STRUCTURAL MODELS FOR BETA LYRAE-TYPE DISKS

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ABSTRACT

Equilibrium structural models are computed for a thick, self-gravitating disk in a binary system. Accretion onto the star is limited by the star's rapid rotation (the system is a double-contact binary). The potential formulation is taken from a previous paper, and represents the gravitational potential as that of a massive wire. Corrections to the stellar structure differential equations for the distorted geometry are applied, and the equations are integrated and solved by the fitting point method. The energy is supplied by viscosity. Energy transfer is by convection, and is appreciably superadiabatic throughout the disk. A mass of 0.5 $M_{\odot}$ is assumed. Representative results are: "central" temperature, 67000 K; "central" pressure, 5×10^{11} dynes/cm^2; "equal volume" radius, 17 $R_{\odot}$; luminosity, 5×10^3 $L_{\odot}$. The model "radius" is in excellent agreement with the observational value for β Lyrae. The model luminosity is slightly higher than the available rate of expenditure of gravitational energy, indicating that a lower disk mass (perhaps 0.25 $M_{\odot}$) should be tried.

I. INTRODUCTION

Over the past several decades, many questions have arisen concerning various unusual features of the β Lyrae system. Underlying all of these is the central question: Why is β Lyrae so different from other semi-detached, mass transferring binaries? We would like to know how β Lyrae is able to maintain a geometrically thick, opaque disk (viz. Huang, 1963; Wilson, 1974) about its more massive component, while a system such as U Cep cannot. Of course, one can point to the rather large rate of period change, which places β Lyrae in the rapid phase of mass transfer. However there are grounds to suspect that the explanation runs deeper, as the predicted time scale for collapse of the disk onto its central star is quite short compared to the historical interval of accurate β Lyr observations (≈ 65 years). There is now fairly uniform agreement regarding the existence of a massive (≈ 10 to 15 $M_{\odot}$)

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main sequence star at the center of the β Lyr disk. Had the disk ceased to exist or had it even become substantially thinner at any time during the star's observational history, the system would have brightened considerably and become optically prominent in the constellation Lyra. Since there is no record of such an occurrence, one can conclude that the disk is quite stable in its large-scale aspects. In this paper, the suggestion (Wilson, 1979, 1981) is adopted that centrifugally limited rotation of the central star is responsible for the disk's persistence, so that β Lyr is a double-contact binary (Wilson, 1979). Note that Packet (1981) has shown that relatively little mass transfer may be needed to spin an accreting star up to its centrifugal limit.

In the past two years, Plavec (1980) has called attention to a class of binaries which have very extensive circumstellar disks and are similar in many ways to β Lyr. These he calls the W Serpentis stars. Members of the class include SX Cas, RX Cas, AR Pav, W Ser, V367 Cyg, W Cru, V356 Sgr, and β Lyr, all of which show spectacular spectroscopic phenomena, including evidence for high velocity ejection of matter from the entire system. Our fundamental question may now be widened, and we ask: Why are the W Serpentis stars so different from other semi-detached, mass transferring binaries? A plausible reason is that they all are in the double-contact phase.

It seems appropriate to place β Lyr, the most thoroughly observed W Ser star, at the focus of our attention, so a brief review of its overall features follows. The optically dominant member of β Lyr is a mass-losing giant star, apparently rotating synchronously with the orbit, and filling its Roche lobe. The spectral type is approximately B9 II and well-defined radial velocity curves exist (Sahade, et al. 1959), which give a mass function of 8.5 $M_\odot$. Primary eclipse in optical passbands is the eclipse of this star. The other component has long been regarded as enigmatic. In the optical region it shows no line spectrum (probably because of rotational line broadening), but we know that it does emit significant light - comparable in amount to that of the B9 star - because there is a well defined secondary eclipse. Also its luminosity has been estimated by fairly detailed modeling of the light variation (Wilson, 1974; Wilson and Lapasset, 1981). The mass of this object has been difficult to determine, although there now seems to be unanimous agreement that it is more massive than the B9 star by a factor of at least 2.5, and probably 6 or more (see Wilson, 1974 for a summary of mass ratio determinations). Here we assume a mass for this component of about 12 $M_\odot$.

