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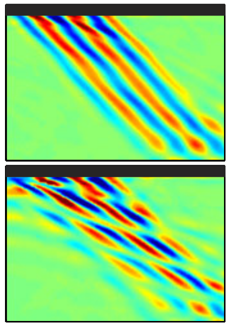
The wave instability pathway to turbulence



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One way that large-scale oceanic internal waves transfer their energy to small-scale mixing is through parametric subharmonic instability (PSI). But there is a disconnect between theory, which assumes the waves are periodic in space and time, and reality, in which waves are transient and localized. The innovative laboratory experiments and analysis techniques of Bourget *et al.* (*J. Fluid Mech.*, vol. 723, 2013, pp. 1–20) show that theory can be applied to interpret the generation of subharmonic disturbances from a quasi-monochromatic wave beam. Their methodology and results open up new avenues of investigation into PSI through experiments, simulations and observations.

Key words: internal waves, parametric instability

1. Introduction

The stratification of the abyssal ocean is generally believed to be maintained by mixing resulting from the breakdown of large-scale internal waves generated by surface processes or by the motion of tides over the ocean floor (Munk & Wunsch 1998). But it is not well understood how the energy from these waves cascades from large scales to sufficiently small scales that it efficiently dissipates. Far from boundaries and neglecting the influence of currents, eddies and other waves, the dominant mechanism in the abyss for the energy transfer to small scales is through parametric subharmonic instability (PSI). This describes a nonlinear resonant interaction through which a primary wave excites pairs of waves whose frequencies and wavenumbers add up to the frequency and wavenumber, respectively, of the primary wave (Hasselmann 1962).

The theory for PSI is based upon the assumption that the primary wave is monochromatic in space and time. Furthermore, the predicted growth rate of PSI is typically quite small compared to the frequency of the wave. And so, although all plane internal waves are known to be unstable to PSI (Lombard & Riley 1996), it is not obvious that PSI occurs in realistic scenarios.

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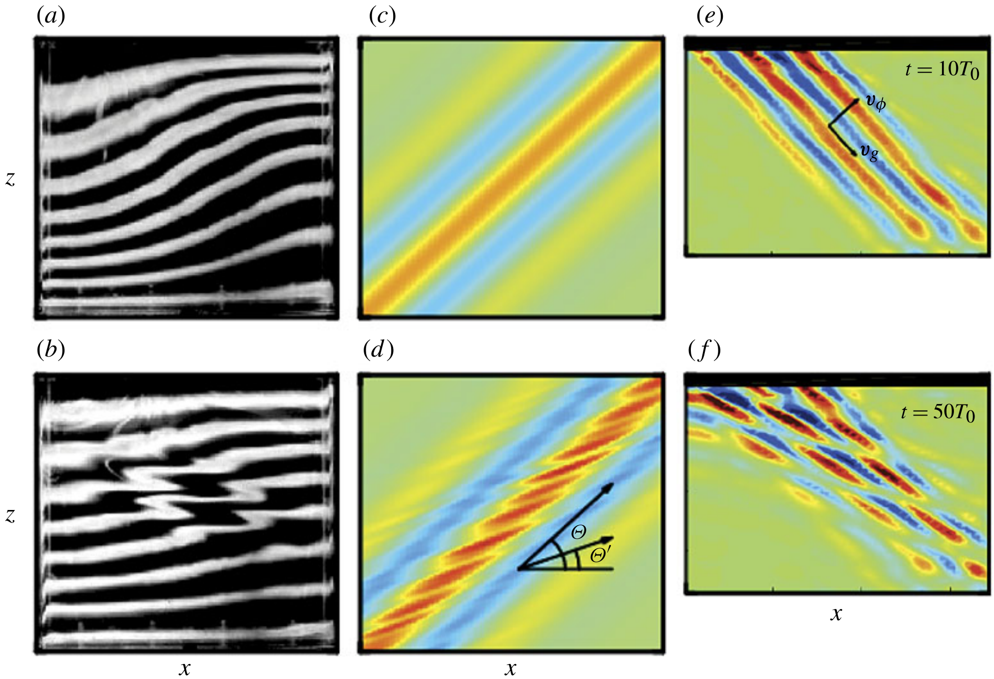

FIGURE 1. The development of PSI in the x–z plane observed at two times in three circumstances: (*a*,*b*) the displacement of dye-lines in experiments with a vertically oscillating rectangular tank at $t = 1200$ and 1220 s (Benielli & Sommeria (1998); (*c*,*d*)) perturbations of the spanwise vorticity field computed in simulations of a wave beam in a doubly periodic domain, shown initially and after 40 buoyancy periods (Clark & Sutherland (2010); (*e*,*f*) perturbations of the vertical density gradient in experiments on a wave beam emanating from a camshaft wave generator, shown after 10 and 50 wave periods (Bourget *et al.* 2013).

