FLUCTUATIONS OF THE MICROWAVE BACKGROUND RADIATION

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Investigations of small scale angular fluctuations and the spectrum of the microwave background radiation is one of the main methods of studying the large scale structure of the Universe. Figure 1 shows the principal stages of the evolution of the Universe. Today we can directly observe galaxies, clusters of galaxies and quasars in the redshift range $z \leqslant 3.5$ by optical, radio and X-ray astronomy. These observations show that significant density perturbations $\delta\rho/\rho > 1$ are present on mass scales $M < 10^{16}\ M_{\odot}$. The Universe is essentially uniform $\delta\rho/\rho < 1$ on large scales $M >> 10^{16}\ M_{\odot}$.

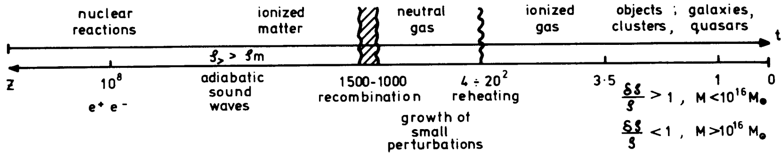

Figure 1. A schematic diagram showing the evolution of the Universe.

We can see from Figure 1 that at $z \sim 10$ annihilation of electron-positron pairs which were numerous at that epoch and nuclear reactions associated with helium synthesis took place in the Universe, which was filled with matter and radiation at the temperature $T_r = 2.7(1+z)$K. At the epoch $z \sim 1500$ recombination of hydrogen took place. This is an important moment: up till that time the optical depth of the Universe to Thomson scattering was very large but after that the Universe became transparent. In principle, by observing the relict radiation we can obtain direct information about the state of the Universe at the moment $z \sim 1000$. After that photons of the relic radiation do not interact with matter.

Thus the matter in the Universe was ionized before $z \sim 1500$, and afterwards became neutral. However, we do not see Lyman-α absorption

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bands in the spectra of distant quasars. This means that the matter must be ionized at $z < 3.5$. We do not know when this secondary ionization took place. If the gas was ionized up to a redshift $z \gtrsim 10-20$, the optical depth due to the Thomson scattering was very large. These scatterings smooth out the fluctuations by a factor $e^{-\tau_T}$ that existed up to that time and generate new ones.

According to the present view, the observed large scale structure of the Universe was formed as a result of gravitational instability and the growth of small density perturbations, $\delta\rho/\rho \ll 1$, that existed at the early stages of the expansion $z \gg 10^{10}$. These perturbations had a broad mass spectrum and somewhere at $z > 1$ $\delta\rho/\rho$ became larger than one which resulted in the formation of observed objects. Silk (1967) first noted that the existence of density perturbations at the epoch of recombination must lead to fluctuations of the relict radiation. The fluctuations are proportional to the amplitude of perturbations, and, therefore, by observing them it is possible to determine $\delta\rho/\rho$ at the moment of recombination (or reheating) and also the epoch of the formation of galaxies or (more accurately) of clusters of galaxies.

Three sources of the fluctuations will be considered below:

(1) Perturbations of the matter density $\delta\rho/\rho$ and velocity v at the epoch of recombination, $z \sim 1000$ - primaeval fluctuations.

(2) Perturbations of density $\delta\rho/\rho$ and velocity v at the epoch of reheating at $z \sim 11\ \Omega^{-1/3}$(if the intergalactic matter was ionized at that time, of course) - secondary fluctuations.

(3) Observed sources: radio sources, clusters of galaxies, young galaxies, non-uniformities of hot intergalactic gas (IGG), protoclusters of galaxies, etc.

1. PRIMAEVAL FLUCTUATIONS

There are three types of density perturbations: adiabatic, enthropy (isothermal) and vortex perturbations. The last type will be described in detail in the lecture of L. M. Ozernoy and therefore I will only discuss in detail the first two types.

