© International Astronomical Union 2015 doi:10.1017/S1743921315004457

Two Centuries of Solar Polarimetry

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Abstract. In 1811, François Arago observed the disk of the Sun with his "lunette polariscopique". From the absence of detectable polarization compared with his laboratory observations of glowing solids, liquids, and flames he concluded that the Sun's visible surface is an incandescent gas. From this beginning, thanks to orders of magnitude technology improvements, a remarkable amount of what we know about the physics of the Sun has continued to flow from solar polarimetry. This short review compares some selected polarimetric discoveries with subsequent recent observations to illustrate the tremendous progress of solar polarimetry during the last two centuries.

Keywords. Polarization, instrumentation: polarimeters, Sun: general, history and philosophy of astronomy

1. Introduction

This is an abbreviated review of selected highlights in the history of solar polarimetry. After placing solar polarimetry in the context of polarimetry in general, five key discoveries are reviewed and compared with their recent status. I emphasize the earlier history since much of it is little known. There are many extensive histories of polarization measurements and of solar physics, but no comprehensive history of solar polarimetry to the best of my knowledge. This brief review does not fill that gap.

Many historians consider the origin of polarimetry to be 1669 when Rasmus Bartholin published a 60 page description of double refraction in calcite crystals. This beginning was followed in 1672 by Christiaan Huygens who studied narrow rays of sunlight in calcite and formulated his 1678 theory of double refraction. Huygens' work was not published until 1690 after he visited Newton and realized that his priority was in jeopardy.

More than a century passed with little progress. But this changed in 1808 when French army engineer Étienne-Louis Malus discovered polarization of sunlight reflected by a window of the Luxembourg palace in Paris. Malus built a simple polarimeter using tilted glass plates as polarizers and introduced the term polarization. He is noted for discovering the Law of Malus describing the cosine-squared variation of the intensity of light transmitted by two linear polarizers whose axes are oriented at various angles to each other.

Stimulated by Malus, the future head of state of France and leader of scientific culture François Arago (Lequeux 2008) began his studies of polarization. Among many findings Arago discovered rotary polarization of light transmitted along the optical axis of quartz crystals. Using this discovery he built a polariscope consisting of a tube with a Rochon polarizing beam splitter at the eye end and a suitable quartz crystal at the far end (Fig. 1). This polariscope produces two adjacent images that are identical if polychromatic light from a viewed object is not linearly polarized. If the light is linearly polarized the optical activity of the quartz rotates shorter wavelengths more than longer wavelengths with the result that the adjacent orthogonally-polarized images show complimentary colors. By orienting and tipping a plane glass plate in front of the polariscope to angles

Table 1. Selected events in polarimetry.

| Year | Event |
|------|--|
| 1669 | R. Bartholin publishes monograph on double refraction in calcite |
| 1690 | C. Huygens publishes model of double refraction based on 1672 observations of sunbeams in calcite |
| 1777 | AM. de Rochon invents quartz crystal beam-splitting polarizer |
| 1807 | EL. Malus translates Huygens model into algebra, wins big prize |
| 1808 | Malus discovers polarization by reflection of sunlight from a window; introduces term polarization |
| 1809 | F. Arago builds a simple mica+calcite polariscope, confirms Malus' cos² law |
| 1811 | F. Arago discovers optical activity of quartz |
| 1811 | F. Arago builds quartz+Rochon beam splitter polarimeter |
| 1819 | F. Arago and A. Fresnel assert transverse nature of polarized light |
| 1820 | W. Wollaston invents a new quartz crystal polarizer |
| 1832 | A. Fresnel publishes equations of reflection and transmission of polarized light |
| 1845 | M. Faraday discovers magnetic rotation of plane of polarization |
| 1852 | G. Stokes introduces Stokes parameters |
| 1875 | J. Kerr discovers birefringence of isotropic material induced by electric and magnetic fields |
| 1896 | P. Zeeman discovers splitting of spectrum lines in magnetic field |
| 1924 | W. Hanle describes magnetic depolarization of anisotropically scattered light |
| 1928 | E. Land invents sheet polarizer |
| 1933 | B. Lyot describes narrow band birefringent filter |
| 1937 | Y. Ohman designs and operates first birefringent filter |
| 1943 | H. Mueller develops matrix calculus for polarization calculations |

