

## ORIGIN AND EVOLUTION OF CONTACT BINARIES OF W UMa TYPE

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### ABSTRACT

Using angular momentum loss estimates from single star studies, it is shown that detached binaries are good candidates as progenitors of contact binaries. Three theories constructed for contact binary evolution (DSC, TRO and AML, see Fig. 1) are discussed. The DSC and TRO theories require a contact binary formation mechanism which produces unequal components (fission) while the AML theory can start from equal components (initially detached binaries). In all theories the end-product is a single star.

The stability of unequal entropy models was studied using a formula (2) which couples the energy transfer with the depth of contact and with the entropy difference. The models experience cyclic behaviour on a time scale of  $10^6$ - $10^7$  years (Fig. 2) and the contact never breaks even on the nuclear time scale. This is the important consequence of a formula of type (2). Similar behaviour (with similar formula) is expected also for TRO models and even for DSC models if the discontinuity can be preserved during one cycle period.

The DSC and TRO theories, which at first sight look quite different, are in fact complementary. The most probable contact binary theory is perhaps a suitable combination of all three. In this theory angular momentum loss is the new and important factor which may manifest itself in the UV- and X-ray activity observed in W UMa stars.

### I. PROGENITORS OF W UMa STARS

Two different origins of W UMa stars have been proposed:

1. The fission of a rapidly rotating star at the end of the pre-main sequence contraction phase. This process produces roughly the correct amount of angular momentum (Roxburgh, 1966), and small mass ratios (Lucy, 1977) as are in fact observed.

2. Evolution from a detached or semidetached binary by angular momentum loss. Huang (1966) suggested that magnetic torques could bring together the separate components of a detached binary. This is Schatzman's (1962) and Mestel's (1968) mechanism of magnetic braking.

We have studied this second formation mechanism in a more quantitative way and tried to estimate its consequences for close binary statistics (the details will be published elsewhere by Vilhu, 1981b). In order to do this we need three rather poorly known factors: a) the initial distribution of physical parameters such as the mass ratio and the period, b) the angular momentum loss rate and c) the life time of the contact phase.

Lucy and Ricco (1979) conclude that short period binaries of small mass are formed by a mechanism that would create binaries with equal components and identify this with the hierarchical fragmentation scheme during the final dynamical collapse of a rotating protostar (Bodenheimer, 1978). A similar conclusion has also been reached by Kraicheva et al. (1979) and they also claim that most probably the initial period distribution is flat (in the sense  $dN/d\log P = \text{const}$ ) with a short period cut-off somewhere around 2 days and a long period cut-off around  $10^6$  days (the exact size of this last number is not essential).

An important piece of evidence for the braking of rotation in solar type stars comes from the observations of rotational velocities in clusters. Using the sun, solar type dwarfs in Hyades, Pleiades and in some other clusters, Skumanich (1972) and Smith (1979) find that the surface rotation (as well as  $\text{Ca}^+$ -emission) decays as the inverse square root of the age:  $v_{\text{rot}} \propto t^{-1/2}$ . Assuming (as seems reasonable) that this behaviour is connected with the rotation itself (by e.g. a rotationally driven dynamo in the outer convective zone and subsequent braking by a magnetic co-rotating wind), we can easily derive a formula for the decline of the spin angular momentum (assuming solid body rotation).

Chromospheric  $\text{Ca}^+$ -emission correlates with the rotational velocity and during the last few years similar activity-rotation relationships have been extended to much higher rotational velocities. This gives us an important argument in favour of an increasing angular momentum loss rate when the rotational period decreases, but the exact form of this behaviour is unknown. For this reason we have parametrized it by  $\alpha$  ( $\alpha = 3$  for periods longer than 3 days which is the revolution period of a typical Pleiades solar type dwarf):

$$dJ_{\text{spin}}/dt \approx 2 \cdot 10^{41} (P/3)^{-\alpha} \text{ g cm}^2 \text{ s}^{-1} \text{ year}^{-1} \quad (1)$$

where  $P$  should be expressed in days.

Next we couple this angular momentum loss rate with the orbital angular momentum assuming that (due to tidal effects) the spin and

orbital motions are synchronized. In this way the orbital angular momentum serves as a reservoir for the spin losses.