(a) Adiabatic perturbations of density

Before the epoch of recombination these perturbations were sound waves in which the densities of matter and radiation increase and decrease together. The evolution of these perturbations has been described in a number of publications (Sunyaev and Zeldovich 1970, Peebles and Yu 1970, Doroshkevich, Zeldovich and Sunyaev 1977).

Let us write down the perturbations of density and their velocities in terms of a Fourier integral.

$$\frac{\delta\rho}{\rho} = \frac{1}{(2\pi)^3}\int a_k\, e^{i\underline{k}\cdot\underline{r}}\, d^3k \quad ; \quad \frac{\underline{u}}{c} = \frac{1}{(2\pi)^3}\int \underline{b}_k\, e^{i\underline{k}\cdot\underline{r}}\, d^3k$$

In this case

$$\overline{\left(\frac{\delta\rho}{\rho}\right)^2} = \frac{1}{(2\pi)^3}\int a_k^2\, d^3k$$

The mass contained within a perturbation of wavelength $\lambda = 2\pi/k$ is equal to

$$m = \frac{4\pi}{3}\left(\frac{\pi}{k}\right)^3\left(\frac{2c}{H_0}\right)^3 \Omega\, \rho_{crit}$$

where H_0 is Hubble's constant.

If, at a sufficiently early time ($z >> 10^8$), the spectrum of fluctuations was of power law form $a_k \propto k^n$ then the evolution of the spectrum in the radiation dominated stages of expansion results in a spectrum of fluctuations after recombination of the form shown in Figure 2, i.e. $a_k^* = c_k a_k$

$$c_k = \frac{\sin kR_J}{kR_J}\, \exp\left\{-\frac{kR_c}{2}\right\}$$

for $R < R_J$ and $c_k = 1$ for $R > R_J$ (see Doroshkevich et al. 1977). Here R_J is the Jeans' wavelength at the moment of recombination

$$M_J \approx 10^{17}(\Omega h^2)^{-2}\, M_\odot$$

and R_c is the scale on which according to Silk viscous dissipation of the perturbations is important $M_c \approx 10^{13}$–$10^{15}\ M_\odot$.

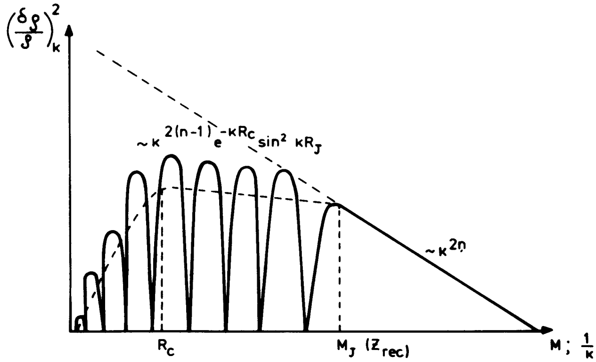

Figure 2. The spectrum of adiabatic density perturbations after recombination. The initial spectrum is assumed to be a power law $a_k \propto k^n$. The spectrum of entropy perturbations does not change and remains of power law form with the same spectral index.

After the epoch of recombination fluctuations of density grow on all scales having mass $M > 10^5\ M_\odot$ according to the law

$$\frac{\delta\rho}{\rho}(z) \propto t^{2/3} \propto (1+z)^{-1}$$

until the time when $\delta\rho/\rho$ is of order unity and the formation of gravitationally bound systems begins. We note that for cases $\Omega < 1$ the perturbations grow rapidly only up to redshift $z \sim \Omega^{-1}$. The subsequent growth is very slow (see Sunyaev 1971).

Perturbations of density and velocity are related by the continuity equation

$$\frac{\partial}{\partial t}\left(\frac{\delta\rho}{\rho}\right) = -\operatorname{div}\,\underline{v}$$

from which it follows that fluctuations in velocity grow according to the law $v \propto (1+z)^{-1/2} \propto t^{1/3}$. This equation gives a simple relation between a_k and b_k.

