\left[\frac{1}{(\lambda_{NM} - \lambda_{LM})(\theta_{NM} - \theta_{NA})} - 1\right](\hat{p}_M - \hat{p}_A) \tag{56}$$

On the demand side, changes in import barriers have to be treated differently from changes in barriers to exports because their effects occur through different channels. We represent import barriers such as tariffs by the cost factors τ_{HM} and τ_{HA}, both terms being greater than or equal to 1. Assuming that foreign producer prices are given, import prices are increased by these factors. The change in the home market price index for good j when import barriers change is then

$$\hat{P}_{Hj} = sd_{Hj}\hat{p}_j + (1 - sd_{Hj})\hat{\tau}_{Hj} \tag{57}$$

and the overall direct impact of import barriers on relative prices in the home market is

$$\hat{\tau}_H = (1 - sd_{HM})\hat{\tau}_{HM} - (1 - sd_{HA})\hat{\tau}_{HA} \tag{58}$$

The change in home market demand is

$$\begin{aligned}\hat{q}_{Hj} &= -\beta(\hat{p}_j - \hat{P}_{Hj}) - \gamma(\hat{P}_{Hj} - \hat{P}_H) + \hat{Q}_H \\ &= -[\beta(1 - sd_{Hj}) + \gamma sd_{Hj}]\hat{p}_j + (\beta - \gamma)(1 - sd_{Hj})\hat{\tau}_{Hj} + \gamma\hat{P}_H + \hat{Q}_H\end{aligned} \tag{59}$$

so

$$\begin{aligned}\hat{q}_{HM} - \hat{q}_{HA} = &-[\beta(1 - sd_{HM}) + \gamma sd_{HM}]\hat{p}_M + [\beta(1 - sd_{HA}) + \gamma sd_{HA}]\hat{p}_A \\ &+ (\beta - \gamma)\hat{\tau}_H\end{aligned} \tag{60}$$

We represent barriers to exports such as tariffs in the foreign markets similarly by the cost factors τ_{FM} and τ_{FA}, with both terms greater than or equal to 1. We assume that home firms have negligible shares of the foreign markets so that tariffs faced by exports have no effect on the price indices of the goods concerned, but just raise the purchaser prices of the home-firm products sold in the foreign markets. Equation (48) thus becomes

$$\hat{q}_{FM} - \hat{q}_{FA} = -\beta(\hat{p}_M - \hat{p}_A) - \beta\hat{\tau}_F \tag{61}$$

where

$$\hat{\tau}_F = \hat{\tau}_{FM} - \hat{\tau}_{FA} \tag{62}$$

To simplify the exposition, and without affecting the conclusions below about the effects of changes in trade policy, we assume that the shares of home-firm supply sold in the home market are the same in both sectors ($ss_{HM} = ss_{HA} = ss_H$). The economy-wide relative demand elasticity is then, as in (54), an average of the market-specific elasticities, while the trade policy effects are weighted by the common measure ss_H of home-firm home-market orientation:

$$\hat{q}_M - \hat{q}_A = -\epsilon(\hat{p}_M - \hat{p}_A) + (\beta - \gamma)ss_H\hat{\tau}_H - \beta(1 - ss_H)\hat{\tau}_F \tag{63}$$

Combining this demand response with the supply response in (56), we get the 'openness-conditioned output–trade-cost elasticity':

$$\hat{q}_M - \hat{q}_A = \frac{(1 - (\lambda_{NM} - \lambda_{LM})(\theta_{NM} - \theta_{NA}))[(\beta - \gamma)ss_H\hat{\tau}_H - \beta(1 - ss_H)\hat{\tau}_F]}{(\lambda_{NM} - \lambda_{LM})(\theta_{NM} - \theta_{NA})\epsilon/\sigma + (1 - (\lambda_{NM} - \lambda_{LM})(\theta_{NM} - \theta_{NA}))} \tag{64}$$

The directions of the effect of trade policy changes are unambiguous and the same as in HOS. A rise in the tariff on imported manufactures, for example, makes $\hat{\tau}_{HM}$ and therefore $\hat{\tau}_H$ positive, and the coefficient of $\hat{\tau}_H$ in (64) is positive, so the increase in protection raises the demand for and therefore output of manufactures. A rise in the foreign tariff on manufactured exports, by contrast, would make $\hat{\tau}_F$ positive, and the rise in the price of foreign sales would result in a fall in the output of manufactures.

