## Symbols

$d_{i j}^{*}$ alternative description of the components of the magnetic part of the rescaled Weyl tensor, page 264
$\left(\boldsymbol{e}_{\boldsymbol{a}}, \Gamma_{\boldsymbol{a}}{ }^{\boldsymbol{b}} \boldsymbol{c}, \Xi, s, L_{\boldsymbol{a b}}, d^{\boldsymbol{a}}{ }_{\boldsymbol{b} \boldsymbol{c} \boldsymbol{d}}, T_{\boldsymbol{a} \boldsymbol{b}}\right)$ unknowns in the frame version of the standard conformal field equations, page 196
$\left(\boldsymbol{e}_{\boldsymbol{a}}, \hat{\Gamma}_{\boldsymbol{a}}{ }^{\boldsymbol{b}}{ }_{\boldsymbol{c}}, \hat{L}_{\boldsymbol{a} \boldsymbol{b}}, d^{\boldsymbol{a}}{ }_{\boldsymbol{b} \boldsymbol{c} \boldsymbol{d}}, T_{\boldsymbol{a} \boldsymbol{b}}, \Xi, d_{\boldsymbol{a}}\right)$ unknowns in the frame version of the extended conformal field equations, page 205
$\left(\boldsymbol{e}_{\boldsymbol{A} \boldsymbol{A}^{\prime}}, \Gamma_{\boldsymbol{A} \boldsymbol{A}^{\prime} \boldsymbol{B C}}, \Xi, s, L_{\boldsymbol{A} \boldsymbol{A}^{\prime} \boldsymbol{B} \boldsymbol{B}^{\prime}}, \phi_{\boldsymbol{A B C D}}, T_{\boldsymbol{A} \boldsymbol{A}^{\prime} \boldsymbol{B} \boldsymbol{B}^{\prime}}\right)$ unknowns in the spinorial version of the standard conformal field equations, page 199
$\left(\boldsymbol{e}_{\boldsymbol{A} \boldsymbol{A}^{\prime}}, \hat{\Gamma}_{\boldsymbol{A} \boldsymbol{A}^{\prime} \boldsymbol{B} \boldsymbol{C}}, \hat{L}_{\boldsymbol{A} \boldsymbol{A}^{\prime} \boldsymbol{B} \boldsymbol{B}^{\prime}}, \phi_{\boldsymbol{A B C D}}, T_{\boldsymbol{A \boldsymbol { A } ^ { \prime } \boldsymbol { B } \boldsymbol { B } ^ { \prime }}}, \Xi, d_{\boldsymbol{A \boldsymbol { A } ^ { \prime }}}\right)$ unknowns in the spinorial version of the extended conformal field equations, page 208
$(\boldsymbol{o}, \boldsymbol{\iota})$ spin basis in index-free notation, page 66
$\left(\hat{\Sigma}_{\boldsymbol{A} \boldsymbol{A}^{\prime} \boldsymbol{B} \boldsymbol{B}^{\prime}}, \hat{\Xi}^{\boldsymbol{C}}{\boldsymbol{D A A ^ { \prime } \boldsymbol { B } \boldsymbol { B } ^ { \prime }}}, \hat{\Delta}_{\boldsymbol{C} \boldsymbol{C}^{\prime} \boldsymbol{D} \boldsymbol{D}^{\prime} \boldsymbol{B} \boldsymbol{B}^{\prime}}, \hat{\Lambda}_{\boldsymbol{B} \boldsymbol{B}^{\prime} \boldsymbol{C} \boldsymbol{D}}, \delta_{\boldsymbol{A A ^ { \prime }}}, \varsigma_{\boldsymbol{A} \boldsymbol{A}^{\prime} \boldsymbol{B} \boldsymbol{B}^{\prime}}, \gamma_{\boldsymbol{A} \boldsymbol{A}^{\prime} \boldsymbol{B} \boldsymbol{B}^{\prime}}\right)$ zero quantities in the spinorial extended conformal field equations, page 208
$\left(\hat{\Sigma}_{\boldsymbol{a} \boldsymbol{b}}, \hat{\Xi}^{c}{ }_{\boldsymbol{d a b}}, \hat{\Delta}_{\boldsymbol{c d b}}, \Lambda_{\boldsymbol{b} \boldsymbol{c} \boldsymbol{d}}, \delta_{\boldsymbol{a}}, \gamma_{\boldsymbol{a b}}, \varsigma_{\boldsymbol{a b}}\right)$ zero quantities in the frame extended conformal field equations, page 205
$(\mathcal{M}, \boldsymbol{g})$ generic spacetime, page 45
$(\mathcal{U}, \varphi)$ coordinate chart, page 28
$\left(\Sigma_{\boldsymbol{A \boldsymbol { A } ^ { \prime } \boldsymbol { B } \boldsymbol { B } ^ { \prime }}}, \Xi^{\boldsymbol{C}}{\boldsymbol{D A \boldsymbol { A } ^ { \prime } \boldsymbol { B } \boldsymbol { B } ^ { \prime }}}, Z_{\boldsymbol{A \boldsymbol { A } ^ { \prime } \boldsymbol { B } \boldsymbol { B } ^ { \prime }}}, Z_{\boldsymbol{A \boldsymbol { A } ^ { \prime }}}, \Delta_{\boldsymbol{C D B} \boldsymbol{B}^{\prime}}, \Lambda_{\boldsymbol{B} \boldsymbol{B}^{\prime} \boldsymbol{C D}}, Z, M_{\boldsymbol{A \boldsymbol { A } ^ { \prime }}}\right.$ ) zero quantities in the spinorial version of the standard conformal equations, page 199
$\left(\Sigma_{\boldsymbol{a b}}, \Xi^{\boldsymbol{c}}{ }_{\boldsymbol{a} \boldsymbol{a} \boldsymbol{b}}, Z_{\boldsymbol{a b}}, Z_{\boldsymbol{a}}, \Delta_{\boldsymbol{c} \boldsymbol{d} \boldsymbol{b}}, \Lambda_{\boldsymbol{b} \boldsymbol{c} \boldsymbol{d}}, Z, M_{\boldsymbol{a}}\right)$ zero quantities in the frame version of the standard conformal field equations, page 196
$\left(g_{a b}, \Xi, s, L_{a b}, d^{a}{ }_{b c d}, T_{a b}\right)$ unknowns in the metric standard conformal field equations, page 191
$\left(h_{i j}, s_{i j}, \zeta, \varsigma\right)$ unknowns in the conformal static equations, page 511
$\left(o^{A}, \iota^{A}\right)$ spin basis in abstract index notation, page 71
(u,r, $\theta^{\mathcal{A}}$ ) Bondi coordinates, page 236
$(x(\tau), \boldsymbol{\beta}(\tau))$ conformal geodesic with parameter $\tau$, page 127
$\left(x^{\mu}\right)$ local coordinates in a four-dimensional manifold, page 28
$\left[\nabla_{a}, \nabla_{b}\right]$ commutator of covariant derivatives, page 39