In the cases, which are of most interest to us, the most important contribution to the fluctuations results from scattering of radiation by moving electrons (resulting from the velocity field of the perturbation) (see Sunyaev and Zeldovich 1970). In fact, because of the Doppler effect, the temperature of the scattered radiation depends on the direction of motion of the electrons, $T(\alpha) = T_0(1 + v/c \cos\alpha)$.

For a continuous medium with variable degree of ionization we have

$$\frac{\Delta T}{T} = \int_0^\infty \frac{u_1(z)}{c}\, e^{-\tau(z)} \frac{d\tau}{dz}\, dz$$

where

$$d\tau = \sigma_T N_e(z)\, c\, \frac{dt}{dz}\, dz$$

is the optical depth to Thomson scattering; $u_1(z)$ is the projection of the velocity vector onto the line of sight and the exponent $e^{-\tau(z)}$ takes into account the smoothing out of the fluctuations which originate in regions with large optical depth. The function $e^{-\tau} d\tau/dz$ is easy to derive knowing the law of change of the electron density during recombination. This function has a sharp maximum at $z = 1100$ and halfwidth $\Delta z/z \approx 7\%$. Fluctuations originate mainly in the region of this maximum.

This function determines the scale $M \approx 10^{15}\ M_\odot$: if the mass of the perturbation is smaller, then a great number of wavelengths can be located within this zone. The velocity of matter changes direction many times (see Figure 3) and the effect decreases strongly due to these anticorrelations. For perturbations on larger scales the velocity does not change within this zone and we obtain the full effect. In this case recombination is rather fast and by observing the fluctuations in the radiation we can obtain an overall picture of density perturbations at the epoch of recombination.

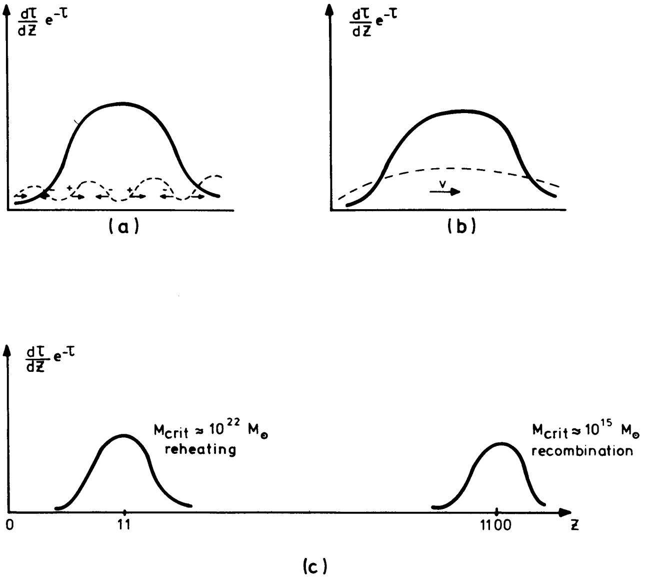

Figure 3. Illustrating how fluctuations of the relic radiation are formed. The bell-like figures show the regions in which the main effects are formed. At the bottom of top Figure 3(a) sinusoidal perturbations of velocity with small wavelength are shown. In this case the velocity changes sign many times in a bell zone, and therefore the effect is reduced. In Figure 3(b) the case of long wavelength velocity perturbations is presented. Effect appears with full strength. In Figure 3(c) two zones are shown where the perturbations are formed.

Results of detailed calculations (Sunyaev and Zeldovich 1970, Doroshkevich et al. 1977) are shown in Figure 4. The abrupt damping of the fluctuations on scales $M \sim 10^{13}$-10^{14} $M_\odot$ is connected with viscous damping of the amplitude of the fluctuations (see Figure 2). It can be seen that the most interesting scales for observations are 2' - 20' (the exact position of the break in the dependence $\Delta T/T(\theta)$ depends on Ωh^2). On small angular scales primaeval fluctuations are quite small and we can only obtain information about the spectrum of fluctuations on scales $M \gtrsim 10^{13}$ $M_\odot$.