The occurrence of PSI for internal waves in continuously stratified fluid has been investigated mainly through laboratory experiments. Internal waves modes in a rectangular tank were shown to excite subharmonic disturbances (Benielli & Sommeria 1998). For example, by vertically oscillating a rectangular tank filled with uniformly stratified fluid, low-mode internal waves were generated (figure 1*a*). After 20 min (approximately 400 buoyancy periods) disturbances developed near the centre of the tank, oscillating with half the forcing frequency of the low-mode wave (figure 1*b*). These experiments showed that the waves need not be plane periodic: PSI occurred for modes as well as for propagating waves. Nonetheless, the primary waves under investigation were still spatially monochromatic in the sense that the horizontal and vertical wavenumbers were fixed. And so, the occurrence of PSI may not be so surprising (McEwan & Plumb 1977).

Several experiments have studied internal wave beams created by oscillating bodies (Sutherland & Linden 2002; Peacock *et al.* 2009). In these circumstances, the waves had fixed frequency but the beam was composed of a superposition of waves with a range of wavenumbers restricted only in the sense that the ratio of vertical to horizontal wavenumber was fixed. In none of these experiments was the occurrence of PSI evident. Possibly this was because of viscous attenuation of the beam. PSI may also have been retarded because only one across-beam wavelength spanned the width

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of a beam. Using shadowgraphs, the appearance of PSI in a wave beam generated by moderately large-amplitude oscillations of a cylinder was first demonstrated qualitatively by McEwan & Plumb (1977). Indirectly, PSI was argued to occur in an experiment with a large cylinder oscillating with large amplitude (Clark & Sutherland 2010). In this case, a wave beam was generated from an oscillatory turbulent patch that surrounded the cylinder. That an internal wave beam could exhibit PSI was supported by numerical simulations, as shown in figure 1(*c*,*d*). In these simulations the Gaussian amplitude-envelope of the beam contained two across-beam wavelengths. The implication was that wavepackets must contain more than one wavelength in order for PSI to occur.

2. Overview

One breakthrough of Bourget *et al.* (2013) is the use of a new wave generation mechanism that creates a beam with three across-beam wavelengths spanning its width. Thus they are able to observe the onset of instability in the beam, as shown in figure 1(*e*,*f*). The generator operates by fixing a sequence of rectangular plates to a camshaft (Gostiaux *et al.* 2007). As the camshaft rotates, the plates move back and forth providing an oscillatory forcing in time. If the camshaft is set up with a stepwise sinusoidal variation of displacement along its length, then the collective motion of the plates is a waveform that propagates in one direction. Thus this versatile tool can create quasi-monochromatic wavepackets of arbitrary spatial extent.

The waves are visualized and disturbances measured using a non-intrusive method called synthetic schlieren (Dalziel, Hughes & Sutherland 2000). The other breakthrough of Bourget *et al.* (2013) is the application of novel analysis methods that separate the signal of the primary beam and the subharmonic waves it excites. For example, at an arbitrary point in the tank measurements of the perturbation vertical density gradient versus time can be Fourier transformed. This is convolved with a windowing function that produces a time–frequency spectrum, showing a peak frequency associated with the primary beam and the growth in power at later times of the subharmonic beams, each observed with a different frequency.

Bourget *et al.* (2013) go on to construct separate snapshots of the primary beam and its subharmonics through an adaptation of the Hilbert transform methods devised by Mercier, Garnier & Dauxois (2008). Here the Fourier-transformed fields are filtered to pass a selected frequency and the result is then inverse Fourier transformed back to temporal space. This can be done for time signals extracted from each horizontal and vertical location in the field of view. Putting the filtered results back together, Bourget *et al.* (2013) produce snapshots of disturbances having the frequency only of the primary wave or of one of the two subharmonically excited waves. Thus one can examine the spatial as well as temporal frequency of each wave component in the resonant triad.

Finally, their observations of instability in a wave beam are compared directly with the theory for PSI, which assumes the primary and subharmonic waves are perfectly periodic in space and time. For a primary wave of known frequency, ω_0, and wavenumber vector, $\boldsymbol{k}_0$, a wide range of subharmonic wave pairs exists for which their frequencies sum to ω_0 and their wavenumber vectors sum to $\boldsymbol{k}_0$. From theory they compute the subharmonic wave pairs that grow at the fastest rate. Consistent with theory, they find that the observed subharmonic waves are indeed amongst the fastest growing.

3. Future

The new experimental and analysis tools have shown how the PSI theory developed for plane waves can indeed make predictions about the stability of quasi-monochromatic wavepackets. Now the challenge is to determine the range of validity of theory. What is the minimum number of wavelengths inside a wavepacket necessary for PSI to occur? If a wavepacket is transient (and so not monochromatic in time), what duration must it have for PSI to occur? If a wavepacket propagates through a background with currents, eddies and other waves, will PSI be the dominant mechanism for breakdown or will interactions with the transient background control the energy cascade?

Even if PSI develops from a quasi-monochromatic wave, it is still not known from theory specifically what pair of subharmonic waves is generated. For example, Bourget *et al.* (2013) identified three distinct classes of subharmonic wave pairs with comparable growth rates. But only one of these was observed to develop to substantial amplitude. Ultimately the goal is to assess whether PSI operates on large-scale waves generated in the ocean by storms or by tidal flow over bottom topography. The signature of PSI seemed evident in observations of near-inertial waves in the Luzon Strait (Alford 2008). Through the application of time–frequency and Hilbert filters to their data and in future endeavours, the occurrence of PSI may be made more conclusive.

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