Our working model for the dim but massive component will be that of a main sequence star surrounded by a thick disk which entirely obscures it from direct view. Perhaps "ring" might be a better word than "disk", since the structure is certainly thick, while the term "disk" usually describes a highly flattened object. However, we retain the name "disk" here because real progress in understanding began with Huang's (1963) paper, which made it clear for the first time that we are dealing with a relatively flattened circumstellar "disk". While Huang's disk was actually much too thin for its predictions to be in quantitative agreement with the light curves, the "Huang disk model" is well known and should continue to be remembered for its start in the right direction.

Our lack of physical models for β Lyr-type disks is a reflection of the fact that we do not understand the accretion process in the rapid phase of mass transfer in sufficient detail. This is a very unsatisfactory state of affairs, for it is now well accepted that mass and angular momentum are not conserved in mass exchange episodes. If these quantities are not conserved, computations of binary star evolution must eventually follow mass transfer events in detail. We must understand what is happening within β Lyrae type disks because spectroscopic observations show that mass is ejected from the disks and from the systems, and all later evolutionary stages are thereby affected.

This paper is intended as a start toward full modelling of β Lyr type disks. Certainly one should be able to answer simple questions concerning the order of magnitude of physical variables within the disks. What are reasonable masses, internal pressures, and temperatures? What is the relative importance of one source of luminosity compared to another? Is energy transferred primarily by radiation or by convection? Can we say anything about the rotation law? What are the approximate distributions of mass and other physical variables? Toward these ends we adopt the following strategy. The disk is to be approximated by an equilibrium structure. If we can establish the form of the potential field, and define a reasonable mode of luminosity generation, it should be possible to compute static structural models in the same manner as for single stars, provided that we apply corrections for the non-spherical geometry. A logical basis for the potential has been given in an earlier paper (Wilson, 1981, hereafter Paper 1). Essentially, the idea is to represent each binary component as a mass point, with the disk mass concentrated into a circular wire, centered on one of the stars. Rotation velocity is to be constant on cylinders so as to give a conservative potential. Numerical experiments have shown that the wire model gives a satisfactory approximation to the disk-mass potential. Note the analogy of the wire model with the Roche point mass approximation for stars. The centrifugal potential is computed from a rotation law in which angular velocity varies as u^n, with u the distance from an axis of rotation and n a parameter which turns out to be constrained in several ways by various features of the problem (Paper 1). Actually, two regimes with different n are required if the solutions are to be fully consistent. Figure 1 shows a computed disk equilibrium figure (cf. Paper 1).

While in a formal sense the present computation of disk models assumes static equilibrium, this is only a simplification, needed to keep the first efforts computationally managable. In reality the situation would be one of dynamical equilibrium, with matter continually accreting from disk to star, but with that flow balanced by a return injection from star to disk. Probably the latter occurs at zero latitude, while the former occurs slightly above and below zero latitude.