The size of the effects of trade policy changes on relative output, however, depends in an ambiguous way on the openness of the economy. For import tariffs, the ambiguity is in $ss_H\hat{\tau}_H$, whose two elements pull in opposite directions. In the example of a tariff on manufactures, the more open is the economy, the larger is the share of imports $(1 - sd_{HM})$ in the home market, which increases $\hat{\tau}_H$, the effect of the tariff on demand in the home market for home-produced manufactures. In contrast, the more open is the economy, the smaller is the share of the home market ss_H in home-firm sales, so a tariff-induced rise in home sales has less effect on overall sales.

For a given $ss_H\hat{\tau}_H$, however, openness has an unambiguous effect on the size of the outcome. In a more open economy, ϵ in (64) is larger, so the coefficient of $ss_H\hat{\tau}_H$ is smaller. A given increase $ss_H\hat{\tau}_H$ in the protection of home manufacturers draws resources into that labour-intensive sector, and restoring factor market equilibrium requires a rise in wages relative to land rent, which raises the relative cost and therefore the relative price of manufactures, reducing relative demand for manufactures. This demand effect is larger the larger is ϵ, and a larger demand effect moderates the impact of protection on domestic prices and on relative outputs.

The ambiguity of the effect of a country's openness on the size of the impact of changes in barriers to exports is somewhat different, because the effect comes through a different route. With greater openness, the larger value of $(1 - ss_H)$ amplifies the impact of a foreign tariff change but, as with import tariffs, the larger value of ϵ diminishes the final effect on relative output, so the overall effect of greater openness on the size of the outcome could be in either direction.

As in the previous subsection, it is reassuring to check what happens to (64) if the degree of production differentiation is very small. If β in the numerator and ϵ in the denominator both tend towards infinity, the supply response approaches that in the second term of (23) as expected.

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Appendix: Comparison with Sato (1967)

Focusing on elasticities of demand for the varieties of two goods sold in a particular market by a particular country, this appendix compares the algebra of this paper with that of Sato (1967), used in Wood (2012) and Rotunno and Wood (2020).

Sato (1967) was the first person to formulate and analyse a two-level CES production function, for which, in an appendix, he derives 'partial elasticities of substitution' that link the relative use and relative prices of two factors used at the lower level to produce intermediate inputs that are combined with others at the upper level to produce the final good. The elasticity he derives is a weighted harmonic mean of the elasticities of substitution between factors in the production of intermediate inputs and between intermediate inputs in the production of final output, where the weights are factor shares in the cost of intermediates and shares of intermediate inputs in the cost of the final good. Sato's equation (p. 217) is

$$\sigma_{ij} = \frac{\dfrac{1}{\theta_i^{(r)}} + \dfrac{1}{\theta_j^{(s)}}}{\dfrac{1}{\sigma_r}\left(\dfrac{1}{\theta_i^{(r)}} - \dfrac{1}{\theta^{(r)}}\right) + \dfrac{1}{\sigma_s}\left(\dfrac{1}{\theta_j^{(s)}} - \dfrac{1}{\theta^{(s)}}\right) + \dfrac{1}{\sigma}\left(\dfrac{1}{\theta^{(r)}} + \dfrac{1}{\theta^{(s)}}\right)} \tag{A1}$$

where 'σ_s represents the elasticity of substitution within the sth input group, … σ is the elasticity of substitution among input groups, … $\theta^{(s)}$ is the relative expenditure share of the sth class of factors, and $\theta_j^{(s)}$ the relative share of the jth element of the sth class in total expenditure' (pp. 202–203).

Wood (2012) and Rotunno and Wood (2020) adapt the Sato elasticity to a two-level CES utility function of the type used in this paper, but with $\theta_j^{(s)}$ rewritten as the product of $\theta^{(s)}$ and the share of the jth element in the sth class of factors. In the notation of this paper, equation (A1) thus becomes, for a single market k

$$\epsilon_k = \frac{\dfrac{1}{sd_{kM}s_{kM}} + \dfrac{1}{sd_{kA}s_{kA}}}{\dfrac{1}{\beta_{kM}}\left(\dfrac{1}{sd_{kM}s_{kM}} - \dfrac{1}{s_{kM}}\right) + \dfrac{1}{\beta_{kA}}\left(\dfrac{1}{sd_{kA}s_{kA}} - \dfrac{1}{s_{kA}}\right) + \dfrac{1}{\gamma_k}\left(\dfrac{1}{s_{kM}} + \dfrac{1}{s_{kA}}\right)} \tag{A2}$$