$[[\boldsymbol{\xi}, \boldsymbol{\eta}]]$ antisymmetric product of $\boldsymbol{\xi}, \boldsymbol{\eta} \in \mathfrak{S}$, page 65
$[\boldsymbol{g}]$ conformal class of the metric $\boldsymbol{g}$, page 113
$[\boldsymbol{u}, \boldsymbol{v}]$ commutator of the vector fields $\boldsymbol{u}$ and $\boldsymbol{v}$, page 34
$\alpha_{\boldsymbol{a}}, \beta_{\boldsymbol{a}}, \omega_{\boldsymbol{a}}, \ldots$ components of the covectors $\boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\omega}, \ldots$ with respect to the frame $\left\{\boldsymbol{e}_{\boldsymbol{a}}\right\}$, page 51
$\alpha_{a}, \beta_{a}, \omega_{a}, \ldots$ generic covectors in abstract index notation
$\approx$ diffeomorphism between sets, page 27
$\bar{\xi}^{A^{\prime}}, \bar{\eta}_{A^{\prime}}, \ldots$ complex conjugates of the spinors $\xi^{A}, \eta_{B}, \ldots$, page 72
$\beta^{2}$ norm of the covector $\tilde{\boldsymbol{\beta}}$, page 134
$\beta_{a}$ covector associated to a conformal geodesic in abstract index notation, page 127
$\boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\omega}, \ldots$ generic covectors in index-free notation
$\boldsymbol{\beta}$ covector associated to a conformal geodesic in index-free notation, page 127
$\chi$ Weingarten map, page 56
$\boldsymbol{D}$ generic three-dimensional connection in index-free notation
d rescaled conformal geodesic covector, page 201
$\boldsymbol{\delta}$ Euclidean metric on $\mathbb{R}^{3}$, page 143
$\boldsymbol{e}, \boldsymbol{e}_{\boldsymbol{A B}}$ space spinor irreducible components of the frame vector $\boldsymbol{e}_{\boldsymbol{A} \boldsymbol{A}^{\prime}}$, page 104
$\ell$ three-dimensional Lorentzian metric on the conformal boundary of an anti-de Sitter-like spacetime, page 456
$f$ covector defining a Weyl connection in index-free notation, page 119
$f$ unphysical conformal geodesic covector, page 201
$\boldsymbol{g}$ generic four-dimensional Lorentzian metric tensor in index-free notation
$\boldsymbol{g}^{\sharp}$ generic contravariant four-dimensional Lorentzian metric tensor in index-free notation
$\boldsymbol{g}_{\mathscr{E}}$ standard metric on the Einstein cylinder, page 144
$\gamma$ metric in the quotient manifold, page 141
$\boldsymbol{h}$ generic (negative definite) Riemannian three-dimensional metric
$\hbar$ standard metric on the unit 3 -sphere, page 142
$\boldsymbol{K}$ extrinsic curvature tensor of a hypersurface in index-free notation, page 61
$\boldsymbol{k}$ intrinsic metric of compact two-dimensional surfaces
$\boldsymbol{M}, \boldsymbol{N}, \ldots$ generic higher rank tensors in index-free notation
$\boldsymbol{N}$ tangent to the generators of null infinity
$\boldsymbol{\nabla}, \overline{\boldsymbol{\nabla}}$ generic linear connections in index-free notation, page 38
$\boldsymbol{\nu}$ unit normal to a hypersurface $\mathcal{S}$, page 54
$\boldsymbol{\omega}, \boldsymbol{\omega}^{\boldsymbol{A B}}$ space spinor irreducible components of the frame covector $\boldsymbol{\omega}^{\boldsymbol{A} \boldsymbol{A}^{\prime}}$, page 104
$\boldsymbol{\partial}_{\mu}$ coordinate basis vector
$\boldsymbol{Q}$ transition tensor between connections in index-free notation, page 42
$\boldsymbol{q}$ intrinsic metric of null infinity
$\boldsymbol{\Sigma}$ torsion tensor of a connection $\boldsymbol{\nabla}$ in index-free notation, page 39
$\boldsymbol{\sigma}$ standard metric on the unit 2-sphere
$\sigma_{\mathbf{L}}(\boldsymbol{\xi})$ symbol of a differential operator $\mathbf{L}$, page 252
$\boldsymbol{t}$ vector field generating a timelike congruence
$\boldsymbol{\tau}$ vector counterpart of the spinor $\tau_{A A^{\prime}}$, page 102
$\boldsymbol{v}, \boldsymbol{u}, \boldsymbol{w}, \ldots$ generic vectors in index-free notation

ऽ shear tensor, page 226
$\boldsymbol{z}, \boldsymbol{\zeta}$ deviation vector and covector, respectively, page 135
$\mathcal{L}_{\boldsymbol{h}}$ conformal Killing operator of the metric $\boldsymbol{h}$, page 257
$\breve{\mathbf{u}}$ perturbation quantity in an evolution system
$\chi_{(A B) C D}$ spinorial counterpart of the Weingarten tensor
$\chi_{A B}$ spinorial counterpart of the acceleration vector

- composition of functions, page 36

】 disjoint union of sets
$\delta(i)$ Dirac's delta, page 279
$\Delta_{\boldsymbol{h}}$ Laplacian operator of the Riemannian metric $\boldsymbol{h}$
$\delta_{\mu}{ }^{\nu}, \delta_{a}{ }^{b}, \delta_{i}{ }^{j}, \delta_{\alpha}{ }^{\beta}, \delta_{A}{ }^{B}, \delta_{\boldsymbol{A}}{ }^{\boldsymbol{B}}, \delta_{\boldsymbol{a}}{ }^{\boldsymbol{b}}, \delta_{\boldsymbol{i}}{ }^{\boldsymbol{j}}$ Kronecker's delta