We mention some general conclusions derived from this detached  $\rightarrow$  contact treatment (for details see Vilhu, 1981b):

1. The process produces roughly the correct mean field density (0.5 % - 2 % of all stars of the same spectral type are contact binaries) if the contact life time  $\tau_{\text{cont}}$  is about  $5 \cdot 10^8$  years.
2. Contact binaries can be produced by this process in old ( $\approx 5 \cdot 10^9$  y) as well as in intermediate age ( $\approx 5 \cdot 10^8$  y) clusters.
3. A smooth period cut-off around 2 days in the initial period distribution produces roughly equal numbers of evolved and non-evolved contact systems.
4. The ratio of low mass binaries (excluding W UMa's) with periods less than two days to those with periods greater than two days is about 1/60.
5. It is difficult to choose the "best" values for the parameters involved, but we make the choice (which may not be unique):  $\alpha = 1.5$ ,  $\tau_{\text{cont}} = 5 \cdot 10^8$  years and  $dN/d \log P$  in the initial period distribution goes from  $P=2$  days linearly to zero at  $P=1$  day. If the contact life time turns out to be e.g. 10 times longer, then practically the same conclusions hold if we had 10 times fewer initial systems in the period interval 1 - 10 days.

These results demonstrate that the values of the parameters involved need not be extremely peculiar for the process to work. It is perhaps just a coincidence that the angular momentum loss rate, as computed from our basic formula (1) with  $\alpha = 1.5$ , is roughly the same ( $10^{43} \text{ g cm}^2 \text{ s}^{-1} \text{ year}^{-1}$ ) as the AML theory predicts (see Vilhu and Rahunen, 1980 and 1981; Rahunen 1981a).

## II. THREE THEORIES FOR CONTACT BINARY EVOLUTION

So far three different theories for contact binaries have been proposed, on the basis of which discussion on evolutionary paths is possible. We consider these theories and especially their predictions in the period-colour diagram shown in Fig. 1 where Eggen's (1961, 1967) period-colour relation for W UMa stars together with theoretical boundaries for zero-age and evolved systems are plotted. It is worth noting that the blue zero-age systems with small mass ratios fall outside the observed region (see Vilhu, 1981a). The evolutionary possibilities as predicted by the different theories are also shown.

The contact discontinuity theory (DSC) traces back to Biermann and Thomas (1972 and 1973) and to Vilhu (1973) who were able to

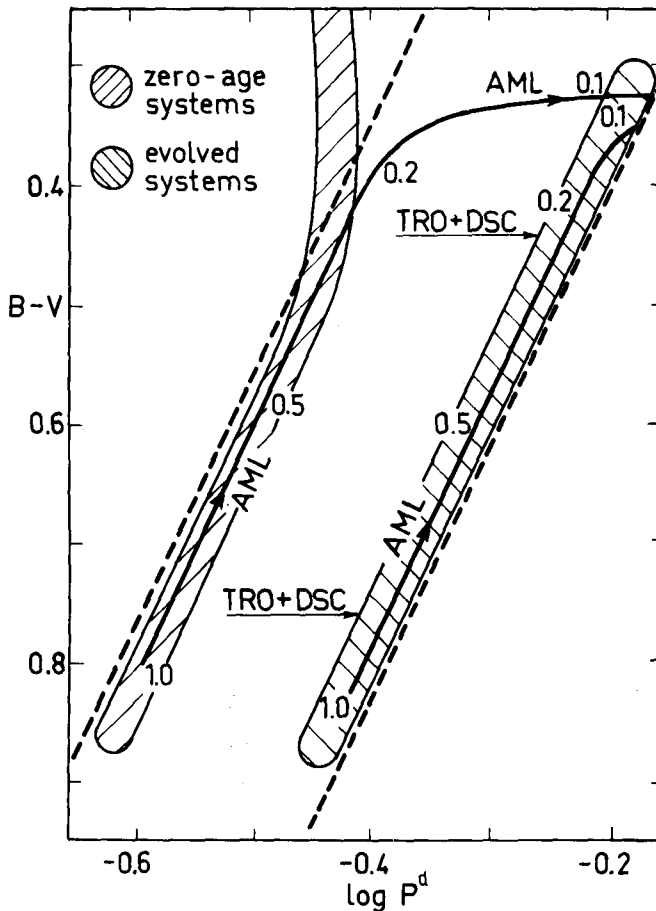


Figure 1. The period-colour diagram for contact binaries of W UMa type. The dashed lines show the observed boundaries. The shaded regions show the positions of thermal equilibrium models with different mass ratios (shown by the numbers 1.0-0.1). Evolutionary tracks of the three main theories DSC (contact discontinuity), TRO (thermal relaxation oscillations) and AML (angular momentum loss) are shown by the heavy lines.

construct zero-age models by allowing unequal entropies for the component stars. Their thermal equilibrium models evolve towards more extreme mass ratios on the primary's nuclear time scale. These models were thought to represent the interiors of the component stars. However, it was clear that the models needed an exterior common envelope to produce the observational property of W UMa stars: both components have nearly the same surface temperatures. However, an ad hoc-assumption was made that this common envelope does not have any crucial influence on the internal structure of the stars.