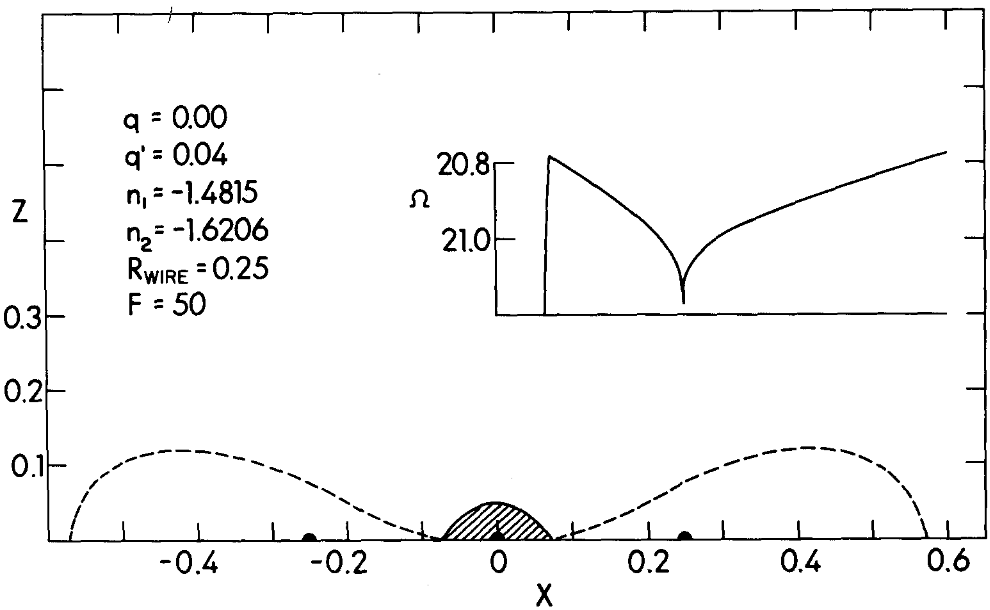

Figure 1. A cross-section of the surface equipotential for the case used in this paper. The hatched area is the critically rotating central star and the three half-dots along the x-axis depict the locations of mass concentrations used in computing the potential. The inset shows the potential as a function of x.

II. INPUT TO THE PROBLEM - GROSS CHARACTERISTICS OF THE DISK

In order to be sure that our disk dimensions are reasonable, we adopt those of the β Lyrae disk, as estimated in earlier papers (Wilson, 1974; Wilson and Lapasset, 1981). According to the disk potential model (Paper 1), the full potential is determined when the star masses, disk mass, disk radius, and disk thickness have been specified. The numbers for our β Lyr calculation are R_{disk} = 0.57a, Z_{max} (half-thickness) = 0.12a, M_{disk} = 0.04 M_{star} and M_{disk} = 0.5 $M_\odot$. We take the mass of the other star as zero so as to make these first computations axisymmetric. It is shown in Paper 1 that the disk mass should be non-neglible, although it is not clear that 0.5 $M_\odot$ is a particularly good estimate. Hopefully, it is of the correct order of magnitude (cf. arguments in Paper 1). The relative chemical abundances by mass are taken to be X = 0.70, Y = 0.27, and Z = 0.03. More detailed abundances needed when solving for the ionization equilibrium were taken from Allen, 1973. The same source was used for ionization potentials and partition functions. We cannot expect the rotation law to be Keplerian because thermal gas pressure support is important in the "z" dimension (the disk is quite thick), so it certainly would be important also in the radial dimension. Only part of the radial support would therefore

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come from rotation.

Is the disk radiative or convective? Since it has no nuclear luminosity, one might think that radiation could easily carry the energy flux. However on two grounds we know that it must have a luminosity which is enormous for a 0.5 $M_{\odot}$ (or so) object. First, we have the directly observable luminosity, which is of the same order as that of the B9 star (Wilson, 1974). Second, the considerable thickness of the disk requires a large luminosity to provide adequate thermal pressure support. If we substitute approximate values for the physical variables into the Schwarzschild criterion for convective instability, we find that the disk will be convective by a large margin, and (drawing on the results of the computed models) full structural computations show that it will be convective throughout. Energy transfer by convection is an advantage if the disk is to be an equilibrium structure because (non-nuclear) energy will not be generated at constant rates on equipotentials. Therefore convection, as a mechanism for lateral redistribution of energy, will improve the correspondence between the real disk and the model. On the other hand, there are difficulties in understanding how the necessary differential rotation can be maintained in the presence of convection. However there is no choice - order of magnitude calculations show that the disk will certainly be convective. Therefore we compute our first models in the spirit of making a beginning. Hopefully something will be learned from them, even if it is only a rough indication of where to go from here.