$\delta_{\alpha \beta}$ components of the three-dimensional Euclidean metric in Cartesian coordinates, page 47
$\delta_{\boldsymbol{A} \boldsymbol{B}}$ Sen connection on a timelike conformal boundary, page 471
$\delta_{\boldsymbol{i j}}$ components of a three-dimensional Riemannian metric with respect to an orthonormal basis, page 45
$\dot{\gamma}(\mathrm{s})$ tangent vector to a curve, page 30
$\dot{\boldsymbol{x}}(\mathrm{s})$ alternative notation for the tangent vector to a curve, page 30
$\epsilon= \pm 1$ encodes the causal character of a hypersurface, page 54
$\epsilon_{a b c \boldsymbol{c}}$ components of the volume form with respect to an orthonormal basis
$\epsilon_{\boldsymbol{A B}}, \epsilon^{\boldsymbol{A B}}$ components of the spinors $\epsilon_{A B}, \epsilon^{A B}$ with respect to a spin basis, page 71
$\epsilon_{A^{\prime} B^{\prime}}, \epsilon^{A^{\prime} B^{\prime}}$ complex conjugates of the spinors $\epsilon_{A B}, \epsilon^{A B}$
$\epsilon_{A A^{\prime} B B^{\prime} C C^{\prime} D D^{\prime}}$ spinorial counterpart of the volume form, page 78
$\epsilon_{A B C D E F}$ spinorial counterpart of the three-dimensional volume form, page 105
$\epsilon_{a b c d}$ volume form of a metric $g_{a b}$, page 49
$\epsilon_{A B}, \epsilon^{A B}$ antisymmetric spinors, page 67
$\equiv$ definition
$\eta_{\boldsymbol{A B C D}}$ components of the electric part of the Weyl spinor, page 373
$\eta_{a b}$ components of a four-dimensional Lorentzian metric with respect to an orthonormal basis, page 45
$\eta_{\mu \nu}$ components of the Minkowski metric tensor in Cartesian coordinates, page 47
व, $\overline{\bar{\delta}}$ eth and eth-bar operators, page 241
exp exponential map, page 275
$\Gamma$ geodesic distance, page 276
$\gamma(\mathrm{s})$ curve in a manifold with parameter s, page 30
$\Gamma_{\boldsymbol{a}} \boldsymbol{c}_{\boldsymbol{b}}$ connection coefficients of $\boldsymbol{\nabla}$ with respect to $\left\{\boldsymbol{e}_{\boldsymbol{a}}\right\}$
$\gamma_{i}{ }^{\boldsymbol{j}} \boldsymbol{k}$ connection coefficients of the three-dimensional connection $\boldsymbol{D}$ with respect to the frame $\left\{\boldsymbol{e}_{\boldsymbol{i}}\right\}$, page 59
$\Gamma_{\mu}{ }^{\nu}{ }_{\lambda}$ Christoffel symbols of the metric $\boldsymbol{g}$ in the coordinates $\left(x^{\mu}\right)$
$\Gamma_{\boldsymbol{A}^{\prime} \boldsymbol{A}^{\prime}}{ }^{\boldsymbol{B} \boldsymbol{B}^{\prime}} \boldsymbol{C}_{\boldsymbol{C}}{ }^{\prime}$ spinorial counterpart of the connection coefficients $\Gamma_{\boldsymbol{a}}{ }^{\boldsymbol{b}} \boldsymbol{c}$, page 82
$\Gamma_{\boldsymbol{A} \boldsymbol{A}^{\prime}}{ }^{\boldsymbol{B}} \boldsymbol{C}_{\boldsymbol{C}}$ reduced spin connection coefficients, page 82
$\Gamma_{\boldsymbol{A B C D}}$ space spinor counterpart of the reduced spin connection coefficients $\Gamma_{\boldsymbol{A} \boldsymbol{A}^{\prime} \boldsymbol{C D}}$, page 107
$\gamma_{\boldsymbol{A}}{ }^{\boldsymbol{C}}{ }_{\boldsymbol{D}}$ reduced spatial spin connection coefficients, page 109
$\gamma_{\boldsymbol{A B}}{ }^{\boldsymbol{C D}}{ }_{\boldsymbol{E F}}$ spinorial counterpart of the three-dimensional connection coefficients $\gamma_{i}{ }^{j}{ }_{k}$, page 109
$\grave{\Omega}, \check{\Omega}$ massless and, respectively, massive part of the conformal factor associated to Euclidean initial data sets, page 529
$\hat{\boldsymbol{\nabla}}$ generic Weyl connection in index-free notation, page 119
$\hat{\Gamma}_{a}{ }^{\boldsymbol{b}}{ }_{c}$ connection coefficients of a Weyl connection $\hat{\boldsymbol{\nabla}}$, page 119
$\hat{\Gamma}_{\boldsymbol{A} \boldsymbol{A}^{\prime}}{ }^{\boldsymbol{B}}{ }_{\boldsymbol{C}}$ reduced Weyl connection spin coefficients, page 206
$\hat{\nabla}_{a}$ generic Weyl connection in abstract index notation, page 119
$\hat{\rho}^{c}{ }_{\text {dab }}$ Weyl connection algebraic curvature, page 203
$\hat{\rho}_{\boldsymbol{A B C C}} \boldsymbol{C}^{\boldsymbol{D}} \boldsymbol{D}^{\prime}$ Weyl connection reduced spinorial algebraic curvature, page 207
$\hat{P}^{c}{ }_{d a b}$ Weyl connection geometric curvature, page 203
$\hat{P}_{\boldsymbol{A B C C}}{ }^{\boldsymbol{D}} \boldsymbol{D}^{\prime}$ Weyl connection reduced spinorial geometric curvature, page 207 $\kappa$ conformal factor associated to the construction of the cylinder at spatial infinity, page 541
$\Lambda$ Newman-Penrose Ricci scalar, page 87
$\lambda$ cosmological constant, page 2
$\Lambda_{(\boldsymbol{A B C D})}, \Lambda_{\boldsymbol{A B}}$ irreducible components of the spinorial Bianchi equation, page 351
$\langle\boldsymbol{\omega}, \boldsymbol{v}\rangle$ action of the covector $\boldsymbol{\omega}$ on the vector $\boldsymbol{v}$