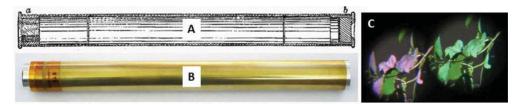


Figure 1. (A) Arago's diagram of his polariscope (B) Replica of Arago's polariscope constructed by the author and demonstrated at the symposium. (C) Replica's image of sunlit plant leaves (color version in on-line copy of this figure).

that eliminate the color difference, the direction and amount of linearly polarized light can be measured with a sensitivity approaching 10^{-2} . If this polarimeter is equipped with a suitable objective lens and eyepiece, we have Arago's "lunette polariscopique".

Visual polarimeters of this general type with clever improvements were used during much of the 19th century and into the early 20th century. Arago and others made important discoveries about the Sun and in many other areas of physics. However, visual detection limited sensitivity to about 10^{-2} . The introduction of photographic and especially photoelectric detection enabled greatly improved polarimetric sensitivity. Solar sensitivity close to 10^{-7} has been achieved (Kemp et al. 1987) and exoplanet research has pushed stellar imaging polarimetry to 10^{-6} . Moreover, spatial and spectral resolution and ranges have increased by many orders of magnitude making polarimetry a crucial tool for contemporary astrophysics. Table 1 is a truncated time line of significant events in the history of polarimetry and Table 2 is a time line for solar polarimetry.

2. Polarimetry of the Solar Disk

2.1. First Observations 1811

When Arago turned his lunette polariscopique to the solar disk he found no detectable polarization anywhere, especially near the limb. In the laboratory Arago observed that light emitted by incandescent solids and liquids was linearly polarized when viewed at an angle to their surfaces. But flames showed no detectable polarization. Based on his laboratory results, he concluded that the visible solar surface was an incandescent gas

Table 2. Selected early events in solar polarimetry.

| Year Event | | |
|--------------|--|--|
| 1811 | F. Arago concludes Sun's surface is gaseous based on non-polarization | |
| 1812 | J. E. Bérard reports solar IR polarization properties similar to visible | |
| 1842 | F. Arago and V. Mauvais observe polarization of corona at eclipse | |
| 1851 | E. Edlund observes that coronal polarization is radial | |
| 1858 | E. Liais confirms solar corona is radially linearly polarized | |
| 1879 | A. Schuster models coronal polarization as scatter from tiny particles | |
| 1908 | G. E. Hale discovers Zeeman splitting in sunspots | |
| 1930 | B. Lyot measures polarization of corona outside of eclipse | |
| 1932 | B. Lyot measures polarization in prominence emission lines | |
| 1934 | B. Lyot measures polarization of prominence continuous spectrum | |
| 1941 | R. Redman observes polarization in wings of a resonance line near solar limb | |
| 1946 | Three groups observe strongly circularly polarized solar radio storm | |
| 1948 | B. Lyot detects weak linear polarization near the solar limb | |
| 1952 | K. O. Kiepenheuer measures solar Zeeman splitting photoelectrically | |
| 1952 | H.D. and H.W. Babcock build first solar magnetograph | |
| 1959 | V. Stepanov and A. Severny build first vector magnetograph | |
| 1964 | E. Mogilevsky et al. measure polarization of coronal emission line | |

(Arago 1830,1865a). He did not report this until 1824 (Arago 1824) though he dated his conclusion as 1811 in his chronology of principal astronomical discoveries (Arago 1865b). His result was not uniformly accepted as some astronomers believed the Sun was solid or liquid well into the 19th century.