The amplitude of intensity fluctuations depends on the amplitude of density perturbations at the epoch of recombination, i.e. on the time of the formation of the clusters of galaxies. Figure 4 shows the case $\Omega = 1$ and $\delta\rho/\rho(10^{15}\ M_\odot) = 1$ at $z_0 = 3$. It is evident that at the epoch of recombination, $\delta\rho/\rho \approx (1+z_0)/(1+z_{rec}) \approx 1/35Q$. If clusters were

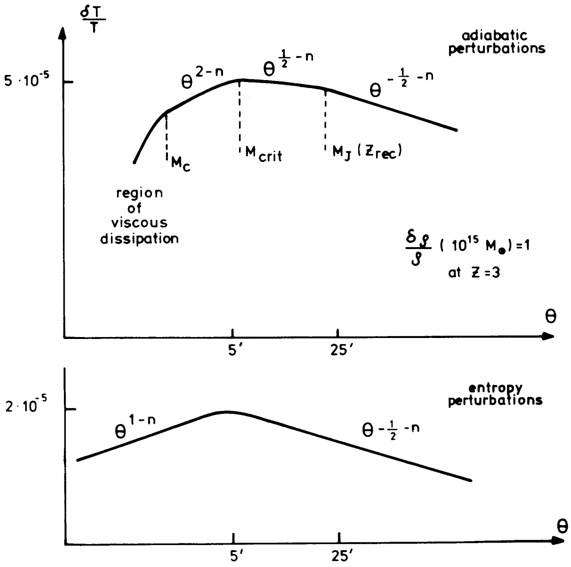

Figure 4. The spectrum of background fluctuations corresponding to the power law spectrum of primaeval perturbations in the case of adiabatic and entropy perturbations.

formed earlier, then $\delta\rho/\rho$ must have been greater at the epoch of recombination and, therefore, $\Delta T/T$ must be greater on this scale. In the case of the open world model $\Omega << 1$, perturbations grew only at epochs $z > \Omega^{-1}$ and therefore the initial perturbations could be even greater in this case. It is interesting to note that, in this case, at $z \gtrsim \Omega^{-1}$ the objects with $\delta\rho/\rho(z = \Omega^{-1}) > 1$ are formed, but perturbations with $\delta\rho/\rho < 1$ are frozen and do not grow any further.

At present the counts of galaxies and radio sources do not give information about the present amplitude of density perturbations on mass scale $M > 10^{16-17} M_\odot$. Therefore observations of fluctuations of the background radiation on corresponding angular scales may give valuable information about their spectrum. In principle, it is possible that the amplitude of the perturbations on this scale is small. In this case the fluctuations must also be small on the scale $\theta \approx 20'-30'$. The angular scale of fluctuations is related to the mass of perturbation by the relation $\theta = 5' M_{15}^{1/3} \Omega^{2/3}$.

(b) Entropy perturbations

Before recombination entropy perturbations were density perturbations of protons and electrons ρ_m in a background of constant radiation density ρ_r. Let us note that before recombination (at $z > 10^4\ \Omega h^2$) the radiation density $\rho_r = \varepsilon_r/c^2 = 4 \times 10^{-34}(1+z)^4$ g cm^{-3} exceeded the density of matter $\rho_m = 5 \times 10^{-30}\Omega h^2(1+z)^3$ g cm^3. While the primaeval plasma was ionized, entropy perturbations were frozen and their spectrum on the scale $M > 1\ M_\odot$ practically did not change. For this reason they differ greatly from adiabatic and vortex perturbations, in particular the viscous dissipation of perturbations occurs only for $M < 1\ M_\odot$ compared to the damping scale $M \sim 10^{13}$-$10^{14}\ M_\odot$ in the case of the adiabatic perturbations. After recombination, the radiation is weakly coupled with the matter. Therefore, the growth of entropy perturbations begins on all the scales $M \gtrsim 10^5\ M_\odot$ due to gravitational instability. In this case velocity perturbations that lead to fluctuations of the relict radiation are generated in accordance with the continuity equation as in the case of adiabatic perturbations. For equal amplitudes at the moment of recombination entropy perturbations lead to the fluctuations on the scale 5' about 2-2.5 times smaller than adiabatic perturbations (Sunyaev and Zeldovich 1970, Zentsova and Chernin 1977, Doroshkevich et al. 1977). Essentially, this is connected with the fact that in the first case at the epoch of recombination density perturbations that existed before the recombination generate velocity perturbations, whereas in the second case velocity perturbations, which were already great before the recombination, generate density perturbations.

