CORRESPONDENCE.

ON THE DETERMINATION OF AN AVERAGE LIFE OFFICE.

To the Editor of the Journal of the Institute of Actuaries.

SIR,—I was unavoidably absent from the reading of the second part of Mr. Sutton's paper at the Institute in April last, and thus lost the pleasure of hearing it and the interesting discussion which followed it. Had I been present, I should probably have taken part in the discussion, as I have lately given some attention to the subjects of which Mr. Sutton treated; but having been precluded from doing so, I trust you will now afford me the opportunity of making a few remarks.

The aim of Mr. Sutton appears to have been to prepare a hypothetical company, which might fairly represent an average office for purposes of valuation; and he has endeavoured to reach it in several distinct ways.

1st. By considering the lives existing at the close of the H^M observations; see Table 1 of his paper.

2nd. By summarizing the Board of Trade returns of 19 companies; see Table 5.

3rd. By dividing the Board of Trade Returns into the two classes, A and B, described in his paper, and examining each class separately; see Table 11.

4th. By investigating the mortality experience of the Scottish offices to 31 Dec. 1863; see Tables 12 and 13.

It will be observed that in all the four processes only the amounts remaining assured, or the policies existing at a given date, are taken into account: that is, the course which has been followed is analogous to forming a mortality table from an enumeration of the people living in a fluctuating population, without taking note of the deaths as well. The results are various hypothetical offices, which fairly represent each the average position of the group of companies from which it was formed at the date the observations closed. They show us the average condition of the respective groups, as regards *age and flow of new business* during the past, as well as in every other respect. But, from the nature of the method on which they have been constructed, it seems to me impossible to make experiments with them for companies of different ages, because the age of a company has a greater effect on the proportionate amounts assured at various periods of life than probably any other circumstance whatever; but of this fact Mr. Sutton has not made mention. Very likely, when Mr. Sutton's paper is concluded, we shall see that an average office such as he has suggested is perfectly adapted to his purpose: but for the object I had in view in my late paper, it would have been quite unsuitable. I wanted to illustrate the valuations of an average office, not of average age, but of a variety of definite ages; and although at first I thought of making use of the numbers "existing" in the Mortality Experience, as Mr. Sutton has done, I soon discovered the idea to be unworkable,

and was compelled to prepare mortality tables for policies, by cal-
culating the ratios at each age for each age at entry of the
" exposed to risk " and "terminated".

All the average offices constructed by Mr. Sutton represent com-
panies of *average* age which have reached their present position by
average rate of increase of new business. Both Mr. Manly's hypo-
thetical company and that suggested by myself can be made to repre-
sent, on the other hand, a company of *any* age that may be named,
which from the beginning has transacted yearly a uniform amount of
new business; and in Table W of my paper I further introduced a
means whereby correction can be made for fluctuations in the rate of
influx of policies. My " model office ", therefore, can be used at will
to illustrate a company of any assigned age which has grown to its
present position at any assigned rate or rates of progress.

The following table, No. 1, shows the effect exercised by the age
of an office, doing a uniform annual amount of new business, on the
proportion at risk at various present ages. It has been formed by
summarizing Table S, " Model office " of my paper, and taking at
various stages of the history of the company the percentages of
assurances existing in each quinquennial period of life to the total
assurances existing at all ages.

TABLE 1.

Showing the Proportions, in Companies of Various Ages, at risk at
various Present Ages, on the assumption that the Total Amount
at risk is 100.

Present Age of Lives.	AGE OF OFFICE, IN YEARS.										Present Age of Lives.
	19	24	29	34	39	44	49	54	64	74	
20 to 24	2·18	1·88	1·70	1·58	1·50	1·45	1·41	1·40	1·40	1·39	20 to 24
25 „ 29	7·37	6·38	5·74	5·33	5·06	4·88	4·78	4·73	4·69	4·69	25 „ 29
30 „ 34	12·80	11·08	9·98	9·26	8·79	8·48	8·31	8·21	8·15	8·15	30 „ 34
35 „ 39	16·75	14·50	13·07	12·13	11·51	11·11	10·88	10·75	10·67	10·67	35 „ 39
40 „ 44	17·93	16·30	14·69	13·64	12·94	12·50	12·23	12·08	12·00	11·99	40 „ 44
45 „ 49	15·57	16·06	15·10	14·01	13·30	12·84	12·56	12·42	12·33	12·32	45 „ 49
50 „ 54	11·56	13·25	14·01	13·53	12·84	12·40	12·13	11·99	11·91	11·90	50 „ 54
55 „ 59	7·58	9·35	10·93	11·80	11·65	11·26	11·01	10·88	10·81	10·80	55 „ 59
60 „ 64	4·54	5·85	7·37	8·77	9·65	9·71	9·50	9·39	9 32	9·32	60 „ 64
65 „ 69	2·41	3·23	4·28	5·52	6·68	7·50	7·65	7·56	7·51	7·51	65 „ 69
70 „ 74	·96	1·46	2·05	2·84	3·76	4·68	5·36	5·53	5·49	5·49	70 „ 74
75 „ 79	·29	·51	·80	1·15	1·65	2·23	2·85	3·31	3·44	3·44	75 „ 79
80 „ 84	·06	·13	·23	·36	·53	·76	1·04	1·35	1·65	1·65	80 „ 84
85 & over	...	·02	·05	·08	·14	·20	·29	·40	·63	·68	85 & over

The figures are sufficiently striking to make it worth while to
ascertain the average ages of the companies in Mr. Sutton's Classes
A and B, so that we may more easily judge with which columns his
several average companies should be compared. Assuming that, taken
one with another, companies are founded in the middle of the year
of their establishment, the average age of the offices in Class A on