2.2. Recent Status

Continuing a French leadership tradition in solar polarimetry, a striking improvement in sensitivity was realized by Bernard Lyot when photomultiplier tubes became available. Using a one arc min field of view centered one arc min from the solar limb he detected linear polarization perpendicular to the limb. The amount of polarization measured ranged from 2×10^{-5} to 3.9×10^{-4} (Lyot 1948). The disk center showed no polarization greater than 2×10^{-5} . Subsequent broad spectral band measurements were made by Dollfus & Leroy (1960) and showed the polarization to increase with smaller wavelengths and distances from the limb. Very useful comparisons of observations with theory were made in the 1970s using spectral resolution sufficient to resolve true continuum (Leroy 1977).

In the 1970s solid-state detectors became available having high quantum efficiencies and other desirable properties. It then became possible to make spectrally resolved polarimetric surveys of sunlight from near the solar limb. The first surveys covered relatively large wavelength regions but were noisy (Stenflo et al. 1983a,b). A novel CCD detector designed for high polarimetric precision observations (ZIMPOL) was used by Gandorfer (2000, 2002, 2005) to produce superior atlases of the linearly polarized solar spectrum. Stenflo (2005) used these observations to produce the current state of the art measurements of continuum polarization near the solar limb included in Fig. 2 along with 3D radiative transfer models of predicted polarization from Trujillo-Bueno & Shchukina (2009).

3. Polarimetry of the Solar Corona

3.1. First Observations 1842-1858

Since the solar corona was a mystery it was natural to attempt to detect and measure its polarization. The total solar eclipse of 8 July 1842 provided such an opportunity. F. Arago observed from near his birthplace, Perpignan, along with V. Mauvais. Both saw definite indications of polarization using different types of polariscopes. Not only the corona but also skylight near the Sun and across the moon's disk showed maximum

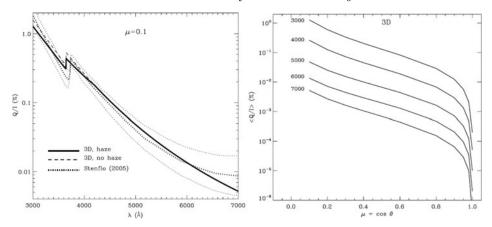


Figure 2. (Left) Comparison of wavelength variation of linear polarization of the continuum near the solar limb. Solid line shows results from a 3D model compared with Stenflo's (2005) bold dotted line measurements. The light dotted lines are approximate upper and lower limits due to observational scatter. (Right) Model results for center-to-limb variation at indicated wavelengths. Figures from Trujillo Bueno & Shchukina (2009) ©AAS. Reproduced with permission.

polarization when the polariscope was oriented horizontally. However, two observers located at Narbonne saw no signs of polarization. To further complicate matters, observing from Milan, A. Gabba saw polarization while G. Majocchi was not able to confirm the observation (perhaps because he was busy attempting to photograph the eclipse). Such contradictory results continued from several subsequent total eclipses including results from such notable observers as Carrington, Langley, Lockyer, and Pickering (see compilation by Ranyard 1879). Separating the polarization of atmospheric light from that of the corona was a troublesome complication. At the eclipse of 1851, E. Edlund (1860) did this and found that the coronal light was radially polarized (tangential vibration). This result was confirmed at the 1858 eclipse by E. Liais (1859). By 1870 the observational consensus was that the corona was radially linearly polarized. Schuster (1879) then assumed the corona consisted of small scattering particles (electrons were yet to be discovered) and worked out the basic theory of coronal polarization.

3.2. Recent Status

The brilliant Lyot (1930) invented a successful coronagraph and combined it with his sensitive visual polarimeter. He made his polarimeter about ten times more sensitive than previous visual polarimeters by adding a barely detectable amount of bias polarization. Then small source polarizations could be detected as changes when the bias polarizer is rotated between orthogonal orientations. Observing in the clear skies of Pic du Midi he made plots of the amount of coronal polarization 80 arc sec above the limb as a function of position angle such as shown in Fig. 3.

Continued progress in instrumentation led to polarimetric K-coronameters starting with Wlérick & Axtell (1957). Regular synoptic observations started in 1964 (Hansen et al. 1969) and continue to the present with improvements that take advantage of technology advances. Figure 3 includes a recent image taken with HAO's new K-cor instrument (de Wijn et al. 2012).

