## REFERENCES

Nye, J. F. 1960. The response of glaciers and ice-sheets to seasonal and climatic changes. *Proceedings of the Royal Society of London*, Ser. A, Vol. 256, No. 1287, p. 559–84.
Nye, J. F. 1963[a]. On the theory of the advance and retreat of glaciers. *Geophysical Journal of the Royal Astronomical Society*, Vol. 7, No. 4, p. 431–56.
Nye, J. F. 1963[b]. The response of a glacier to changes in the rate of nourishment and wastage. *Proceedings of the Royal Society of London*, Ser. A, Vol. 275, No. 1360, p. 87–112.
Nye, J. F. 1965[a]. The frequency response of glaciers. *Journal of Glaciology*, Vol. 5, No. 41, p. 567–87.
Nye, J. F. 1965[b]. A numerical method of inferring the budget history of a glacier from its advance and retreat. *Journal of Glaciology*, Vol. 5, No. 41, p. 589–607.

## DISCUSSION

W. D. Hibler III: Would it be possible to use a complete explicit scheme rather than iterating, or is this too slow?

E. D. Waddington: A forward time-differencing scheme with no iteration for the velocity would require small time steps and could still be subject to numerical instability. Time steps in the model in this paper can be an order of magnitude larger than in such a scheme, while using only two or three iterations. I have not investigated the use of an (explicit) leap-frog scheme.

W. F. Budd: Nye's flux equations are based on a flow law of the form of the flow law of ice and a sliding law of the Weertman type. The problem is that when applied to different glaciers, different values of the parameters are found for best fit. Another approach is to use flow and sliding relations which give better fit to real glaciers, in which case the comparison with the Nye equations may be inappropriate. For example, have you considered how to handle double-valued sliding relationships?

Waddington: Examination of double-valued sliding relationships is certainly a problem worth doing. I have been using the Nye equations strictly as a quantitative test of the numerical aspects of the model.

# WAVE OGIVES AS SECOND-ORDER EFFECTS IN THE CREEP OF LARGE ICE MASSES

*By* F. M. Williams

(Glaciology Division, Inland Waters Directorate, Fisheries and Environment Canada, Ottawa, Ontario K1A 0E7, Canada)

Abstract. This paper presents a mathematical analysis of the steady-state motion of a large ice mass. The purpose of the study is to determine the parameters which govern the motion of the ice and to establish the relationships between these parameters and the observable features of the motion. The study considers both land-based glaciers and floating ice shelves as different cases of the same problem. The particular features of the motion which are considered in this study are the surface waves or ogives which appear on both glaciers and ice shelves.

Previous studies have obtained useful information on the behaviour of the glacier by assuming very simple distributions of the stress and strain in the ice, but these analyses cannot account for some of the interesting surface features. In the present paper it is shown that a

https://doi.org/10.3189/S0022143000015161 Published online by Cambridge University Press

more accurate analysis of the ice movement gives rise to features in the stress and strain distributions which may be regarded as corrections to the simpler solutions and hence are referred to as second-order effects.

The paper considers an ice mass of uniform thickness inclined at an arbitrary angle to the horizontal. Bottom conditions are chosen to represent ice–rock and ice–water conditions respectively. Since only two-dimensional motion is considered, the input data and the results are expected to apply near the centre line of the glacier. The analysis assumes a flow law which represents a large class of incompressible materials, including those of Glen, and which allows for variation in the material properties with depth in the ice. A complete set of governing equations and boundary conditions is presented.

The solution to the governing equations is represented by a power-series expansion in a small parameter or correction factor. Where assumptions concerning the nature of the motion are made in order to simplify the solution, these assumptions are justified on both mathematical and physical grounds. The analysis results in a fourth-order differential equation for the correction velocities. This equation is reduced by conventional analytical methods. In particular, the solution which corresponds to the wave-like surface features is sought.

The study demonstrates that the nature of the surface features is sensitive to some of the parameters and relatively insensitive to others.

## DISCUSSION

J.-P. Benoist: Do you know the works of Reynaud ([1978]) and Martin ([1978]) of the Laboratoire de Glaciologie (Grenoble) who explain the ogives by the seasonal variations of mass balance of the glacier? If the glacier has a large accumulation zone and becomes very narrow at the beginning of the ice fall, any balance variations will be amplified and will give kinematic waves. What do you think about this interpretation?

F. M. Williams: There have been several explanations offered for ogives which involve seasonal variations in the accumulation and flow rates. These theories would be more satisfactory if the relation between annual flow rate and ogive spacing were consistent from glacier to glacier which, apparently, it is not.

W. F. Budd: The approach is very promising, but one of the difficulties is the large change in ice thickness from very thin ice over the ice fall to thick ice at the base. Do you think your technique can handle this? (The ogive wavelength seems to be some several times the ice-fall thickness rather than the thickness of the thick ice.)

Williams: The analysis presented here is for a mass of ice of approximately constant thickness. It may be taken to model that part of the glacier which begins at the foot of the ice fall and extends down-stream. The analysis places no restrictions on the thickness of the ice in the ice fall. The change in ice thickness from the ice fall to the down-stream glacier certainly has an effect on the flow below the ice fall. This effect is transmitted, in mathematical terms, through the velocity profile at the up-stream end of the study section.

## REFERENCES

Martin, S. [1978.] Analyse et reconstitution de la série des bilans annuels du Glacier de Sarennes, sa relation avec les fluctuations du niveau de trois glaciers du Massif du Mont-Blanc (Bossons, Argentière, Mer de Glace). *Zeitschrift für Gletscherkunde und Glazialgeologie*, Bd. 13, Ht. 1–2, 1977, p. 127–53.

Reynaud, L. [1978.] Glacier fluctuations in the Mont Blanc area (French Alps). *Zeitschrift für Gletscherkunde und Glazialgeologie*, Bd. 13, Ht. 1–2, 1977, p. 155–66.