2. SECONDARY FLUCTUATIONS GENERATED DURING REHEATING OF THE INTERGALACTIC GAS

At $z < 3.5$ the IGG is highly ionized (Gunn and Peterson 1965). At $z < 5$ it is easy to ionize the IGG by the UV radiation of quasars, young galaxies and "pancakes". In this case the temperature of gas is close to $T = 10^4$ K. The density of gas increases rapidly with z and its effective emission measure increases correspondingly: to preserve the high degree of ionization at $z \sim 10$-20 enormous luminosities, $L \sim 10^{46-47}$ erg s^{-1} and high densities (of the order of the present density of galaxies) of UV sources are necessary (Weymann 1966, Arons and Wingert 1972, Sunyaev and Zeldovich 1972, Doroshkevich and Shandarin 1975, Ozernoy and Chernomordik 1976, Sunyaev 1976). There are models according to which the gas was ionized by shock waves originating in the explosions of quasars or by subcosmic rays (Ginzburg and Ozernoy 1965) and also by shock waves generated in the formation of "pancakes" - protoclusters of galaxies (Sunyaev and Zeldovich 1972). In these models the gas has to be heated up to a temperature $T \sim 10^6$ K. At $z > 10$ these models meet with enormous energetic difficulties. We must take the problem of reheating very seriously because a large optical depth of the IGG to Thomson scattering in the range $z \approx 10$-30 may lead to strong damping of primaeval fluctuations of the relic radiation. If the bulk of the matter in the Universe was in a gaseous state and ionized at that time, it is easy to find the optical depth to scattering.

$$\tau = \int \sigma_T N_e c\, dt \approx 0.03\, \Omega^{1/2} h\, z_{max}^{3/2}$$

At $z \approx 11\ \Omega^{-1/3}$ we have $\tau \approx 1$; in order to ensure strong damping of the primaeval fluctuations it is necessary that $\tau \gtrsim 3$-5, i.e. $z_{max} \approx (25$-$30)\ \Omega^{-1/3}$. It is exceedingly difficult to ensure such early complete ionization of the IGG. However, let us suppose that this was the case, and calculate the magnitude of secondary fluctuations. At the epoch $z \sim 10$-20, $\delta\rho/\rho$ did not exceed unity on the scale of clusters of galaxies (and on larger scales). These perturbations grew in accordance with the continuity equation and there were also perturbations in the velocity field. It is easy to calculate fluctuations arising from scattering on moving matter, according to the same scheme as in the previous paragraph. In the case under consideration the function $e^{-\tau}\, d\tau/dz$ has a peak at $z \sim 12\ \Omega^{-1/3}$ and the whole effect accumulates in this zone. The scale $M \approx 10^{22}\Omega^{-2}M_\odot$ is the characteristic mass in this case. It is analogous to $M \approx 10^{15}\ M_\odot$ in the case of primaeval fluctuations (see Figure 3). The amplitude of the effect might be very great since the velocity after the epoch of recombination increases due to gravitational instability, roughly $v/v_0 \approx \sqrt{(1+z_r)/(1+z)} \approx 10$ times. However, we do not know the amplitude of perturbations $\delta\rho/\rho$ in the region of such large masses: they may be small and in this case the fluctuations are small as well.