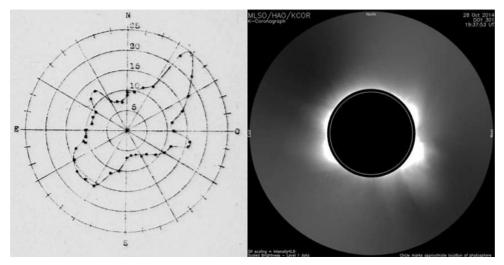


Figure 3. (Left) Broad-band polarization of corona 80 arc sec above the limb measured on 31 July 1930 by Lyot (1930) at different position angles. Units: 0.001. (Right) K-corona image taken on 28 October 2014 by HAO's new K-cor instrument. Courtesy of the Mauna Loa Solar Observatory, operated by the High Altitude Observatory, as part of the National Center for Atmospheric Research (NCAR). NCAR is supported by the National Science Foundation.

4. Polarimetry of Solar Prominences

4.1. First Observations 1851-1934

The striking appearance of brilliant prominences at total solar eclipses attracted efforts to measure the polarization of their light. Visual observations at the eclipses of 1851, 1860 and 1868 were contradictory at first but eventually favored no detectable polarization (see Ranyard 1879). Lyot developed colored liquid/glass filters that isolated the D_3 and $H\alpha$ lines, and the continuum between them. With these filters and his coronagraph he started measuring prominence polarization in August 1932 (Lyot 1934, 1937). He found 15% in the faint continuous spectrum with orientation always perpendicular to the limb. The D_3 and $H\alpha$ lines showed less polarization, 0.9-1.5% and 0.8-1.2% respectively but with non-radial alignments up to 40 and 30 deg respectively. The orientations were systematically opposite in the north and south hemispheres. We now know that he was observing rotation by the Hanle effect.

Sensitive photoelectric spectropolarimetric observations of prominence magnetic fields were undertaken in the 1960s and 1970s. These (including my own thesis observations) naively assumed that disk-based techniques would be successful with prominence emission lines. Circular polarization signals in the blue and red line wings were nulled by shifting the spectrum line synchronously with polarization modulation. The required nulling shift was interpreted as Zeeman splitting due to the average line-of-sight magnetic flux density. In quiescent prominences field strengths of up to a few tens of gauss were inferred while up to a few hundred gauss was estimated for active region prominences. Line profile anomalies and strong linear polarization were ignored. These were serious omissions. It was just luck that the inferred field strengths turned out to be about right.

4.2. Recent Status

The reader should consult the excellent review by Lites (2013) and references therein. In the late 1970s and into the 1990s linear polarization measurements averaged across various bright prominence emission lines exploited the Hanle effect to permit estimates



Figure 4. (Top to bottom) An active disk filament observed at three different wavelength offsets from the center of the CaII 854.2 nm line. (Left to right) Intensity, line of sight magnetic flux density, same but masked to the filament area. Display range is ± 195 G.

of field strength and direction. Similar to Lyot's results, measurements by Leroy et al. (1977) typically showed about 2% polarization with a deviation of about ± 15 deg from parallel to the limb when using the D_3 line. Reduction of such measurements to magnetic field strength and direction required some assumptions, principally that the field was horizontal. For quiescent prominences in polar regions the inferred field strengths of a few gauss were similar to the earlier Zeeman results.

By the start of the 21st century, technical improvements made full Stokes polarimetry of resolved emission lines in prominences and the corresponding absorption lines in ondisk filaments the observation of choice since both Zeeman and Hanle effects could be determined. Advances in theoretical modeling allowed development of inversion codes to convert the polarimetry into magnetic field strength and direction (e.g. Orozco Suárez et al. 2014). A recent observation of an on-disk filament is shown in Fig. 4. Notably, earlier Zeeman and Hanle results that suggested smooth spatial variations of the ~ 50 G magnetic field are also seen here (Harvey 2012).