$\left.\langle\boldsymbol{t}\rangle^{\perp}\right|_{p}$ subspace orthogonal to $\boldsymbol{t}$, page 55
$\langle\boldsymbol{t}\rangle$ one-dimensional subspace spanned by $\boldsymbol{t}$, page 55
$\langle\langle\boldsymbol{\xi}, \boldsymbol{\eta}\rangle\rangle$ Hermitian product of $\boldsymbol{\xi}, \boldsymbol{\eta} \in \mathfrak{S}$, page 94
$\llbracket \nabla_{a}, \nabla_{b} \rrbracket$ modified commutator of covariant derivatives, page 40
$\mathbb{H}^{n} n$-dimensional half Euclidean space, page 29
$\mathbb{R}^{+}$non-negative real numbers
$\mathbb{R}^{2}$ Euclidean plane
$\mathbb{R}^{n} n$-dimensional Euclidean space
$\mathbb{S}^{2} 2$-sphere
$\mathbb{S}^{3}$ three-dimensional unit sphere, page 142
$\mathbf{A}^{*}$ transpose of the complex conjugate of the matrix $\mathbf{A}$
$\mathbf{A}^{3}$ normal matrix in an initial boundary value problem, page 314
$\mathbf{A}^{\mu}$ symmetric matrices in a symmetric hyperbolic system, page 294
d exterior derivative (differential), page 31
$\mathbf{d} x^{\mu}$ coordinate basis covector
$\mathbf{L}$ generic differential operator
$\mathbf{L}^{*}$ formal adjoint of the differential operator $\mathbf{L}$
$\mathbf{L}_{\boldsymbol{h}}$ Yamabe operator, page 256
T map associated to the prescription of boundary conditions in an initial boundary value problem, page 314
$\mathbf{u}, \mathbf{v}, \mathbf{w}, \ldots \mathbb{C}^{N}$-valued functions
$\mathcal{B}_{a}(p)$ ball of radius $a>0$ centred at the point $p$
$\mathcal{C}_{p}$ null cone at a point $p \in \mathcal{M}$, page 45
$\mathcal{C}_{p}^{+}, \mathcal{C}_{p}^{-}$future and, respectively, past null cone at a point $p \in \mathcal{M}$, page 45
$\mathcal{D}$ a generic derivation, page 30
$\mathcal{D}_{A B}$ Sen connection of $\nabla_{A A^{\prime}}$ induced by $\tau_{A A^{\prime}}$, page 105
$\mathcal{E}$ corner in an initial boundary value problem, page 314
$\mathcal{G}$ generic lens-shaped domain, page 301
$\mathcal{H}_{k}$ standard hyperboloids, page 154
$\mathcal{I}$ cylinder at spatial infinity, page 542
$\mathcal{I}^{0}$ intersection of the cylinder at spatial infinity with a Cauchy initial hypersurface, page 542
$\mathcal{I}^{ \pm}$critical sets where null infinity touches spatial infinity, page 542
$\mathcal{M}, \mathcal{N}$ generic (unphysical) spacetime manifolds
$\mathcal{N}, \mathcal{N}^{\prime}$ initial null hypersurfaces in a characteristic problem, page 320
$\mathcal{N}_{i}$ complex null cone at $i$, page 522
$\mathcal{N}_{\mathbb{C}}(i)$ complexification of the null cone through $i$, page 532
$\mathcal{P}$ covariant derivative in the direction of $\tau_{A A^{\prime}}$, page 105
$\mathcal{Q}$ generic quotient manifold, page 141
$\mathcal{R}$ generic subset of a hypersurface $\mathcal{S}$
$\mathcal{S}$ generic hypersurface on a manifold $\mathcal{M}$
$\mathcal{T}$ timelike boundary, page 314
$\mathcal{U}, \mathcal{V}$ generic open subsets of a manifold or $\mathbb{R}^{n}$
$\mathcal{U}_{\mathbb{C}}$ complexification of a neighbourhood $\mathcal{U}$ of the point at infinity, page 532
$\mathcal{Z}$ intersection of initial null hypersurfaces in a characteristic problem, page 320
$\mathfrak{S}$ complex vector space, page 65
$\mathfrak{S}(\mathcal{M})$ spin structure (spin bundle) over $\mathcal{M}$, page 81
$\mathfrak{S}(\mathcal{S})$ space spinor structure over a three-dimensional manifold $\mathcal{S}$, page 101
$\mathfrak{S}^{*}$ dual of the complex vector space $\mathfrak{S}$, page 65
$\mathfrak{S}^{\bullet}(\mathcal{M}), \mathfrak{S}_{A}(\mathcal{M}), \mathfrak{S}^{A}(\mathcal{M}), \mathfrak{S}_{A A^{\prime}}{ }^{B}(\mathcal{M}), \ldots$ various spin bundles over $\mathcal{M}$
$\mathfrak{S}^{\bullet}$ spin algebra, page 66
$\mathfrak{S}^{A}, \mathfrak{S}_{A}, \ldots$ alternative notation for the vector spaces $\mathfrak{S}, \mathfrak{S}^{*}, \ldots$, page 66
$\mathfrak{S}^{A^{\prime}}, \mathfrak{S}_{A^{\prime} B^{\prime}}, \ldots$ complex conjugates of the spaces $\mathfrak{S}^{A}, \mathfrak{S}_{A B}, \ldots$, page 72
$\mathfrak{T}^{\bullet}(\mathcal{M})$ tensor bundle over $\mathcal{M}$, page 34
$\mathfrak{T}^{a}(\mathcal{M})$ alternative notation for the tangent bundle over $\mathcal{M}$, page 36
$\mathfrak{T}^{a_{1} \cdots a_{k}}{ }_{b_{1} \cdots b_{l}}(\mathcal{M})$ alternative notation for the tensor bundle over $\mathcal{M}$, page 36
$\mathfrak{T}_{a}(\mathcal{M})$ alternative notation for the cotangent bundle over $\mathcal{M}$, page 36
$\mathfrak{X}(\mathcal{M})$ set of of scalar fields over $\mathcal{M}$, page 30
u background quantity in an evolution system
$\mathscr{C}$ generic cut of null infinity
$\mathscr{C}_{\star}$ fiduciary cut of null infinity