31 Dec. 1863—the date to which the returns of the 20 companies to the Institute were made up—was 34 years: and that of the offices in Class B, 40·7 years:* while the average age of the 20 companies taken together was 37·3 years. It is probably a fair assumption to make that on the average 10 years elapsed from 31 Dec. 1863 to the dates of the valuations from which Mr. Sutton's tables on Board of Trade data were constructed; so that at the date of those valuations, the offices in Class A would be on the average 44 years old, those in Class B 50·7 years, and all the offices combined 47·3 years. These are the average ages of the companies themselves, but not of their business. It is evident that if the annual amount of new business transacted by a company be progressively increasing, then the character of its existing business corresponds to that of a younger company whose new business has been stationary; and that the greater the rate of increase, the more markedly is this effect produced. Taken in the aggregate, the 20 companies have, no doubt, I think, been decidedly developing, but it is difficult to say to what extent the development practically reduces their average age. I do not attempt to estimate the amount of reduction, but merely point out that it unquestionably exists, and probably to an important extent.

In his Table 3, Mr. Sutton compares the position of the 20 companies, in regard to the proportions at risk at present ages, as at 31 Dec. 1863, and at the date of their first valuations deposited with the Board of Trade about 10 years later: and he seemingly attributes the discrepancies principally to a tendency of the average sum assured to increase with the age at entry. A glance at the above table shows that he has omitted a material circumstance which tells in the same direction, in ignoring the 10 years that elapsed between the two returns. Also his comparison in Table 11 of Classes A and B would seem to be similarly vitiated, but not quite to the same degree; for, as above shown, there is a difference of nearly 7 years in the average ages of the offices in the two classes;—a difference, too, that must have been exaggerated by the comparative rates of the flow of new business. Anyone inspecting the list of offices in each class must observe that the offices in Class A are those which, taken collectively, have grown rapidly; while the offices in Class B have been in the aggregate in this respect less progressive. For an illustration of this fact we need not go beyond Mr. Sutton's Table 8. The figures there displayed show that at the first valuation after the Act of 1870 came into operation, the companies in Class A had each on the average £6,518,470 of assurances existing, while those in Class B had each only £3,441,063: and that, although the offices in Class B were on the average 6·7 years older than those in Class A.

From the above considerations, it is evident that the comparisons Mr. Sutton has instituted between the average offices he devised and those of Mr. Manly and myself, are likewise untrustworthy. He has taken, for instance, my "model office" at its full age, 75 years, and placed it alongside of hypothetical companies, whose ages, it will be perceived, should be taken at not more, but rather considerably less,

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* The age of the London Assurance Corporation is not taken at 143 years, but at only 75 years, which is the age at which the business becomes stationary of a company transacting a uniform amount annually.

than 47, 44, 51 and 37 years respectively. The consequence of his method of procedure has been to exaggerate the comparative amounts assured at the old ages in Mr. Manly's office and mine, at the expense of the young ages; and although he alleges, and probably with reason, that the amounts at the young ages in my office are too large, he thus makes them appear too small. Turning to his Table No. 15, it will be seen that, even after he has made corrections for the small amount of policies taken out at young ages, the amounts of assurances existing in his average office formed from the H^M experience, are at the young ages larger and at the old ages smaller than the corresponding amounts in my model office when it is taken as 75 years old. This result is exactly what, on the theoretical grounds above indicated, might have been anticipated. Mr. Sutton promises, on a future occasion, to examine the figures more closely, with a view to considering in detail the causes of the variations. I trust he will see his way to include in the investigation the effect of the age of an office on the sums remaining assured at the different periods of life.

In Table 13, Mr. Sutton has given a valuable statement of the proportions of sums assured effected at various ages in the Scotch Life Offices Mortality Experience for the year ending 31 Dec. 1863. It is very interesting to observe how little these proportions vary in different offices and at different times. My model office was formed from the mortality experience of twenty companies, English and Scotch, not for one year, but during the whole of their duration. It takes account of the numbers of the policies, and not of the amounts assured; but yet the percentages given in the following statement are remarkably close to those supplied by Mr. Sutton. The variations displayed would not exercise any appreciable effect on the comparative valuations of an average company by different tables.

TABLE No. 2.

Showing Proportions of Sums Assured at Various Ages.

Ages at Entry.	Scotch Offices, by Mr Sutton.*	20 Offices, as shown in my Model Office.
20–24	7·47	6·97
25–29	17·05	17·76
30–34	19·33	21·04
35–39	19·34	18·41
40–44	14·68	13·81
45–49	10·77	9·45
50–54	5·89	6·23
55–59	3·20	3·51
60–64	1·74	1·97
65–69	·53	·85

* The figures in this column are borrowed from Mr. Sutton's Table No. 13. There appears to be a discrepancy between his Tables Nos. 13 and 16. The column in the latter table headed "Scotch Life Office Mortality Experience for year ending 31 Dec. 1863," would seem from the former table to relate to assurances *existing* at the end of 1863, and not to those *effected* in that year.

Mr. Sutton is no doubt right in his conclusion that the average amount of assurance effected on each life, increases with the age of

the entrants; but I think he has possibly given to the increase too high a ratio; at least, the above Table (No. 2) would seem to teach us so.   His figures relate to *sums assured;* mine to *policies;* yet the differences are not great.   Had I known where to find data to work upon, I should probably have taken account of the unequal amounts of the policies at different ages at entry in constructing my "model office"; and it may be worth while to point out that, by reducing the proportionate amounts of assurances effected at the younger ages, we should have increased the already large reserves demanded by the "Analyzed Mortality Tables", in comparison with those required by each of the other mortality tables included in Table V of my paper.

I remain,

Yours obediently,

London, June 1877.   GEORGE KING.