5. Disk Spectropolarimetry

5.1. First Observations 1908

Motivated by a suspicion that sunspots might harbor strong magnetic fields, G. E. Hale (1908) looked for and found polarization of many solar absorption lines consistent with a Zeeman effect produced by kilogauss magnetic fields. Early measurements were made from photographed slit spectra of sunspots using analyzers for circular and linear polarization. By 1917 a daily program of visual measurements of umbral Zeeman splitting was underway at Mt. Wilson that continues to this day. The early results of this program led to the discovery of the 22-year Hale magnetic cycle. A hunt for a large-scale, general solar magnetic field was pursued but was defeated by the poor sensitivity of the photographic observing method. As is well-known, this quest finally met with success thanks to the genius of Babcock & Babcock (1952) through their use of photomultiplier detectors, fast circular polarization modulation with synchronous demodulation, and servo-controlled scanning of the solar disk and display of the results.

5.2. Recent Status

In 1960 there were few instruments to measure polarization with solar disk absorption lines. Today, most research solar telescopes are equipped with spectropolarimetric

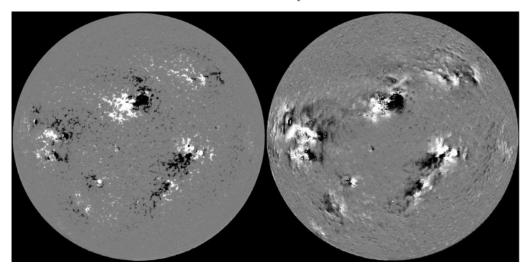


Figure 5. (Left) VSM/SOLIS circular polarization measurements interpreted as magnetic flux density observed with the near-photospheric wings of the CaII 854.2 nm line. Display range ± 300 G. (Right) Same but using the chromospheric core of the line and the photospheric component subtracted. Note signatures of field spreading with height. Display range ± 50 G.

instrumentation. Breakthrough polarimetric capabilities came with the MDI/SOHO, HMI/SDO, and SOT/Hinode space instruments. Since adaptive optics and other image improvement techniques became available in the late 1990s there has been a strong emphasis on high spatial and temporal resolution spectropolarimetry with the aim of comparing observations with radiative MHD models. At the other end of temporal and spatial scales, sustained full-disk observations over long time periods continue to provide crucial information about the solar magnetic cycle and evolution of solar activity. Real-time availability of these observations is important for space weather predictions. Relatively unexploited are regular observations of the chromospheric magnetic field such as shown in Fig. 5.

6. Polarimetry of Solar Radio Emission

6.1. First Observations 1946

Major technology advances during World War II led to the post-war development of solar radio astronomy and polarimetry (cf. Wielebinski 2012). When a large active region appeared in July 1946, Martyn (1946), Appleton & Hey (1946), and Ryle & Vonberg (1946) nearly simultaneously reported observing strong circular polarization in a metric radio noise storm from the region. These results confirmed a non-thermal origin for solar metric radio emission.

6.2. Recent Status

Polarimetry continues to be a crucial diagnostic of solar radio emissions. Besides informing the physics of various radio emission processes, the varying opacity of the solar atmosphere with wavelength presents a way of probing the magnetic field structure with height well into the corona. A spectacular recent example of solar radio polarimetry with the Long Wavelength Array(LWA; http://www.phys.unm.edu/lwa/lwatv.html) is shown in Fig. 6.

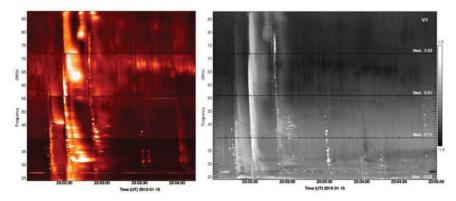


Figure 6. Three minutes of solar radio spectra from 25 to 87 MHz obtained with the LWA showing intensity (left) and circular polarization (right). Courtesy of S. White.

7. Prospects and Summary

As in many fields of science, it is clear from the history of solar polarimetry that observational advances result from application of new technology. Equally important are physically sound models to interpret observed phenomena.