$\mathscr{E}$ extension operator of functions between Sobolev spaces, page 308
$\mathscr{I}$ part of the conformal boundary that is a hypersurface, page 178
$\mathscr{I}^{ \pm}$future and, respectively, past null infinity
$\mathscr{N}_{i}^{+}, \mathscr{N}_{i}^{-}$null cones generated by the null geodesics through $i$, page 531
$\mathscr{N}_{u}$ outgoing null hypersurface associated to the retarded time $u$
$\mathscr{R}_{\boldsymbol{h}}$ linearised Ricci operator, page 289
$\mathscr{Z}$ generic intersection of null infinity with a null hypersurface
int $\mathcal{A}$ topological interior of the set $\mathcal{A}$, page 397
i square root of -1
$\mu_{\boldsymbol{A B C D}}$ components of the magnetic part of the Weyl spinor, page 373
$\nabla_{\boldsymbol{a}}$ covariant directional derivative in the direction of $\boldsymbol{e}_{\boldsymbol{a}}$, page 51
$\nabla_{\boldsymbol{u}} \boldsymbol{v}$ covariant derivative of $\boldsymbol{v}$ with respect to $\boldsymbol{u}$, page 38
$\nabla_{a}, \bar{\nabla}_{a}$ generic linear connections in abstract index notation, page 38
$\nabla_{\boldsymbol{A} \boldsymbol{A}^{\prime}}$ directional spinorial covariant derivative, page 82
$\nabla_{A A^{\prime}}, \tilde{\nabla}_{A A^{\prime}}, \ldots$ spinor covariant derivatives, page 81
$\nabla_{A B}$ space spinor counterpart of $\nabla_{A A^{\prime}}$, page 105
$\Omega$ generic three-dimensional conformal factor
$\oplus$ direct sum
$\otimes$ tensor product between tensors or tensor spaces
$\overline{\mathcal{A}}$ topological closure of the set $\mathcal{A}$, page 394
$\|\mathbf{u}\|_{\mathcal{S}, m}$ Sobolev norm of order $m$ of a function over $\mathcal{S}$, page 306
$\partial \mathbb{H}^{n}$ boundary of the $n$-dimensional half Euclidean space, page 29
$\partial \mathcal{M}$ boundary of $\mathcal{M}$
$\phi$ unphysical conformally coupled scalar field, page 216
$\phi_{0}$ radiation field in the asymptotic characteristic problem on a cone, page 500
$\Phi_{A B A^{\prime} B^{\prime}}$ spinorial counterpart of the trace-free Ricci tensor, page 89
$\Phi_{a b}$ trace-free Ricci tensor of a connection $\nabla_{a}$ in abstract index notation, page 48
$\phi_{A B}$ unphysical Maxwell spinor, page 215
$\Pi$ generic distribution, page 55
$\left.\Pi\right|_{p}$ hyperplane induced by a distribution at a point $p \in \mathcal{M}$, page 55
$£_{\boldsymbol{v}}$ Lie derivative in the direction of $\boldsymbol{v}$, page 37
$\Psi_{A B C D}$ Weyl spinor, page 87
$\rho$ boundary-defining function, page 285
$\rho$ polar radial coordinate, page 514
$\rho^{\alpha}$ three-dimensional unit position vector, page 514
$\rho^{\boldsymbol{C}} \boldsymbol{D A A A}^{\prime} \boldsymbol{B B}^{\prime}$ reduced spinorial algebraic curvature, page 198
$\rho^{c}{ }_{d a b}$ components of the algebraic curvature, page 195
$\rho^{A A^{\prime}}$ spatial spinor used to introduce a $1+1+2$ spinor formalism, page 464
$\boldsymbol{R i c}, \boldsymbol{R i c}[\boldsymbol{g}]$ Ricci tensor of a connection $\boldsymbol{\nabla}$ in index-free notation, page 48
Riem Riemann curvature tensor of a connection $\boldsymbol{\nabla}$ in index-free notation, page 40
Schouten, Schouten $[\boldsymbol{g}]$ Schouten tensor of a connection $\boldsymbol{\nabla}$ in index-free notation, page 48
$\sigma$ Newman-Penrose spin connection coefficient corresponding to $\Gamma_{\mathbf{0 1} \mathbf{1}^{\prime} \mathbf{0}}$ $\sigma_{\boldsymbol{A} \boldsymbol{A}^{\prime}}, \sigma_{\boldsymbol{a}} \boldsymbol{A A}^{\prime}$ spacetime Infeld-van der Waerden symbols, page 74
$\Sigma_{a}{ }^{c}{ }_{b}$ components of the torsion tensor with respect to an orthonormal frame, page 53
$\sigma_{i}{ }^{\boldsymbol{k}}{ }_{\boldsymbol{j}}, \Pi^{\boldsymbol{k}}{ }_{\boldsymbol{l i j}}, \pi_{\boldsymbol{k l i j}}$ components of the three-dimensional torsion, geometric and algebraic curvatures, page 264
$\sigma_{\boldsymbol{i}}{ }^{\boldsymbol{A B}}, \sigma_{\boldsymbol{i}}^{\boldsymbol{i}}$ 的 spatial Infeld-van der Waerden symbols, page 99
$\Sigma_{a}{ }^{c}{ }_{b}$ torsion tensor of a connection $\nabla_{a}$ in abstract index notation, page 39
$\simeq$ equality at the conformal boundary
$\square$ D'Alembertian operator, page 89
$\square_{A B}$ box commutator, page 89
$\stackrel{\star}{\sim}$ equality at a fiduciary cut of null infinity
$\tau_{A A^{\prime}}$ privileged timelike spinor inducing a space spinor formalism, page 102
$\Theta$ conformal factor associated to a conformal geodesic, page 132
$\theta=\left(\theta^{\mathcal{A}}\right)$ local coordinates on $\mathbb{S}^{2}$
$\Theta_{A B C D}$ space spinor counterpart of the components of the Schouten tensor of a Weyl connection, page 373