If there was a power law spectrum of perturbations after recombination, $a_k \propto k^P$ the secondary fluctuations, formed due to reheating, are equal to

$$\frac{\Delta T}{T} \approx 3\times 10^{-7} M_{15}^{1/3-P/3} \propto \theta^{1-P} \; ; \qquad \frac{\Delta T}{T} \approx 7\times 10^{-4}\, M_{15}^{-1/6-P/3} \propto \theta^{-P-1/2}$$

corresponding to small ($M < 10^{22}\ M_\odot$) and large scales respectively (Sunyaev 1977). The amplitude depends strongly on the spectral index P. It is assumed that $\delta\rho/\rho(M = 10^{15}\ M_\odot) = 1$ at $z = 4$. The angular dimensions are determined by the relation $\theta \approx 5'\ M_{15}^{1/3}\ \Omega^{2/3}$ as before. In the stage after recombination both adiabatic and entropy perturbations are of potential form and do not differ except for their spectra. Observations of fluctuations on large angular scales $\theta \sim 30' - 10^\circ$ may give valuable information about the spectrum of initial large-scale perturbations with small amplitude even at the present time which have not led to the formation of gravitationally bound objects.

Vortex perturbations at this stage lead to

$$\left\langle \frac{\Delta T}{T} \right\rangle \approx 5.6 \left\langle \frac{v_0}{c} \right\rangle M_{21}^{1/6} \text{ and } \left\langle \frac{\Delta T}{T} \right\rangle \approx 5.6 \left\langle \frac{v_0}{c} \right\rangle$$

respectively, for small ($M < 10^{21}\ M_\odot$) and large masses (Sunyaev 1977) where v_0 is the present vortex velocity on scale M. At the epoch of $z \sim 10$, vortex velocities were $(1+z) \approx 10$ times greater. An interesting situation arises when the heating occurred at $z \approx 12$. In that case the optical depth due to the scattering is small, $\tau \sim 1$ and the initial fluctuations are damped only e times, whereas secondary fluctuations are formed with full amplitude. Thus both the epoch $z \approx 1000$ and $z \approx 10$ contribute to the observed fluctuations.

3. OBSERVABLE OBJECTS AS SOURCES OF FLUCTUATIONS

(a) Radio sources

Longair and Sunyaev (1969) have shown that radio sources observed at long wavelengths (ν = 408 MHz) and already in catalogues must lead to noticeable fluctuations in centimetre wavelength band ($\delta T/T \sim 10^{-5}$-10^{-6}). The main contribution to fluctuations is given by the sources whose density on the sky is close to one source per beam width of the radiotelescope. This estimate is somewhat uncertain since it requires extrapolation of the results of the counts of sources.

(b) Clusters of galaxies

As a result of the Compton effect the presence of hot gas in a rich cluster of galaxies must lead to a decrease in brightness temperature of the relic radiation in the direction to the cluster (Sunyaev and Zeldovich 1972b),

$$\frac{\Delta T}{T} = -2\,\frac{kT_e}{m_e c^2}\,\sigma_T N_e \ell \approx 10^{-4}$$

This effect has been observed in the last few years by Parijskij, Gull and Northover and Partridge. It is evident that all rich clusters taken together must be a source of fluctuations of the microwave background. Estimates show that $\Delta T/T \approx 10^{-5}$ can be expected on an angular scale of 10'-20'.

The magnitude of the effect does not depend on the redshift at which the object is located; it is only the angular dimensions that change. Hence protoclusters of galaxies in which the gas was heated by shock waves, generated at the formation of pancakes, might also make a contribution to the fluctuations. Fluctuations may also arise as a result of inhomogeneities in the temperature and density of the gas at the epoch of reheating of the intergalactic gas.