Although we expect the energy transport to be convective, we cannot expect adiabatic convection because temperatures and densities are too low. The outer convection zone of a normal star such as the sun is superadiabatic only in its very outermost part - perhaps half of one percent of the stellar radius. However our disk will have internal temperatures of the order of a hundred times smaller and densities of the order of 10^3 to 10^4 times smaller than those of a star (viz. estimates in Paper 1), so that we expect it to be superadiabatic throughout, or almost throughout. Thus the temperature gradient should be computed by means of a full theory of convective transport. The computations here, including solutions for the ionization equilibrium, are based on the method outlined by Baker and Kippenhahn, 1962 (BT), which is a variation on that by Bohm-Vitense, 1958. The convection routine was checked (satisfactorily) against the "Tables of Convective Stellar Envelope Models" by Baker and Temesvary, 1966.

III. COMPUTATIONAL METHOD

Viewing our problem from the standpoint of stellar structure theory, we can write the potential (Paper 1, eqns. 1, 3, 12) in a form which satisfies all the exterior conditions, such as rotational continuity with the central star, which seem logically to apply for β Lyr and the other W Ser stars. The potential so formulated shows sufficient topological similarity with the standard stellar structure problem that a

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one-dimensional formulation becomes natural. Let us make a meridional slice through the disk. There will be a point of maximum density, which should lie on the symmetry axis of the section. This point is the analog of the center of a normal star. We are to integrate the differential equations of stellar structure outward from this point. A similar inward integration from the surface of the disk is to meet the outward integration smoothly, as in the usual fitting point method (Haselgrove and Hoyle, 1956).

Corrections for the distorted geometry can be made very conveniently by the method given by Kippenhahn and Thomas, 1970 (herafter KT). While the KT method was intended for distorted stars, there is no reason not to use it for our disk model. The radial coordinate for the integration is the radius of a sphere having the same volume as the toroidal equipotential. We label this coordinate r_s (for r_{sphere}). We need to compute the volume, surface area, mean acceleration due to gravity, and mean spacing of the level surfaces for equipotentials as they are encountered while integrating the differential equations. Combinations of these functions (viz. KT) are lumped into two correction factors - one, f_t, for the dT/dM_r equation and one, f_p, for the dP/dM_r equation. The continuity equations, dr/dM_r and dL_r/dM_r require no correction factors. The volume, surface area, mean gravity, and mean inverse gravity (proportional to level spacing) were computed, respectively, by

$$V = 4\pi \int_{u_A}^{u_B} zu \, du \tag{1}$$

$$S = 4\pi \int_{u_A}^{u_B} u\sqrt{1 + \left(\frac{dz}{du}\right)^2} \, du, \tag{2}$$

$$\bar{g} = \frac{4\pi}{S} \int_{u_A}^{u_B} gu\sqrt{1 + \left(\frac{dz}{du}\right)^2} \, du, \text{ and} \tag{3}$$

$$\overline{g^{-1}} = \frac{4\pi}{S} \int_{u_A}^{u_B} g^{-1} \, u\sqrt{1 + \left(\frac{dz}{du}\right)^2} \, du. \tag{4}$$

Here u is the distance from the rotation axis, which passes through the center of the star and is normal to the disk-orbit plane. The quantities u_A and u_B are the inner and outer limits of a given (toroidal) level surface, and z is the coordinate, measured vertically to the disk-orbit plane, of a given point on the level surface. Eqn. (1) [supplemented by eqns. 3 and 12] of Paper 1 and its u derivatives provide dz/du, g, and g^{-1}. The integrals in eqns. (1, 2, 3, 4) were evaluated by a seventh order gaussian quadrature. V, S, $\overline{g}$ and $\overline{g^{-1}}$ were then combined into correction factors, as prescribed by KT.