Later Shu, Lubow and Anderson (1976 and 1979) and Lubow and Shu (1977) gave a physical (but heavily criticized) argument for this common envelope concept and published models which are now known as contact discontinuity (DSC) models. Although no evolutionary sequences have been published for DSC models, it seems clear that they (if stable) evolve in the same way as the unequal entropy models by Biermann and Thomas and Vilhu. In the period-colour diagram (Fig. 1) these models move from left to right with slightly decreasing mass ratios on the nuclear time scale of the primary.

In the thermal relaxation oscillation (TRO) theory (Lucy, 1976, Flannery, 1976, Rahunen and Vilhu, 1977, Robertson and Eggleton, 1977, Rahunen, 1981a) the component stars are unable to find thermal equilibrium either in or out of contact. Therefore the system performs oscillations with alternating contact and semi-detached phases. During the contact phase, the thermal contact is good and the mass ratio  $q$  decreases ( $q < 1$  by definition). During the semi-detached phase thermal contact is poor and  $q$  increases. The attempt of the system to reach the nonexistent thermal equilibrium with equal entropies, coupled with the Roche geometry, is the driver for the cycling behaviour.

All constant angular momentum models (i.e. DSC or TRO) need a formation mechanism which can produce unequal components, because with constant angular momentum it is not possible to cover a large range of mass ratios (see e.g. Vilhu, 1981a, Rahunen, 1981a). We identify this mechanism to be the fission mechanism.

In the period-colour diagram (Fig. 1) DSC and TRO models evolve in the same way from the left boundary to the right one on the primary's nuclear time scale. At low mass ratios (where most W UMa stars are) only a relatively small range of mass ratios are reached in one evolutionary sequence.

The angular momentum loss (AML) theory is simply a link between different TRO models (in good thermal contact) at a fixed nuclear evolutionary state of the primary (Vilhu and Rahunen, 1980 and 1981; Rahunen, 1981a). In this theory the cycles are avoided by continuous angular momentum loss. The most suitable contact binary production mechanism for the AML theory is the detached  $\rightarrow$  contact picture as discussed in Ch. I. Whatever the production mechanism is in reality, one AML sequence can cover all the mass ratios observed, which is not

the case with the DSC and TRO theories.

AML models evolve parallel to the boundaries of the period-colour diagram with decreasing mass ratio on the secondary's thermal time scale. The nuclear evolutionary state of the primary at the time when contact was established determines the horizontal distance from the observed boundaries, so that zero-age systems reproduce the left boundary whereas evolved ones populate the right boundary. But in fact also the initially zero-age AML models turn to the right in the period-colour diagram when the mass ratio has decreased sufficiently. This occurs because finally the nuclear time scale of the primary becomes shorter than the thermal time scale of the secondary. Using simple expressions for the time scales [ $\tau_{\text{nuc}} \sim 10^{10} (M/M_{\odot})^{-3.5}$  y and  $\tau_{\text{th}} \sim 3 \cdot 10^7 (M/M_{\odot})^{-3.5}$  y] we find that these are equal when the mass ratio is  $q \sim 0.2$ . After this point the evolution is essentially determined by the nuclear evolution of the primary.

In reality the angular momentum loss may not be so large as the ideal AML models require. In this case also AML models might have cycling behaviour which, however, would not be so violent as with constant angular momentum. But, provided that the time scale of angular momentum loss is shorter than the nuclear time scale of the primary, all the essential features of the AML theory remain. So a mixture of AML and TRO theories is perhaps the best choice for contact binary evolution.

We can summarize some of the features of the three theories as follows:

DSC and TRO theories (constant angular momentum) require a formation mechanism which produces unequal components. We identify this as fission. Otherwise systems with very small mass ratios cannot be obtained. In these theories the systems move across the period-colour diagram (from left to right, see Fig. 1) with only slightly decreasing mass ratio and without much change in colour. The evolution takes place on the primary's nuclear time scale.

