STAR POSITIONS FROM OVERLAPPING PLATES IN THE CAPE ZONE −40° TO −52°

S. V. M. CLUBE
Royal Observatory, Edinburgh, U.K.

Abstract. A modification of the normal regression models used for the determination of star positions is described. Some preliminary results relating to accuracy are given.

I would like to make some brief comments in relation to the systematic errors that may be introduced to photographic star positions as a result of incorrect modelling. The changes in procedure described are being applied to the determination of star positions from overlapping plates of the recent Cape survey, but the results are at present provisional.

Consider first the 'simple' problem of estimating m in the functional relationship $y = mx$ between an independent variable x and a dependent variable y. It is not widely appreciated that the least squares procedure by which m is found differs according to the origin of the residuals $r = y - mx$. Three situations may be distinguished (sampling errors in m are neglected):

(1) The functional relationship is exact (i.e. the model is correct) and there is no error in x, by which it follows that all the error occurs in y: in this case, $\sigma_{rr} = \sigma_{yy}$ and the 'standard' least squares procedure applies.

(2) The functional relationship is exact, but there is error in both x and y, which may or may not be correlated: in this case, $\sigma_{rr} = \sigma_{yy} + 2m\sigma_{xy} + m^2\sigma_{xx}$. There is an extensive literature concerning this case in particular and in more general circumstances, which is not discussed any further here. Case (1) is obviously a special example of case (2).

(3) The functional relationship is not exact (i.e. the model is only approximate), and x and y are both error free. In this case, the adopted relationship is arbitrary but is presumably guided by some physics which is assumed relevant to the situation. The variance σ_{rr} can be written as $x^2\sigma_{mm}$ assuming a normal distribution for the m values though this may be as arbitrary as the adopted functional relationship.

The third case seems to have been identified clearly only very recently in the statistical literature (Fisk, 1967) and is known as a 'regression model of the second kind', but the most general case would be a combination of cases (2) and (3). An example of the most general case arises in the study of stellar kinematics and an appropriate least squares technique of analysis has been developed elsewhere (Clube, 1972). The derivation of plate constants confronts us with a very similar problem. In determining an improved standard coordinate ξ from photographic measures (x, y), we may write each residual

$$r = \xi - (ax + by + c)$$

Gliese, Murray, and Tucker, 'New Problems in Astrometry', 305–306.

and commonly analyse for the plate constants (a, b, c) as if all the residual error originates in ξ, x, y whereas in practice there is likely to be a considerable contribution to r arising from shortcomings in the model (i.e. either too few or too many or wrong terms). A more satisfactory representation of σ_{rr} should reflect the likely deterioration in the model as $(x^2+y^2)^{1/2}$ increases, and we could for example write

$$\sigma_{rr} = \sigma_{\xi\xi} + a^2\sigma_{xx} + b^2\sigma_{yy} + \beta^2(x^2+y^2) \equiv \alpha^2 + \beta^2(x^2+y^2).$$

This then introduces two new 'plate constants' α, β and necessitate an adjustment of the weights of the condition equations contributing to the estimation of the plate constants.

In applying this procedure to the Cape survey plates, it is found initially that $\alpha/\beta \gg x, y$ because the dominant source of uncertainty is due to the meridian circle positions, but subsequently using averaged positions from overlapping plates, $\alpha/\beta \sim$ one half the plate width revealing that modelling errors towards the plate edge are comparable to the photographic errors. As a result, it now becomes possible to introduce an effective criterion by which adjustments to the model may be judged; improvements are those which cause $\beta \to 0$. It is hoped that the joint application of this procedure with the 'overlapping' constraints will result in some control over any systematic errors introduced photographically.

References

Clube, S. V. M.: 1972, *Quart. J. Roy. Astron. Soc.* **13**, 185.
Fisk, P. R.: 1967, *J. Roy. Statist. Soc.* **B29**, 266.

DISCUSSION

Eichhorn: It is very encouraging to see that the method of least squares is beginning to be regarded more critically than used to be the case. There have, in the past, been a lot of sins committed against the spirit of Gauss by applying least squares adjustments in ways that they were not meant to be used.

The problem of more than one observation in any equation of condition has, by the way, been solved by Denning (*Statistical Adjustment of Data*, N.Y. 1942) and extended, in a very general and elegant form, to the case of correlated observations by D. Brown in a paper 'A matrix Treatment of the General Case of Least Squares', published 1955 as Report No. 937 by the Ballistics Research Laboratory at Aberdeen Proving Grounds, Aberdeen, Maryland, U. S.A.

Clube: These papers are certainly elegant and relevant to the plate constants problem, but do not cover the equally serious problem of uncertainties in the physical model.