We must also find a means for computing the energy deposition. From the several likely contributors, let us adopt viscous dissipation for the present. In a simple treatment we may consider this proportional to the local density and to the velocity shear, dv/du. The rate of viscous energy generation per gram will be independent of the density. Averaged over a given toroidal shell between two adjacent level surfaces, it can be shown to be given by

$$\varepsilon_v = \frac{2\Pi \left| n_2 + 1 \right| KF}{u_1^{n1}\, u_2^{(n2 - n1)}\, P} \; \frac{\displaystyle\int_{u_A}^{u_B} \frac{u^{n_2 + 1}}{\left(\frac{d\Omega}{dz}\right)}\, du}{\displaystyle\int_{u_A}^{u_B} \frac{u}{\left(\frac{d\Omega}{dz}\right)}\, du}\ , \tag{5}$$

where u_1, u_2 are the values of u at the disk-star contact point and the disk density maximum, respectively. F is the ratio of the rotation rate to the synchronous rate at u_1, P is the binary orbital period, and Ω is the potential according to eqn. 1 of Paper 1. We leave K, the effective efficiency of viscous energy conversion, as a free parameter, and we shall compute models for several K values.

As in normal stellar structure integrations, it is necessary to make short analytic integrations at the "center" and surface, to avoid singularities. These may be done in the usual way (viz. Schwarzschild, 1958, pp. 114-6) except that we need values of f_p and f_t at the "center" and surface. At the surface these functions vary slowly with r_s and may be computed in the same way as elsewhere within the disk. However, near the density maximum (our "center") they vary rapidly. Furthermore our quadrature schemes for finding f_p and f_t may be expected to lose accuracy near the center. Therefore we need analytic forms for f_t and f_p in the limit of small r_s. The f_t factor for a toroidal equipotential of very small r_s, surrounding the ring-like density maximum, becomes

$$f_t\ (r_s \to 0) = \frac{r_s}{3\Pi u_2}\ . \tag{6}$$

The factor f_p is not so simple, and goes to

$$f_p\ (r_s \to 0) = \sqrt{\frac{3}{2\Pi u_2}}\ \frac{r_s^{5/2}}{\overline{g^{-1}}} \tag{7}$$

near the center. For very small r_s, the wire potential alone determines the force field and $\overline{g^{-1}}$ depends only on r_s, u_2, and q'. The behavior of f_p (eqn. 7) depends on the ratio $r_s^{5/2}/\overline{g^{-1}}$, and for small r_s

one can show that $\overline{g^{-1}}$ is proportional to r_s. Thus f_p is proportional to $r_s^{3/2}$ and approaches zero for small r_s. Unfortunately, without resorting to series approximations, $\overline{g^{-1}}$ cannot be represented by a simple expression which will improve the usefullness of eqn. 7. In future work it may be worthwhile to develop such series, but for now we adopt $f_p = 0$ for the short analytic integration from the center. As the analytic integration covers only a small range of r_s (from 0 to 0.001a) and f_p is indeed small, only a minute error is involved.

We also need an expression for the energy generation rate for the "center" analytic integration. Eqn. 5 simplifies at the center and becomes

$$\varepsilon_{v_c} = \frac{2\Pi\left|(n_2 + 1)\right| KF}{P} \left(\frac{u_2}{u_1}\right)^{n1} . \tag{8}$$

The independent variable for the integrations through the disk will be M_r (mass interior to a given r_s). However M_r is not a good choice for the independent variable in the outer parts of the disk because the derivatives $d(\log P)/d(\log M_r)$, $d(\log T)/d(\log M_r)$, etc. become extremely large and non-linear. Therefore, a switchover is made to r_s as the independent variable in the difficult region, beginning where the surface analytic integration terminates and ending where the density is found to be reasonably high. For the present computations the surface analytic integration extends from $r_s/R = 1.00$ to 0.996, and the Eulerian numerical integration from there to $r_s/R = 0.550$. Here R is the surface value of r_s (not the value of u at the density maximum as in Paper 1). The fitting point was placed at $M_r/M = 0.400$.