$\tilde{\boldsymbol{\eta}}$ Minkowski metric
$\tilde{\boldsymbol{g}}_{\mathscr{E}}$ metric of the anti-de Sitter spacetime, page 159
$\tilde{\boldsymbol{g}}_{\mathscr{S}}$ metric of the Schwarzschild spacetime, page 163
$\tilde{\boldsymbol{g}}_{d S}$ metric of the de Sitter spacetime, page 155
$\tilde{\mathcal{E}}_{k}$ asymptotic ends of asymptotically Euclidean manifold $\tilde{\mathcal{S}}$, page 272
$\tilde{\mathcal{F}}_{a b}$ self-dual Faraday tensor, page 213
$\tilde{\mathcal{M}}$ generic (physical) spacetime manifold
$\tilde{\phi}$ physical conformally coupled scalar field, page 216
$\tilde{\phi}_{A B}$ physical Maxwell spinor, page 215
$\tilde{\varrho}$ density of a perfect fluid, page 219
$\tilde{\varrho}_{\tilde{\mathscr{F}}}$ energy density, page 254
$\tilde{F}_{a b}$ physical Faraday tensor, page 213
$\tilde{j}_{k}$ energy flux vector, page 254
$\tilde{p}$ pressure of a perfect fluid, page 219
$\tilde{T}_{a b}$ physical energy-momentum tensor
$\tilde{u}^{a}$ physical 4-velocity of a perfect fluid, page 219
$\underline{x}$ spatial coordinates $\left(x^{1}, x^{2}, x^{3}\right)$
$\Upsilon_{a}$ logarithmic gradient of a conformal factor, page 116
$\Upsilon_{A A^{\prime}}$ spinorial counterpart of the logarithmic gradient of a conformal factor, page 123
$\varphi^{*}$ pull-back, page 36
$\varphi_{*}$ push-forward, page 36
$\varpi_{\boldsymbol{A} \boldsymbol{A}^{\prime}}$ components of $\varpi_{A A^{\prime}}$ with respect to a spin basis, page 95
$\varpi_{A A^{\prime}}$ Hermitian spinor assocated to a Hermitian inner product, page 95
$\varrho$ conformally rescaled density of a perfect fluid, page 220
$\varrho$ unphysical energy density, page 255
$\boldsymbol{W e y l}, \boldsymbol{W e y l}[\boldsymbol{g}]$ Weyl tensor of a connection $\boldsymbol{\nabla}$ in index-free notation, page 48 $\xi^{A}, \eta_{\boldsymbol{A}}, \ldots$ components of the spinors $\xi^{A}, \eta_{A}, \ldots$ with respect to a spin basis
$\xi^{A}, \eta_{A}, \ldots$ generic spinors in abstract-index notation
$\xi_{\boldsymbol{A B C C}}, \chi_{\boldsymbol{A B C D}}$ real and imaginary parts of $\Gamma_{\boldsymbol{A B C D}}$, page 107
$\Xi_{i j}, S_{i}, S_{i j}, H_{k i j}$ zero quantities associated to the conformal static field equations, page 511
$\zeta_{0}, \ldots \zeta_{4}$ components of the spin-2 zero-rest mass field $\zeta_{A B C D}$, page 551
$\zeta_{A B C D}$ spin-2 zero-rest mass field, page 551
$\left\{\boldsymbol{c}_{i}\right\}$ global orthonormal frame on $\mathbb{S}^{3}$, page 142
$\left\{\boldsymbol{e}_{\boldsymbol{a}}\right\}$ vector basis in index-free notation, page 31
$\left\{\boldsymbol{\omega}^{a}\right\}$ covector basis in index-free notation, page 31
$\left\{\mathcal{S}_{t}\right\}_{t \in \mathbb{R}}$ foliation of $\mathcal{M}$, page 54
$\left\{\boldsymbol{e}_{\boldsymbol{i}}\right\}$ three-dimensional vector basis in index-free notation, page 59
$\left\{\boldsymbol{e}_{\boldsymbol{A} \boldsymbol{A}^{\prime}}\right\}$ alternative index-free notation for the Newman-Penrose null tetrad, page 79
$\left\{\boldsymbol{e}_{\boldsymbol{A B}}\right\},\left\{\boldsymbol{\omega}^{\boldsymbol{A B}}\right\}$ three-dimensional basis and cobasis with spin frame indices, page 109
$\{\boldsymbol{l}, \boldsymbol{n}, \boldsymbol{m}, \overline{\boldsymbol{m}}\}$ Newman-Penrose null tetrad in index-free notation, page 77
$\left\{\boldsymbol{\omega}^{i}\right\}$ three-dimensional covector basis in index-free notation, page 59
$\left\{\boldsymbol{\omega}^{\boldsymbol{A} \boldsymbol{A}^{\prime}}\right\}$ soldering form, page 79
$\left\{\epsilon_{\boldsymbol{A}}{ }^{\boldsymbol{A}}\right\},\left\{\epsilon^{\boldsymbol{A}}{ }_{A}\right\}$ alternative abstract index notation for a spin basis and its dual, page 71
$\left\{\omega^{a}{ }_{a}\right\}$ covector basis in abstract index notation, page 36
$\left\{\omega^{i}{ }_{i}\right\}$ three-dimensional covector basis in index-free notation, page 59
$\left\{e_{\boldsymbol{a}}{ }^{a}\right\}$ vector basis in abstract index notation, page 36
$\left\{e_{i}{ }^{i}\right\}$ three-dimensional vector basis in abstract index notation, page 59
$\left\{l^{a}, n^{a}, m^{a}, \bar{m}^{a}\right\}$ Newman-Penrose null tetrad in abstract index notation, page 77
$\left\{m, m_{\alpha}, m_{\alpha_{1} \alpha_{2}}, \ldots\right\}$ sequence of multipole moments of a static spacetime, page 519
$b_{A B C D}$ Cotton spinor, page 512
$C_{p}^{*}$ characteristic set of a symmetric hyperbolic system at the point $p$, page 297
$C^{\infty}$ class of infinitely differentiable (smooth) functions
$C^{\infty}\left(\mathbb{R}^{3}, \mathbb{C}^{N}\right)$ space of smooth functions from $\mathbb{R}^{3}$ to $\mathbb{C}^{N}$, page 306