but for comparison with the analysis in this paper is better rearranged as

$$\hat{q}_{kM} - \hat{q}_{kA} = \frac{\dfrac{1}{sd_{kM}s_{kM}} + \dfrac{1}{sd_{kA}s_{kA}}}{\dfrac{1}{s_{kM}sd_{kM}}\left[\dfrac{1-sd_{kM}}{\beta_{kM}} + \dfrac{sd_{kM}}{\gamma_k}\right] + \dfrac{1}{s_{kA}sd_{kA}}\left[\dfrac{1-sd_{kA}}{\beta_{kA}} + \dfrac{sd_{kA}}{\gamma_k}\right]}(\hat{p}_M - \hat{p}_A) \tag{A3}$$

in which the elasticity term, with a bit of expansion, would have the standard form of a two-level harmonic mean. The square-bracketed terms are the reciprocals of the harmonic mean elasticities of substitution for each of the goods separately, with $1/\beta_{kj}$ and $1/\gamma_k$ weighted by foreign-firm and home-firm shares of market k for good j. These two elasticities are then combined into a single elasticity that relates the relative sales of home-firm varieties of these goods to relative home-firm prices, with the weights being the relative values of home-firm sales of the two goods.

The counterpart to (A3) in this paper is equations (46) and (49):

$$\hat{q}_{kM} - \hat{q}_{kA} = -[\beta(1 - sd_k) + \gamma sd_k](\hat{p}_M - \hat{p}_A) \tag{A4}$$

where sd_k is somewhere between sd_{kM} and sd_{kA}.

The two square-bracketed terms in (A3) correspond to the averaged square-bracketed term in (A4), the difference being that in (A3) they are reciprocals of harmonic means and in (A4) an average of arithmetic means. The goods share terms, s_{kM} and s_{kA}, which are part of the weighting across levels in equation (A3), are absent from equation (A4), though they appear in the equations from which (A4) is derived.

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More fundamentally, equation (A3) is an exact relationship between changes in relative quantities and relative prices, while (A4) is derived from equations that show only how changes in relative quantities respond to changes in the two goods prices separately. Sato arrives at an exact relationship with relative prices by making stronger assumptions than in this paper, namely (in his original article) that final output 'y and all x_k except x_i and x_j are held constant' (p. 217). In a utility function, the first of these assumptions is equivalent to holding constant the total utility from consuming both goods, while the second assumption permits no variation in the consumption of anything except the varieties of these two goods produced by the country concerned.

Sato's elasticity is thus what Blackorby and Russell (1989) define as a 'Hicks elasticity of substitution'. They also prove that in a single-level CES utility function, the Hicks elasticity is equivalent to the more accurate Morishima elasticity, but not in a two-level CES utility function, with different elasticities at the two levels, even if all the elasticities at the lower level are the same.

These assumptions of the Sato elasticity were explicitly stated in Wood (2012), but the analysis in the present paper makes their limitations clearer. The assumption of a constant total level of utility is unsatisfactory in the context of assessing the effects on a country of variation in its trade policies and endowments. In this context, assuming constant consumption of everything other than this country's varieties of the two goods concerned is also unsatisfactory, because it precludes changes in the quantities consumed of foreign varieties of these two goods. It may reasonably be assumed for a small country that the prices of foreign varieties are constant (other than as a result of changes in its trade policies), but changes in a country's own prices will alter consumption of both the home variety and the foreign varieties of the goods concerned.

The present paper, however, also provides a way of linking relative quantities $\hat{q}_{kM} - \hat{q}_{kA}$ to relative prices $\hat{p}_M - \hat{p}_A$ without Sato's two restrictive assumptions. Equations (46) and (49) set bounds on the elasticity of substitution in a single market. These single-market elasticities can then be combined across home and foreign markets (as in Rotunno and Wood (2020)) to obtain one approximate demand-side elasticity of substitution for the country concerned. The analysis in this appendix thus supports the argument of Wood (2023) and of Rotunno and Wood (2020) that in a country which is more open to trade (with a less protected home market and selling more of its output in foreign markets), the relative sales of the varieties of goods it produces will be more responsive to variation in their relative prices.

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