To illustrate one interaction of models and observations, the 3D models of Trujillo-Bueno & Shchukina (2009) predict that linear polarization should be observed closely associated with granulation even at disk center, as illustrated in Fig. 7. The high levels of observational sensitivity and spatial resolution needed to verify these predictions are very demanding. It is expected that the Daniel K. Inouye Solar Telescope (DKIST) will provide such observations to test the models.

In coronal polarimetry the combination of spectral line and the continuous spectrum is powerful since the line polarimetry provides additional information such as Doppler velocities and Zeeman and Hanle magnetic field measurements. Such observations are discussed elsewhere in these proceedings. However, the optically thin corona poses challenges in finding the location of features along lines of sight. The problem of 3D localization can be reduced by combining both continuum and line emission and polarization measurements. In the future, the Multi Element Telescope for Imaging and Spectroscopy (Fineschi et al. 2012) instrument on board the Solar Orbiter will provide 590-650 nm K-corona polarimetry (Crescenzio et al. 2012) for electron density measurements away from the Sun-Earth line. Combining these measurements with those from K-cor should help reduce the 3D localization problem.

Signals from solar prominences are weak. So large aperture telescopes are required to map their magnetic fields with decent time and spatial resolutions. By using prominence spectrum lines with significant opacity seen against the solar disk lines much more could be learned about these curious features. There are at least 20 ground-based instruments around the world that can do spectropolarimetry of limb prominences and disk filaments. But the number of observations is small. Breakthrough measurements are expected from DKIST and the Japan/US/Europe Solar-C space mission. Fortunately, diagnostic methods are advancing in pace with new instrumental capabilities.

The distinction between disk and off-limb spectropolarimetry is mainly telescopic. Large apertures are required to observe weak signals in both cases. But low scatter and dark background skies for off-limb work are difficult to attain. The large coronagraph planned for the COSMO project (http://www.cosmo.ucar.edu/) is a promising answer to this challenge.

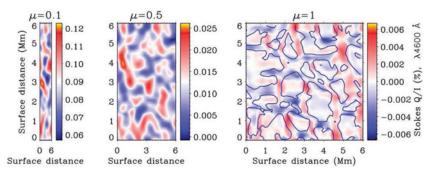


Figure 7. 3D models of continuum linear polarization expected in near limb to disk center observations at high spatial resolution and polarimetric sensitivity. Figures from Trujillo Bueno & Shchukina (2009) ©AAS. Reproduced with permission.

Today's emphasis on high spatial resolution observations should not displace the need for polarimetric observations of large-scale, long-duration magnetic phenomena. In this regard, the Solar Physics Research Integrated Network Group (http://www.solarneteast.eu/) initiative is promising.

New and upgraded radio facilities that can be used to observe the Sun should complement existing facilities such as the Nobeyama Radioheliograph (http://solar.nro.nao.ac.j-p/norh/), the Nançay radioheliograph (http://www.obs-nancay.fr), and the Owens Valley Solar Array (http://www.ovsa.njit.edu/). A Frequency Agile Solar Array (FASR) has long been advocated as a major step forward. The new Chinese Spectral Radio Heliograph (http://srg.bao.ac.cn/csrh.htm) should provide such observations. Though built for other purposes, the LOw-Frequency Array for Radio astronomy (http://www.lofar.org/) and Atacama Large Millimeter/submillimeter Array (http://www.almaobservatory.org/) promise spectacular new results when trained on our nearest star.

Polarimetry is a high dimensionality problem that requires many trade offs to obtain practical results. Technology eases the pain of those trade offs. Rapid advances in detectors and array-optics offer a near-term prospect of revolutionary new multiplexed polarimetric observations that will greatly add to our knowledge of the Sun over much of its atmosphere. Compared with past two centuries, the gains of the next two centuries of solar polarimetry should be fantastic.

Acknowledgements

I thank the SOC for an invitation to look into and report some of the fascinating history of solar polarimetry. The National Solar Observatory is operated by the Association of Universities for Research in Astronomy (AURA), Inc. under a cooperative agreement with the National Science Foundation.

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