Unless some special steps are taken, a program of the type we need will use an enormous amount of computing time, even on a rather fast machine. There are two reasons for this. First and more important, the geometrical correction factors must be computed many times for each integration step. In fact the situation is far worse than one might first think, because the independent variable is not the potential (Ω), but rather r_s, so an additional nested step is required to find $\Omega(r_s)$ by (for example Newton-Raphson) iteration. Naturally, even after Ω is known, many inversions of the potential equation are needed to evaluate eqns. 1, 2, 3, 4, and 5. The second problem concerns the need for doing non-adiabatic convection calculations at every integration step.

A great reduction in computer time is potentially available if the first problem can be circumvented. Note that the entire disk geometry is permanently specified as soon as the potential has been formulated. This point suggests that we establish the functions $f_t(r_s)$, $f_p(r_s)$ and $\varepsilon_v(r_s)$ at the beginning and represent them by simple approximation polynomials for later use. Adequate accuracy was obtained here with cubic polynomials, as the three special functions are all smooth and generally well behaved. Actually, all three special functions are nearly linear over long ranges, so a combination of a straight line and a cubic was used for each. The improvement in running speed was more than a factor of ten. Unfortunately the program is still slow because of the convection calculations and the relatively slow convergence of the fitting point scheme for disks, compared to that for normal star models.

IV. RESULTS AND DISCUSSION

Table 1 lists the main results of the first experiments, with masses and dimensions as given in section II. Figure 2 shows the results in graphical form. The precision (i.e. repeatibility, given the assumptions of the procedure) of the numbers is set by tolerances within the fitting point iteration scheme. For all four physical variables (radius, luminosity, "central" temperature and "central" pressure) the error tolerances were 0.3 percent. Four values of K, the coefficient of viscous energy generation, were tried. Convergence was much faster for the middle two K values than for the high and low value, which could not quite satisfy the 0.3 percent error tolerances. In fact, in all cases convergence was much slower than for a stellar model, and was only reliable within regions of parameter space fairly close (within perhaps 20 percent) of the final answers. Therefore to a certain extent it is necessary to "discover" solutions, although once a solution has been found, it is easy to predict neighboring solutions for other values of K, disk mass, etc. The radius and luminosity are in excellent agreement with the observational values for β Lyr (Wilson, 1974, 1981). The central temperature and pressure agree well with the order of magnitude predictions in Paper 1. It is important to have found solutions over a range of K values. If only eigensolutions were possible, the entire idea would be rendered implausible, as the real disk could not be expected to know the correct physical value of K.

Table 1

Disk Structural Models

K(erg/g)	$\bar{R}_s/R_\odot$	$L/10^3\ L_\odot$	Tc(K)	Pc $\left(10^{11}\ \frac{\text{dynes}}{\text{cm}^2}\right)$
5.0×10^8	15.7	3.7	69,800	7.2
7.0×10^8	17.0	5.1	67,100	4.9
8.0×10^8	17.5	5.9	66,500	4.3
11.0×10^8	18.8	8.1	63,000	1.6

The run of physical variables through the disk is shown in Figures 3 and 4. Application of the Schwarzschild criterion shows the disk to be convective everywhere. Notice the high central condensation, which shows that the wire approximation for computing the potential is a good one, or at least that the overall scheme is self-consistent in this respect. Taken at face value, the coincidence between the observed and theoretical radii and luminosities is encouraging. To put this in perspective, we are finding a radius of the order of 20 times the normal radius for a 0.5 $M_\odot$, chemically uniform object. The corresponding luminosity factor is of the order of 10^5. The apparent agreement with