(c) Young galaxies

According to present ideas (see the paper by B. Tinsley) in the first 10^8 years of their life, the luminosity of the young galaxies was determined by O and B stars. The main part of the energy was emitted in the form of UV radiation (Weymann 1966, Partridge and Peebles 1967). If at this stage there was a lot of gas in the galaxies, they were similar to an aggregate of gigantic HII regions similar to Orion. Under these conditions bremsstrahlung radio emission of gas in the HII zones and hence the luminosity of galaxies at centimetre wavelengths were very great. Such objects may lead to the background fluctuations of the order of

$$\frac{\delta T}{T} \approx 3\times 10^{-4}\left(\frac{10}{1+z}\right)^3\left(\frac{\lambda_0}{3\,\mathrm{cm}}\right)^2$$

on angular scale of the order of $\theta \simeq 10''$ (Sunyaev 1977). At $z = 5$-20 young galaxies must undergo significant clustering, since the amplitude of fluctuations on the scale of clusters of galaxies must be large at this time. Apparently young galaxies may ensure fluctuations of the order of $\Delta T/T \simeq 10^{-5}$ on the scale $\theta \lesssim 10'$.

4. CONCLUSION

Observations of the relic radiation impose stronger and stronger limits on the amplitude of the fluctuations (let us note especially the papers of Parijskij, Carpenter, Gulkis and Sato, and Partridge). The theoretical analysis presented above shows that background fluctuations at centimetre and millimetre wavelength must exist at a level $\Delta T/T \approx 10^{-5}$ independent of whether there was early secondary reheating or not. At this level, fluctuations must be observed, which are associated with known objects - radiogalaxies etc.

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DISCUSSION

Silk: The presence of hot gas produced during the formation of galaxy clusters suggests that limits on distortions of the Wien region of the background radiation spectrum may be able to yield significant constraints on secondary reheating models.

Sunyaev: I agree with you. Observations of distortions of the black-body spectrum in the millimeter and submillimeter band give extremely important restrictions to the temperature of IGG and the time of reheating (see discussion in the paper Ya B. Zeldovich and R. A. Sunyaev, *Astrophys. Sp. Sci.*, 4, 285, 1969).

Jones: In principle we should be able to distinguish the effects due to cosmic fluctuations and discrete sources such as galaxy clusters and radio sources. Can you comment on these?

Sunyaev: The primaeval and secondary fluctuations of the relic radiation do not depend on the frequency. The contribution of radiosources is strongly dependent on frequency. Therefore observations at several frequencies might distinguish these effects. In the case of clusters of galaxies (holes in background), the measurements at $h\nu \gtrsim kT_r$ are necessary, because in the Rayleigh-Jeans part of the spectrum the effect of Comptonization does not depend on the frequency.

Ostriker: What do you believe is the origin of the fluctuations in the distribution of matter and radiation which you assume in your discussion?

Sunyaev: This question will be discussed by Zeldovich in his paper.

Parijskij: You have described many sources of fluctuations of the relic radiation. What is the total fluctuation you expect on summing all these effects?

Sunyaev: I do not think it is possible to obtain a good answer for the total fluctuations because the magnitudes of all the effects I described in my paper are very poorly known.

Chernin: Where would the energy for the reionization come from?

Sunyaev: I agree with you that there are great difficulties with the energy sources in the case of very early reheating ($z > 5$). It seems to me that heating and ionization by shock waves from "pancakes" and quasars or by soft cosmic rays at $z > 5$ is very improbable. These mechanisms give a high temperature and one needs an enormous energy release. However, in the case of ionization of the intergalactic gas (IGG) by U-V radiation from quasars and young galaxies, the temperature of gas might be low, $T \approx 10^4$ K, and the energy requirements are smaller. Even in this case we have great trouble with the sources of the U-V-radiation, because the effective "emission-measure" of the IGG increases rapidly with redshift.

van der Laan: You have suggested a large, even bewildering variety of reasons why we should expect $\Delta T/T \gtrsim 10^{-5}$. The last ten years have demonstrated how difficult and expensive (in telescope time especially) these measurements are. Every measurement is at a particular frequency ν_i and covers usually a small range of angular scales $\Delta\theta_i$. To motivate observers and to persuade committees that allocate telescope time, theorists should specify the value or information content in the (ν,θ) plane as precisely as they can.