$C^{c}{ }_{d a b}$ Weyl tensor of a connection $\nabla_{a}$ in abstract index notation, page 48
$C^{k}$ class of $k$-times differentiable functions
$C^{k}\left(\mathbb{R}^{3}, \mathbb{C}^{N}\right)$ set of $C^{k}$ functions from $\mathbb{R}^{3}$ to $\mathbb{C}^{N}$, page 307
$C^{k}\left([0, T] ; H^{m}\left(\mathbb{R}^{3}, \mathbb{C}^{N}\right)\right)$ set of $C^{k}$ functions from $[0, T]$ to $H^{m}\left(\mathbb{R}^{3}, \mathbb{C}^{N}\right)$, page 307
$D$ bounded open subset of $H^{m}\left(\mathbb{R}^{3}, \mathbb{C}^{N}\right)$ such that for $\mathbf{w} \in D$ the matrix $\mathbf{A}^{0}(0, \underline{x}, \mathbf{w})$ is positive definite bounded away from zero by $\delta>0$ for all $p \in \mathbb{R}^{3}$, page 309
$D(\mathcal{R})$ domain of dependence of $\mathcal{R}$, page 304
$D, \Delta, \delta, \bar{\delta}$ Newman-Penrose directional covariant derivatives, page 92
$D^{ \pm}(\mathcal{A}), D(\mathcal{A})$ future/past and total domain of dependence of a set $\mathcal{A}$, page 392
$d^{a}{ }_{b c d}$ rescaled Weyl tensor, page 188
$d_{\boldsymbol{a}}$ components of the rescaled physical conformal geodesics covector, page 203
$D_{\boldsymbol{i}}$ three-dimensional directional covariant derivative in the direction of $\boldsymbol{e}_{\boldsymbol{i}}$, page 59
$D_{i}$ generic three-dimensional connection in abstract index notation
$D_{\boldsymbol{A B}}$ three-dimensional covariant directional derivative, page 109
$d_{\boldsymbol{i j}}, d_{\boldsymbol{i j k}}$ components of the electric and magnetic parts of the rescaled Weyl tensor, page 261
$D_{A B}$ spinorial counterpart of a three-dimensional Levi-Civita connection $\boldsymbol{D}$, page 106
$F^{\boldsymbol{a}}(x), F^{\mu}(x)$ coordinate gauge source functions, page 339
$f_{\boldsymbol{a}}$ components of the unphysical conformal geodesics covector, page 203
$f_{a}$ covector defining a Weyl connection in abstract index notation, page 119
$F_{\boldsymbol{A} \boldsymbol{B}}(x)$ frame gauge source functions, page 345
$F_{a b}$ unphysical Faraday tensor, page 214
$g^{a b}$ generic contravariant four-dimensional Lorentzian metric tensor in abstract index notation
$G_{a b}$ Einstein tensor of a metric $g_{a b}$
$g_{a b}$ generic four-dimensional Lorentzian metric tensor in abstract index notation
$H^{ \pm}(\mathcal{A}), H(\mathcal{A})$ future/past and total Cauchy horizons of the set $\mathcal{A}$, page 394
$H^{m}\left(\mathbb{R}^{3}, \mathbb{C}^{N}\right)$ Sobolev space of order $m$ of functions from $\mathbb{R}^{3}$ to $\mathbb{C}^{N}$, page 307
$h_{a}{ }^{b}$ projector associated to a distribution $\Pi$, page 55
$h_{\boldsymbol{A B C D}}$ components of $h_{A B C D}$ with respect to a spin frame $\left\{\epsilon_{\boldsymbol{A}}{ }^{A}\right\}$, page 99
$h_{A A^{\prime}}{ }^{B B^{\prime}}$ spinorial counterpart of the projector $h_{\boldsymbol{a}}{ }^{\boldsymbol{b}}$, page 98
$h_{A B C D}$ space spinor counterpart of $h_{A A^{\prime}}{ }^{B B^{\prime}}$ and of a three-dimensional Riemannian metric, page 98
$I$ generic interval in $\mathbb{R}$
$i^{0}$ spatial infinity
$i^{ \pm}$future and, respectively, past timelike infinity
$I^{ \pm}(\mathcal{U})$ chronological future and, respectively, past of a set $\mathcal{U}$, page 391
$J^{+}\left(o, \mathcal{M}^{\prime}\right)$ set consisting of $o$ and all points of $\mathcal{M}^{\prime}$ which can be joined to $o$ by a causal curve in $\mathcal{M}^{\prime}$, page 497
$J^{ \pm}(\mathcal{U})$ causal future and, respectively, past of a set $\mathcal{U}$, page 391
$j_{k}$ unphysical flux vector, page 255
$J_{\boldsymbol{j} \boldsymbol{k}}, J_{\boldsymbol{j}}$ normal components of the rescaled Cotton tensor, page 262
$K_{i j}$ extrinsic curvature tensor of a hypersurface in abstract index notation, page 61
$L_{a b}$ Schouten tensor of a connection $\nabla_{a}$ in abstract index notation, page 48
$l_{i j}$ three-dimensional Schouten tensor, page 60
$p \prec \prec q$ timelike related points, page 391
$p$ conformally rescaled pressure of a perfect fluid, page 220
$p \prec q$ strictly causally related points, page 391
$p \preceq q$ causally related points, page 391
$P^{\boldsymbol{C}}{ }_{\boldsymbol{D A} \boldsymbol{A}^{\prime} \boldsymbol{B B}^{\prime}}$ reduced spinorial geometric curvature, page 198
$P^{c}{ }_{d a b}$ components of the geometric curvature, page 194
$P^{\boldsymbol{C C}}{ }^{\prime}{ }_{\boldsymbol{D}} \boldsymbol{D}^{\prime} \boldsymbol{A} \boldsymbol{A}^{\prime} \boldsymbol{B} \boldsymbol{B}^{\prime}$ spinorial geometric curvature, page 197
$P_{n}^{(\alpha, \beta)}(\tau)$ Jacobi polynomial of degree $n$ with parameters $(\alpha, \beta)$, page 553
