CORRESPONDENCE.

TABLES TO CONVERT q_x TO m_x AND m_x TO q_x

To the Editors of the Journal of the Institute of Actuaries.

SIRS,—Many hours must have been spent by actuaries in converting various values of q_x to the corresponding values of m_x and *vice versa*. Assuming a uniform distribution of deaths over each year of age, the relationship between these two functions is fixed and it is therefore possible to give it in a simple table which applies to any values of q_x and m_x independently of the table of mortality concerned.

As in working to a given number of figures the difference between q_x and m_x is the same for several values of the function the table can be given as a "critical" table, *i.e.*, this difference is tabulated as a correction to the function and only the largest value of the function which produces each correction is inserted.

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3-figure Table

q_x	Correction	m_x
·000		·000
	+0–	
·031		·031
	1	
·054		·055
	2	
·069		·071
	3	
·081		·085
	4	
·092		·097
	5	
·101		·107
	6	
·110		·117
	7	
·118		·126
	8	
·126		·134
	9	
·133		·142
	10	
·139		·150
	11	
·146		·157
	12	
·151		·164
	13	
·157		·171
	14	
·163		·177
	15	
·168		·183
	16	
·173		·190
	17	
·178		·196
	18	
·183		·201
	+19–	
·187		·207

4-figure Table

q_x	Correction	m_x	q_x	Correction	m_x	q_x	Correction	m_x	q_x	Correction	m_x
·0000		·0000	·0614		·0634	·0869		·0908	·1061		·1121
	+0–			+20–			+40–			+60–	
·0099		·0100	·0630		·0650	·0879		·0920	·1070		·1130
	1			21			41			61	
·0172		·0173	·0645		·0666	·0890		·0932	·1078		·1140
	2			22			42			62	
·0222		·0224	·0659		·0682	·0900		·0943	1087		·1149
	3			23			43			63	
·0262		·0266	·0673		·0697	·0911		·0954	·1095		·1159
	4			24			44			64	
·0297		·0302	·0687		·0712	·0921		·0965	·1104		·1168
	5			25			45			65	
·0328		·0334	·0701		·0727	·0931		·0976	·1112		·1177
	6			26			46			66	
·0357		·0363	·0714		·0741	·0941		·0987	·1120		·1186
	7			27			47			67	
·0383		·0391	·0727		·0755	·0951		·0998	·1128		·1196
	8			28			48			68	
·0408		·0416	·0740		·0769	·0960		·1009	·1136		·1205
	9			29			49			69	
·0431		·0440	·0753		·0783	·0970		·1020	·1144		·1214
	10			30			50			70	
·0453		·0463	·0765		·0796	·0980		·1030	·1152		·1223
	11			31			51			71	
·0473		·0485	·0778		·0809	·0989		·1040	·1160		·1232
	12			32			52			72	
·0493		·0506	·0790		·0822	·0999		·1051	·1168		·1240
	13			33			53			73	
·0512		·0526	·0801		·0835	·1008		·1061	·1176		·1249
	14			34			54			74	
·0531		·0545	·0813		·0848	·1017		·1071	·1183		·1258
	15			35			55			75	
0549		·0564	·0825		·0860	·1026		·1081	·1191		·1267
	16			36			56			76	
·0566		·0582	·0836		·0872	·1035		·1091	·1199		·1275
	17			37			57			77	
·0582		·0600	·0847		·0884	·1044		·1101	·1206		·1284
	18			38			58			78	
·0599		·0617	·0858		·0896	·1052		·1111	·1214		·1292
	+19–			+39–			+59–			+79–	
·0614		·0634	·0869		·0908	·1061		·1121	·1221		·1301

In critical cases ascend.

N.B.—*The q_x and m_x columns form independent tables and are not related to each other.*

To obtain the value of m_x corresponding to any value of q_x *add* that correction ($\div$ 1,000 in the 3-figure table or $\div$ 10 000 in the 4-figure table) given in the centre column which is found opposite the interval in the first column in which the particular value of q_x lies ; *if q_x is one of the values given, add the correction next above it.* Similarly to obtain q_x from m_x enter the third column and *deduct* the correction found in the centre column.

Examples : (4-figure table)

$$q_x = \cdot 0473, \quad m_x = \cdot 0473 + \cdot 0011 = \cdot 0484$$
$$q_x = \cdot 0474, \quad m_x = \cdot 0474 + \cdot 0012 = \cdot 0486$$
$$m_x = \cdot 0970, \quad q_x = \cdot 0970 - \cdot 0045 = \cdot 0925$$
$$m_x = \cdot 1149, \quad q_x = \cdot 1149 - \cdot 0062 = \cdot 1087$$

As m_x increases more rapidly than q_x there are bound to be

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values of m_x which do not correspond to any particular value of q_x (*i.e.*, there will be gaps in the values of m_x). On the other hand a value of q_x corresponding to these missing values of m_x can always be found; in fact, two values of m_x at each of these critical points will give the same value of q_x.

The basis upon which the table was constructed is as follows:

$$m = \frac{2q}{2-q} \quad \therefore \quad m - q = \frac{q^2}{2-q} = \text{(say) } n$$

To find the values of q which make $n = \frac{1}{2}, 1\frac{1}{2}, 2\frac{1}{2}$, &c., in order to ascertain the largest values of q which give a correction (*i.e.*, $m - q$) of 0, 1, 2, &c., one must solve for q in terms of n.

$$q = \frac{\sqrt{n^2 + 8n} - n}{2}$$

Similarly for m in terms of n (*i.e.*, $m - q$)

$$m = \frac{\sqrt{n^2 + 8n} + n}{2}$$

The working is shown in the following table:

n (1)	$n(8+n)$ (2)	$\sqrt{(2)} = \sqrt{n^2+8n}$ (3)	$\frac{(3)-(1)}{2} = q_x$ (4)	$\frac{(3)+(1)}{2} = m_x$ (5)
·00005	·000400	·02000	0099	·0100
15	1200	3464	·0172	·0173
25	2000	4472	·0222	·0224
35	2800	5292	·0262	·0266
⋮	⋮	⋮	⋮	⋮

Cols. (4) and (5) are always rounded off to the lower figure in the fourth place, *i.e.*, ·009975 is taken as ·0099. This ensures that this value of q (or m) gives a correction of just less than n (in this case $\frac{1}{2}$), *i.e.*, the correction to the fourth place is $n - \frac{1}{2}$ (in this case 0), hence the rule "In critical cases ascend."

A 5-figure table can be constructed similarly and, in spite of its greater size, it would be well worth the doing if any considerable number of conversions had to be made.

This process is illustrative of the construction of "critical" tables in general which are extremely useful in such circumstances as the present.

I am,

Yours faithfully,

W. A. FORSTER.

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