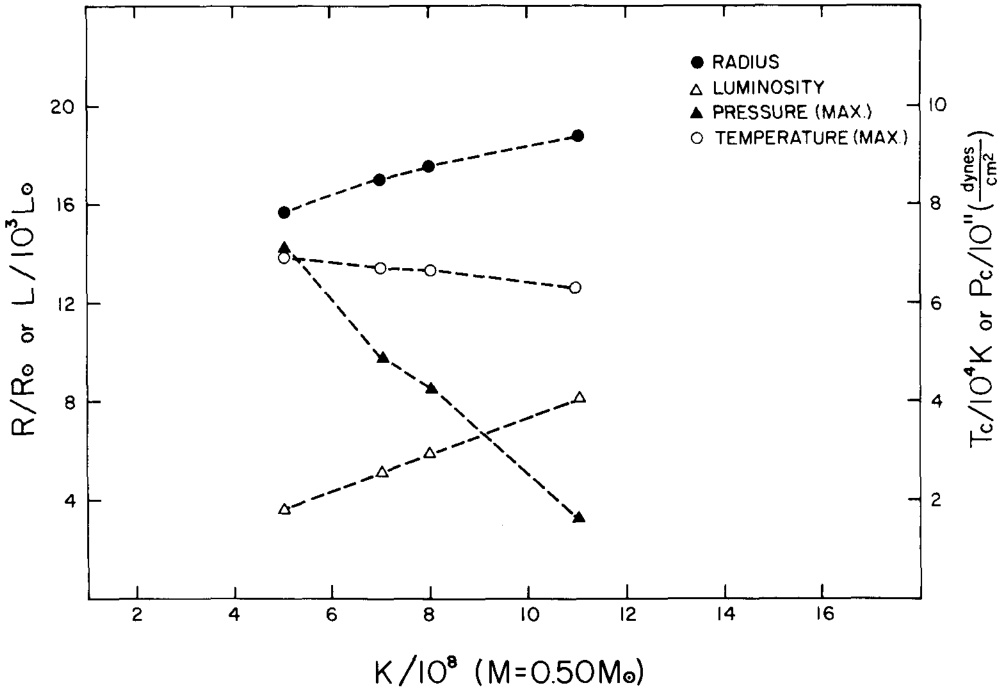

Figure 2. Variation of radius, luminosity, "central" temperature and "central" pressure with K, the factor governing the rate of viscous energy deposition. The solutions for the high and low K-values did not converge as well as for the middle values.

observation does seem remarkable. In fact, as we shall presently see, there is still another coincidence in the numbers which suggests that we are on the right track.

The released viscous energy ultimately comes at the expense of the gravitational energy of the accreting matter, which may be calculated approximately from the relation,

$$\frac{dE}{dt} = \frac{GM}{R}\frac{dm}{dt}\,, \tag{9}$$

where M, R are the mass and radius of the embedded (supposedly main sequence) star, and dm/dt is the mass accretion rate. Reasonable estimates would be $M = 2.4 \times 10^{34}$ g, $R = 2.9 \times 10^{11}$ cm, and $dm/dt = 3 \times 10^{-5}$ $M_\odot$/yr (2×10^{21} g/sec). The last number comes from the rate of period change and can be found in Wilson, 1974. Eqn. 9 then gives $dE/dt \simeq$ 1.1×10^{37} ergs/sec ($2.8 \times 10^3\ L_\odot$) for the accretion luminosity, which differs from our middle theoretical estimates by only a factor of 2. For a slightly less massive disk we should find complete agreement.