Sunyaev: The best wavelength for observations of "real" cosmological fluctuations is the band close to the maximum of the 2.7 K black body spectrum. However, the atmospheric conditions are better in the centimetre band. The best angular scales are indicated in the written version of this paper.

CONSTRAINTS ON THE MEAN DENSITY OF THE UNIVERSE WHICH FOLLOW FROM THE THEORIES OF ADIABATIC AND WHIRL PERTURBATIONS

A. A. Kurskov and L. M. Ozernoy

The aim of this communication is to investigate what constraints to the cosmological parameter $\Omega = 2q_o$ can be obtained if one assumes that primaeval whirl motions or adiabatic density perturbations with an appropriate initial spectrum were responsible for the formation of large scale structure in the Universe. These constraints are readily obtained from the two conditions: (i) an upper limit to small scale temperature fluctuations of the microwave background radiation, and (ii) the requirement that the primaeval perturbations should be large enough in order to produce observed structures.

(a) Adiabatic perturbations. One usually makes the reasonable assumption that the initial metric perturbations are scale-independent: $h = h_i$ = constant ("white noise"). Then using Chibisov's (1972) value for the mass M_d damped at decoupling we are able to show (Kurskov and Ozernoy 1977a) that the condition of isolation ($\Delta\rho/\rho \simeq 1$ at $z \simeq \Omega^{-1}$) for inhomogeneities of maximum amplitude corresponding to masses $M \sim M_d$ is $h_i > 2 \times 10^{-4}\ \Omega^{-5/4}\ h^{-1/2}(1+-3\Omega h^2)$, where $h = H_o/75$ km s^{-1} Mpc^{-1}. Since the r.m.s. temperature fluctuations produced at the epoch of recombination due to the Doppler-effect on potential velocities (Zeldovich and Sunyaev 1970) are $\Delta T/T \sim (\sqrt{3})^{-1}(v_{rec}/c) = 1/3(z_{rec}/z_{eq})^{3/4}\ h_i$ on characteristic mass-scale $M \lesssim M_J(t_{rec})$, one finds for $0.5 < \Omega\ h^2 < 0.7$ that $\Delta T/T \gtrsim 10^{-5}\ \Omega^{-2}\ h^{-2}(1+3\Omega h^2)$ on angular scales $\theta \lesssim 15'-30'$. Comparison with corresponding upper limit $\Delta T/T \lesssim 10^{-4}$ yields $\Omega \gtrsim 0.3[(\Delta T/T)/10^{-4}]^{-1}$* This conclusion cannot be avoided by introducing secondary reheating since otherwise appreciable angular variations of $\Delta T/T \gtrsim 10^{-3}$ on scales $\theta \sim 20^o\ \Omega^{2/3}$ will appear (Kurskov and Ozernoy 1977a) unless the initial metric perturbation spectrum had a cut-off on masses $M << 10^{22}\ M_\odot$ which seems to be rather artificial.

(b) Whirl perturbations. Angular variations of $\Delta T/T$ produced by the Doppler-effect on turbulent velocities (Chibisov and Ozernoy 1969) are the sum of (i) temperature variations at $t = t_{rec}$ weakened by Thompson scattering due to secondary reheating of the cosmic plasma by young galaxies and of (ii) temperature variations produced at the moment of "last scattering" when $\tau \approx 1$. The latter gives the main contri-

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*This lower limit to Ω is apparently even more stringent if $\Omega h^2 >> 0.1$ since in that case appreciable velocities are present on scales $M > M_J(t_{rec})$ where oscillations were always absent.