For the AML theory we find the detached  $\rightarrow$  contact picture as the most natural production mechanism (see Ch. I). The theory can start from a detached system having  $q = 1$  which evolves into contact at zero-age or at some later hydrogen burning stage. The contact system evolves towards  $q \sim 0.2$  on the secondary's thermal time scale and continues towards  $q = 0$  on the primary's nuclear time scale. Zero-age systems move along the left boundary while the evolved ones (if near the end of the main sequence phase) move along the right boundary of the period-colour diagram (see Fig. 1).

In all three theories zero-age models with very small mass ratios ( $q < 0.2$ ) would be located outside the observed region of the period-colour diagram. This suggests that all W UMa stars with rather small mass ratios are partially evolved.

In all theories the most probable end-product of contact evolution is a single star, and the final coalescence takes place presumably after the main sequence. We identify FK Com stars (rapidly rotating giants) as the most probable candidates for these end-products (Webbink, 1976, Bopp and Rucinski, 1981).

### III. ON THE STABILITY OF UNEQUAL ENTROPY MODELS

In the previous chapter (Ch. II) we considered the unequal entropy models of Biermann and Thomas (1972 and 1973) and of Vilhu (1973), as well as the discontinuity models of Shu et al. (1976 and 1979) and of Lubow and Shu (1977). We shall call these models the "DSC theory". Although the physical basis of these models may seem different, they have one common feature: the internal structure of the models inside the Roche lobes is similar. The essential assumption is that the components, which have unequal entropies in their convective envelopes, can maintain thermal equilibrium for a nuclear time scale. Therefore the question of the thermal stability of unequal entropy models is crucial for the DSC theory.

TRO theory at its early stage was criticized for postulating oscillations around a nonexistent equilibrium configuration. However, later Lucy and Wilson (1979) stressed that the oscillations take place around unequal entropy configurations. Therefore the question of the thermal stability of unequal entropy models is crucial for the TRO theory as well.

Hazlehurst and Refsdal (1980) studied the stability problem using "stellar response functions" introduced by Hazlehurst et al. (1977). They found the system of two zero-age stars ( $1 M_{\odot} + 0.6 M_{\odot}$ ) to be unstable with an e-folding time of  $3 \cdot 10^4$  years. Unfortunately they were not able to trace the further evolution with their linear stability analysis. They used a formula which couples the energy transfer ( $\Delta L$ ) with the depth of contact ( $d$ ) and with the entropy difference ( $\Delta S$ ):

$$\Delta L = k d^m \Delta S^n, \quad (2)$$

where  $k$ ,  $m$  and  $n$  are constants. In the lack of a detailed physical theory for the energy transfer this formula may give some physical insight into the problem, if different choices for the parameters  $k$ ,  $m$  and  $n$  are considered. Moreover, without such a formula it is not possible to study the stability problem at all.

We have studied the thermal stability and evolution of a  $1.0 M_{\odot} + 0.6 M_{\odot}$  system using a Henyey-type stellar evolution code (the details will be published by Rahunen, 1981b). We applied as boundary conditions the usual equipotential condition and the formula (2) for the energy transfer. The stability was studied by introducing a small perturbation in the form of an energy pulse from the primary to the secondary. We assumed several values for the transport coeffi-

cients ( $k$ ,  $m$  and  $n$ ), and studied the stability in each case separately.

As an example consider the following case: With luminosity transfer  $\Delta L = 0.144 L_{\odot}$  two ZAMS-stars of  $1.0 M_{\odot}$  and  $0.6 M_{\odot}$  can be brought into contact while still remaining in equilibrium. In this case there is an entropy difference  $\Delta S = 0.813$  leading to different surface temperatures  $\Delta \log T_{\text{eff}} = 0.076$ . Here  $\Delta S$  is expressed as  $\Delta S = -\Delta \ln K$  where  $K$  is the adiabatic constant  $= P/T^{2.5}$ . We may now ask how stable this kind a configuration is against small perturbations if the luminosity transfer is computed from the formula (2). Without detailed model computations it is not at all obvious what will be the result.

Results of computations (with specific values for the parameters  $k$ ,  $m$  and  $n$ ) are shown in Fig. 2. As can be seen the system is thermally unstable and behaves much like the TRO models. After  $10^7$  years steady cycles with a period of  $6 \cdot 10^6$  years seem to be established. The important consequence of the energy transfer formula (2) is that the contact never breaks. On the other hand, the entropy difference is fairly large, the minimum corresponding to  $\Delta \log T_{\text{eff}} = 0.04$ . This minimum is also very sensitive to the choice of the transport coefficients.