$Q_{a}{ }^{b}{ }_{c}$ transition tensor between connections in abstract index notation, page 42
$r$ three-dimensional Ricci scalar, page 60
$R(x)$ conformal gauge source function, page 348
$R, R[\boldsymbol{g}]$ Ricci scalar of a connection $\nabla_{a}$, page 48
$R_{d a b}^{c}$ components of the Riemann tensor with respect to an orthonormal frame, page 53
$R_{c a b}^{d}$ Riemann curvature tensor of a connection $\nabla_{a}$ in abstract index notation, page 40
$r^{k}{ }_{l i j}$ three-dimensional Riemann curvature tensor in abstract index notation, page 60
$r_{A B C D E F G H}$ spinorial counterpart of the three-dimensional Riemann curvature tensor, page 110
$R_{a b}$ Ricci tensor of a connection $\nabla_{a}$ in abstract index notation, page 48
$r_{A C E F G H}, r_{A B C E}$ reduced three-dimensional curvature spinors, page 110
$R_{C C^{\prime} D D^{\prime} A A^{\prime} B B^{\prime}}$ spinorial counterpart of the Riemann curvature tensor, page 86
$R_{C D A A^{\prime} B B^{\prime}}$ reduced Riemann curvature spinor, page 86
$r_{i j}$ three-dimensional Ricci tensor in abstract index notation, page 60
$s$ the Friedrich scalar, page 186
$s_{A B C D}$ spinorial counterpart of the three-dimensional trace-free Ricci tensor, page 110
$s_{i j}$ three-dimensional trace-free Ricci tensor, page 60
$S O(3)$ three-dimensional special orthogonal group
$T(\mathcal{M})$ tangent bundle over $\mathcal{M}$, page 34
$\left.T\right|_{p}(\mathcal{M})$ tangent space at a point $p \in \mathcal{M}$, page 31
$T^{*}(\mathcal{M})$ cotangent bundle over $\mathcal{M}$, page 34
$\left.T^{*}\right|_{p}(\mathcal{M})$ cotangent space at a point $p \in \mathcal{M}$, page 31
$\left.T^{\bullet}\right|_{p}(\mathcal{M})$ tensor algebra at $p \in \mathcal{M}$, page 33
$\left.T_{l}^{k}\right|_{p}(\mathcal{M})$ space of $(k, l)$-tensors at the point $p \in \mathcal{M}$, page 33
$T^{a_{1} \cdots a_{k}}{ }_{b_{1} \cdots b_{l}}$ arbitrary ( $k, l$ )-tensor in abstract index notation
$T_{a b}$ unphysical energy-momentum tensor
$T_{c d b}$ rescaled Cotton tensor, page 189
$u$ retarded time
$U, X^{\mathcal{A}}, \omega, \xi^{\mathcal{A}}$ components of an adapted frame in the asymptotic
characteristic problem, page 482
$u, v$ retarded and, respectively, advanced time coordinates
$u^{\boldsymbol{a}}, v^{\boldsymbol{a}}, w^{\boldsymbol{a}}, \ldots$ components of the vectors $\boldsymbol{u}, \boldsymbol{v}, \boldsymbol{w}$ with respect to the coframe $\left\{\boldsymbol{\omega}^{a}\right\}$, page 51
$u^{a}$ unphysical 4-velocity of a perfect fluid, page 220
$u^{a}, v^{a}, w^{a}, \ldots$ generic vectors in abstract index notation
$v$ norm of a static Killing vector, page 504
$x(\mathrm{~s})$ alternative notation for a curve with parameter s, page 30
$X_{C D A B}, Y_{C D A^{\prime} B^{\prime}}$ curvature spinors, page 86
$Y[\boldsymbol{h}]$ Yamabe invariant, page 280
$Y_{a b c}$ four-dimensional Cotton tensor, page 116
$y_{i j k}$ three-dimensional Cotton tensor, page 118
$y_{i j}$ three-dimensional Cotton-York tensor, page 118
$z_{\boldsymbol{A} \boldsymbol{A}^{\prime}}, z, z_{(A B)}$ spacetime and space spinor components of the spinorial counterpart of the deviation vector of a congruence of conformal geodesics, page 383

* $F_{a b}$ Hodge dual of an antisymmetric tensor $F_{a b}$, page 50
${ }^{*} R_{a b c d}, R_{a b c d}^{*}$ left and, respectively, right duals of the tensor $R_{a b c d}$, page 50
+ Hermitian conjugation, page 96
$\dagger, \stackrel{\ddagger}{ }$ generalised dualisation operations, page 50
\#, ${ }^{\text {b }}$ musical operators, page 44
$\alpha, \beta, \gamma, \ldots$ spatial coordinate indices
$\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{C}, \ldots$ spinor frame indices, page 74
$\boldsymbol{a}, \boldsymbol{b}, \ldots$ spacetime frame indices ranging $\mathbf{0}, \ldots, \mathbf{3}$
$\boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k}, \ldots$ frame indices ranging either $\mathbf{0}, \mathbf{1}, \mathbf{2}$ or $\mathbf{1}, \mathbf{2}, \mathbf{3}$
$\perp$ perpendicular component
$\mu, \nu, \lambda, \ldots$ spacetime coordinate indices
$A, B, C, \ldots$ abstract spinor indices, page 66
$a, b, c \ldots$ abstract spacetime indices
$i, j, k, \ldots$ abstract spatial indices
${ }_{s} Y_{l m}$ spin-weighted spherical harmonics
$\left(a_{1} \cdots a_{l}\right)$ symmetrisation over the indices $a_{1} \cdots a_{l}$, page 36
[ $a_{1} \cdots a_{l}$ ] antisymmetrisation over the indices $a_{1} \cdots a_{l}$, page 36
$\mathcal{A}, \mathcal{B}, \ldots$ arbitrary string of indices
$\left\{a_{1} \cdots a_{l}\right\}$ symmetric trace-free part over the indices $a_{1} \cdots a_{l}$, page 47