One can imagine many discordances between theoretical and observa-

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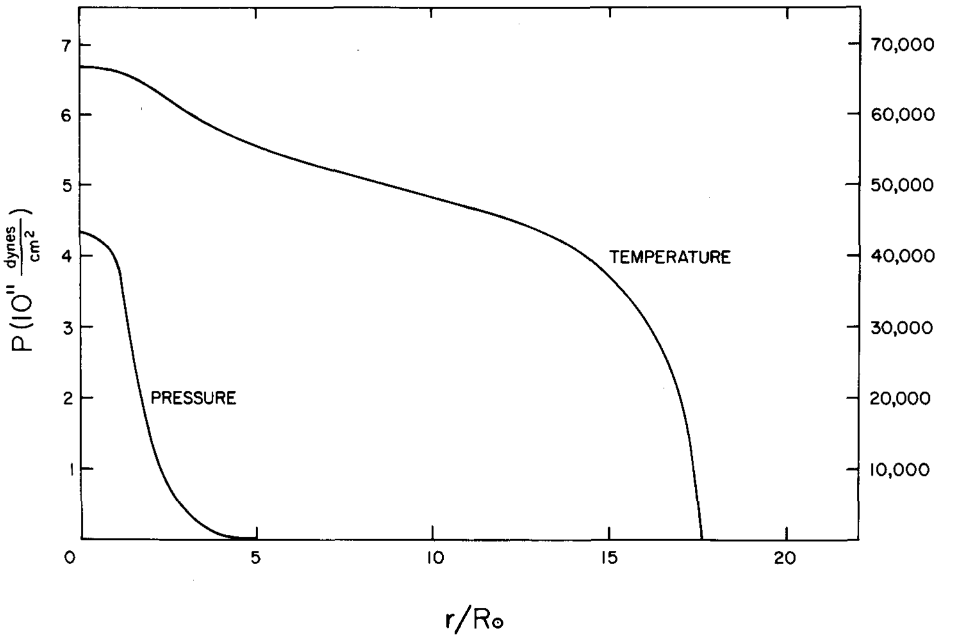

Figure 3. The run of pressure and temperature with "equal volume" radius within the disk.

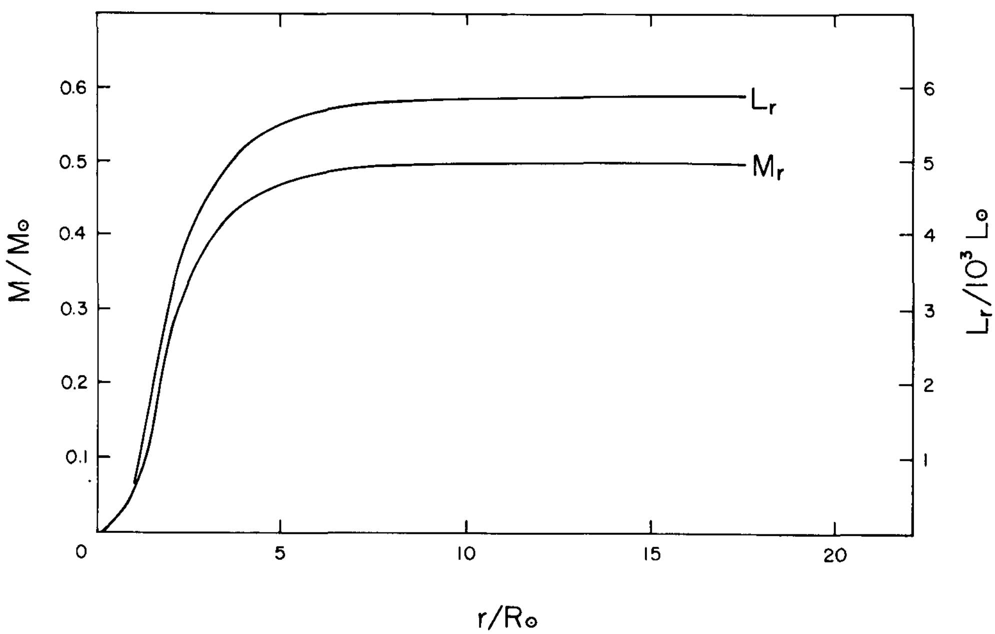

Figure 4. The run of M_r and L_r with "equal volume" radius. Note the rather high "central" condensation, which shows that the wire approximation for computing the potential is a good one.

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tional numbers which might have appeared, and such difficulties were expected. Somehow they have not materialized. Despite the simple nature of the disk structural model, it seems to be - as much to my astonishment as anyone else's - virtually in complete agreement with the observational properties of β Lyr.

The original purpose of this paper was to make estimates of the orders of magnitude of the physical variables within β Lyrae type disks. It seems that the models may now have made that possible for β Lyr itself. In further work it may be possible to predict and account for the characteristics of other members of the W Serpentis class.

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