To see how the system behaves in different stages of nuclear evolution similar unequal entropy models were constructed with various central hydrogen contents of the primary. It turned out that at first the most prominent effect of the nuclear evolution was to shorten the cycle period, but finally at a certain hydrogen content the instability was removed. A stable configuration in thermal equilibrium was found after a reduction of the initial central hydrogen by about 15 % corresponding to an age of about  $10^9$  years. This age is achieved after about 200 cycles. Then such a system evolved on the primary nuclear time scale with decreasing mass ratio and with decreasing entropy difference resembling more and more the A-type W UMa stars.

Although these ZAMS cycling unequal entropy models cannot explain the equal temperatures of W UMa stars, they help us to understand some of their problems. Our computations show that an unequal entropy model is unstable and its subsequent evolution closely resembles the evolution of TRO models. We have a good reason to believe that a slight modification of the energy transfer formula (2) might result in cycles which are consistent with the TRO models published so far. Further, it seems probable that in a similar manner TRO models can be constructed in better agreement with observations (components are in contact permanently).

Our results speak against the assumption of thermal equilibrium in unequal entropy models. However, our results also show that the contact does not necessarily break even on the nuclear time scale. Further, if we do not insist on thermal equilibrium and take the DSC concept more generally, our unequal entropy models can be made much



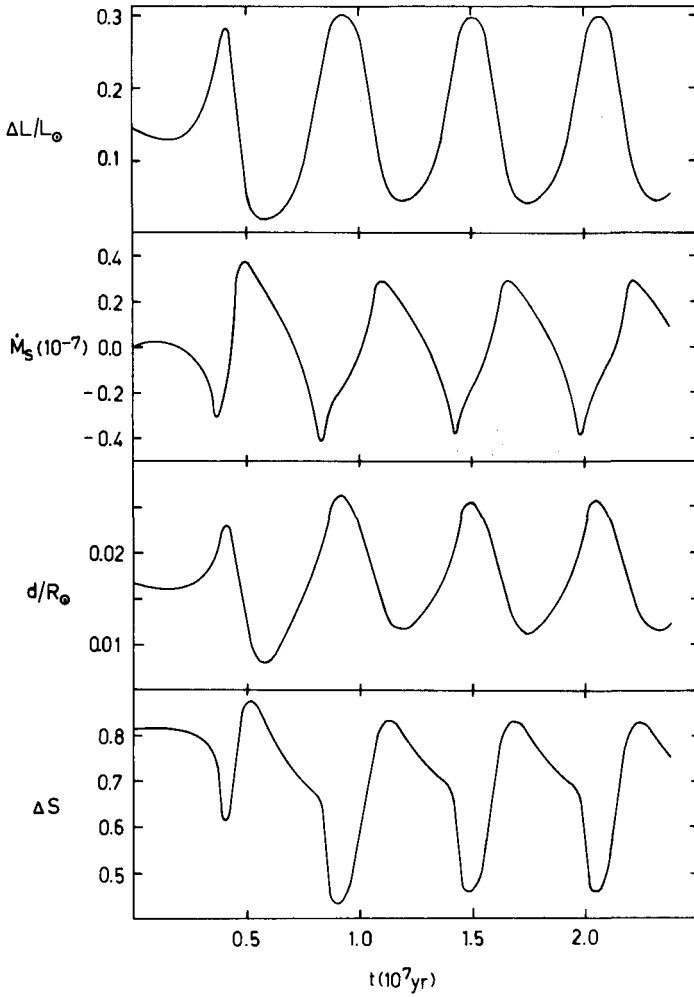


Figure 2. The cycling behaviour of an unequal entropy model with component masses  $1.0 M_{\odot}$  and  $0.6 M_{\odot}$ . The luminosity transfer  $\Delta L$  was computed from the formula  $\Delta L = k d^m \Delta S^n$  with the energy transport coefficients  $m = 3$  and  $n = 1$ .  $\dot{M}_S$  is the rate of mass transfer ( $M_{\odot}/y$ ) from the primary to the secondary.  $d$  is the depth of contact and  $\Delta S$  the entropy difference between the component envelopes.

closer to DSC models assuming a common envelope around the component stars. As far as we know nobody has proved that the contact discontinuity cannot be preserved on the short time scale of one cycle ( $6 \cdot 10^6$  years, see Fig. 2). In fact our models show a tendency to develop a temperature inversion (a contact discontinuity) at a certain stage of the cycle.

In conclusion, we think that the results presented can be applied to improve both DSC and TRO theories which at first sight look quite different but are in fact complementary (see also Shu, 1980). Perhaps it is possible, with the aid of a more refined energy transfer formulation, to unite these theories.

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