ON THE NATURE OF THE NONHYDROSTATIC QUADRUPOLE EXCESS MOMENT OF THE EARTH

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I wish to discuss an effect that is caused by the secular decrease in the Earth's rotation. I shall show that this deceleration induces mass flows across level surfaces and that these flows redistribute temperature and density in the Earth and produce an excess equatorial bulge. This mechanism does not require large lower mantle viscosity, unlike mechanisms discussed by Munk and MacDonald (1960) and McKenzie (1966). Therefore it does not suffer from the difficulties pointed out by Goldreich and Toomre (1969).

It is well known that in a state of hydrostatic equilibrium surfaces of constant pressure, density, and temperature coincide. One of the effects of the mechanism we discuss here is to destroy this coincidence.

The deceleration of the Earth's rotational speed causes its oblateness to decrease. This is proved, for example, by the fact that the current quadrupole moment of the Earth is smaller than the hydrostatic quadrupole moment corresponding to the higher angular speeds the Earth possessed earlier in its history (and which are established by coral-reef observations). The decrease of oblateness is accompanied by a quadrupole flow inside the Earth. In the simple model of an incompressible viscous liquid with $\nu < 3.10^{27}$ cm^2/sec the radial component of the flow velocity across the level surfaces of the hydrostatic model is

$$v_r = -\dot{\omega}\omega \frac{a^2}{g}\left[\frac{r}{a} - \left(\frac{r}{a}\right)^3\right] P_2(\cos\theta) \qquad (1)$$

where $\dot{\omega}$ is the rate of angular velocity change, a is the Earth's radius, g is gravity. [For $-\dot{\omega}\omega$ = const the expression is the same as one obtained by McKenzie (1966).] An important feature of the upper mantle, and possibly of the whole mantle, is the superadiabatic temperature gradient (Magnitsky, 1971). The "excess" temperature gradient decreases with depth in the mantle and practically vanishes in the core. We approximate the "excess" temperature gradient by:

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$$\partial_r T - \partial_r T^a = -\beta \left(\frac{r}{a}\right)^2 \theta(r-r_c) \tag{2}$$

where β is a positive constant, r_c is the radius of the core, $\theta(r-r_c)$ is the theta-function [$\theta(x) = 1$ if $x > 0$, $= 0$ otherwise]. Because the temperature gradient is superadiabatic, the flow induces a temperature disturbance $\delta T = T - T_{eq}$ (where T_{eq} corresponds to the equilibrium temperature distribution) which in general varies along the equilibrium level surfaces.

As the boundary and initial conditions we take

$$\delta T(t,a,\theta) = \delta T(t,r_c,\theta) = 0$$

$$\delta T(t_i,r,\theta) = 0 \quad .$$

For small times, the temperature disturbance grows with time. Ultimately a steady-state equilibrium with thermal conduction is reached. For

$$f = \frac{5}{2}\left(\frac{\omega_i^2}{\omega_o^2} - 1\right) \frac{\omega_o}{\dot{\omega}_o} \frac{K}{a^2} \gtrsim 0.1$$

a steady state is reached in the whole mantle. (Here $\dot{\omega}_2$, ω_o are the initial and present rotation rates and K is the thermal diffusivity.)

$$\delta T = \frac{-\dot{\omega}_o \omega_o \beta a^5}{5gK} \left\{ 0.458 \left(\frac{r}{a}\right)^5 - 0.300 \left(\frac{r}{a}\right)^7 + 0.163 \left(\frac{r}{a}\right)^2 - 0.011 \left(\frac{r}{a}\right)^{-3} \right\} P_2(\cos\theta) \quad . \tag{3}$$

If $f < 0.1$, a steady state is reached only in the vicinity of the boundaries. In the central parts of the mantle $a(1-0.6\sqrt{f}) > r > r_c(1+6f)$, δT continues to rise and

$$\delta T = \left(\frac{\omega_i^2}{\omega_o^2} - 1\right) \frac{\omega_o^2 \beta a^2}{2g} \left[\left(\frac{r}{a}\right)^4 - \left(\frac{r}{a}\right)^5\right] P_2(\cos\theta) \quad . \tag{4}$$

The temperature variation along equilibrium level surfaces produces density disturbances which have the opposite sign,

$$\delta\rho = -\alpha\rho\,\delta T \quad . \tag{5}$$

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In the vicinity of the equator $\delta T < 0$, $\delta\rho > 0$; while near the poles $\delta T > 0$, $\delta\rho < 0$. In other words the flow in the mantle associated with the slowing down of the rotation increases the oblateness of surfaces of constant density relative to equilibrium ones.

Now we can estimate the excess quadrupole moment produced by these density disturbances. The additional quadrupole moment is

$$d_2 = -\int \delta\rho\, r^2 P_2(\cos\theta)dv \quad . \tag{6}$$

(The integration is carried out over the whole Earth.) Commonly the non-dimensional factor J_2 is used which appears in the expansion of the external gravitational field in spherical harmonics and is connected with the quadrupole moment by the relation $D_2 = -Ma^2J_2$. The satellite evidence shows that J_2 exceeds the hydrostatic value J_2^{EQ} by $J_2 - J_2^{eq} \sim 10^{-5}$. Substituting $\delta\rho$ and δT from (3)-(5) into (6) and performing the integration for $f < 0.1$ gives

$$J_2 - J_2^{eq} = \frac{-d_2}{Ma^2} \approx 0.1\, \frac{[(\omega_i^2/\omega_o^2) - 1]\omega_o^2\alpha\beta\rho a^5}{gM}(1-2.5\sqrt{f}+4f) \; . \tag{7}$$

When $f > 0.1$

$$J_2 - J^{eq} \approx 0.02\, \frac{-\dot{\omega}_o\omega_o\alpha\beta\rho a^7}{gMK} \quad . \tag{8}$$

With $M = 6\times10^{27}$ g, $a = 6.4\times10^8$ cm, $g = 10^3$ cm sec^{-2}, $\rho = 3.5$ gm cm^{-3}, $\alpha = 3\times10^{-5}$ grad^{-1}, $\beta = 10^{-5}$ grad cm^{-1}, $-\dot{\omega}_o = 7\times10^{-22}$ sec^{-2}, $\omega_o = 7\times10^{-5}$ sec^{-1}, we have

$$J_2 - J^{eq} = \begin{cases} 10^{-4}\left(\dfrac{\omega_i^2}{\omega_o^2} - 1\right)(1-2.5\sqrt{f}+4f) & \text{for } f < 0.1 \\ 0.6\times10^{-5}/K^{-1} & \text{for } f > 0.1 \end{cases}$$

($f = 0.1$ corresponds to $[(\omega_i^2/\omega_o^2) - 1]K \simeq 0.2$).

The excess quadrupole moment predicted by the second formula coincides with the observed excess for $K \sim 0.6$ cm^2/sec. This value is more than an order of magnitude greater than can be produced by conductive processes. It could be produced, however, by convection. Another possible interpretation of these results is that the temperature disturbance does not reach steady state. Then the observed excess quadrupole moment could persist for 300 million years.

The model under consideration is simple and results presented here are preliminary. But I believe that it is an important mechanism for the creation of the nonhydrostatic excess quadrupole moment of the Earth.

References

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