

frictional heating took over. The value of n would start as a low value, increasing with the increase in ice thickness until a good-sized sheet had been build up, then would probably change very little with time until the ice sheet began to waste appreciably.

J. F. NYE: The equations you are using seem to imply that at the edge of the ice sheet, where the thickness is zero, the outward velocity of the ice is automatically zero. This follows essentially because the sliding velocity is held at zero (and provided the surface slope is not allowed to be infinite). Thus the equations do not allow the ice sheet to expand in size, even though this is what you deduce. Thus, strictly speaking, your results are inconsistent with your equations. However, I imagine that this difficulty can be fairly easily removed by some suitable readjustment of the model in a zone very close to the ice edge. Then probably your results would not be seriously affected. Do you agree?

MAHAFFY: In fact, the finite-difference scheme used tacitly assumes that the mass discharge across the edge is not zero, but is equal to the mass discharge calculated at the previous point. This is equivalent to a boundary condition of a sliding velocity at the edge that is small but finite. You are right in pointing out that, at the edge, this finite-difference scheme does not approximate the analytic equations with a sliding velocity equal to zero.

REFERENCE

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NEAR-SURFACE TEMPERATURES IN THE SUPERIMPOSED ICE ZONE AND LOWER PART OF THE SOAKED ZONE OF POLAR ICE SHEETS

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ABSTRACT. The temperature distribution in a polar glacier is described by the equation of heat conduction,

$$\kappa \nabla^2 \theta + \frac{Q}{\rho C} = \frac{D\theta}{Dt} \quad (1)$$

where κ is the thermal diffusivity of ice, Q is the internal heat generation, ρ is the ice density, and C is the heat capacity. To obtain a solution to this equation, boundary conditions at the surface and bed must be known. The boundary condition at the bed is generally taken to be the temperature gradient in the ice required to conduct the geothermal heat upward into the glacier, with certain modifications where the pressure melting temperature is reached. The boundary condition at the surface is the ice temperature, which is usually assumed to be equal to the mean annual atmospheric temperature. This assumption is incorrect in the ablation area and in the percolation and saturation zones of the accumulation area. In this paper I examine the reasons for the break down of this assumption, and attempt to indicate the magnitude of the error introduced.

The atmospheric temperature at a glacier surface changes seasonally; thus measurements of the "surface" temperature for use with Equation (1) are generally made at some depth, z_0 , in the glacier below which the effect of these seasonal variations is negligible. If the seasonal variation can be represented by a sinusoidal function, this depth is given by:

$$z_0 = \left(\frac{2\kappa}{\omega}\right)^{\frac{1}{2}} \ln \frac{\theta_r}{\Delta} \tag{2}$$

(Carslaw and Jaeger, 1959, p. 65) where ω is the period of the fluctuations (in this case $2\pi/\text{year}$), θ_r is the temperature range from winter minimum to summer maximum, and Δ is the maximum acceptable change in temperature at depth z_0 . For example, in the dry zone of the accumulation area if we take $\kappa = 16 \text{ m}^2/\text{year}$, a value appropriate for unpacked snow, $\theta_r = 30 \text{ deg}$, and $\Delta = 0.4 \text{ deg}$, we obtain $z_0 = 10 \text{ m}$. This is the basis for the common assumption that the 10 m temperature is approximately equal to the mean annual temperature.

In the superimposed ice zone superimposed ice occurs immediately beneath the winter snow cover, and κ for ice at -10°C is about $38 \text{ m}^2/\text{year}$. Furthermore, the temperature fluctuation at the ice-snow interface cannot be approximated by a sinusoidal function because the accumulating snow cover insulates the ice during the winter, and because the temperature rises rapidly in the late spring when percolating melt water reaches the interface. Equation (2) can still be used to calculate an approximate value for z_0 of about 15 m, but due to the non-sinusoidal temperature variation at the ice-snow interface, the temperature at depth z_0 is commonly a few degrees above the mean annual atmospheric temperature.

The magnitude of this difference, which we will call $\Delta\theta$, can be calculated from Equation (1) if it is assumed that convection, internal heat generation, and transverse and longitudinal conduction are negligible, and if the proper boundary condition at the ice-snow interface is known. The equation to be solved is:

$$\frac{\partial^2\theta}{\partial z^2} = \frac{\partial\theta}{\partial t} \tag{1'}$$

The boundary condition at depth z_0 is taken to be the temperature gradient below this depth. The problem is thus reduced to one of determining the temperature of the ice-snow interface as a function of time, $\theta_s(t)$.

In the soaked zone of the snow cover rests on permeable firn rather than on superimposed ice, and melt water percolating down through the snow pack can penetrate some distance into the firn. Upon refreezing, this water releases the heat of fusion, thus warming the firn. Mathematically, this can be represented by adding an internal heat-production term to Equation (1'), thus:

$$\frac{\partial^2\theta}{\partial z^2} + \frac{Q}{\rho C} = \frac{\partial\theta}{\partial t} \tag{1''}$$

Due to this internal heat production, $\Delta\theta$ may be substantially larger in the soaked zone than in the superimposed ice zone or ablation area.

In order to determine $\theta_s(t)$ and $Q(z)$, six 30 m bore holes were drilled on the south dome of the Barnes Ice Cap. Five of these holes were along a flow line extending from the divide to the margin. Temperature measurements were made in each hole in mid-July 1973, and three times in June and July 1974. Two finite-difference calculations were carried out with the use of these data. In one, the first of the 1974 measurements was used as an initial condition, and Equation (1') was integrated, using an assumed form of $\theta_s(t)$ as a boundary condition. The form of $\theta_s(t)$ was varied until reasonable agreement was obtained between calculated and measured profiles at the times of the second and third measurements in 1974. This calculation thus permits an estimate of $\theta_s(t)$ during the critical period of melt from early June to mid-July.

In the second calculation, the July 1973 temperature profile was used as an initial condition, and Equation (1') was again integrated, this time simulating a time period of one year. In this model, snow was allowed to accumulate on the ice surface. The accumulation pattern of the snow cover as a function of time was determined from climatic records from nearby weather stations. The temperature variation at the snow-air interface was also determined from these records, using measured lapse rates to correct for differences in altitude (private communication from R. Barry in 1974). Calculations using this model suggest that three factors have a significant influence on the temperature distribution in the ice. One is the fact that most accumulation occurs in the fall and spring, with relatively little accumulation during the winter. The second is that thermal diffusivity of the snow apparently increases during the early spring as atmospheric temperatures begin to rise. The third and most important factor is that the snow-ice interface remains at 0°C late into the fall due to the presence of melt water that percolated down through the accumulating snow cover in the early fall, and the temperature of this interface rises to 0°C early in the spring, again due to percolation of melt water.

The temperature measurements and climatic records were also used to estimate values for $\Delta\theta$ on the Barnes Ice Cap. In the ablation area and superimposed ice zone $\Delta\theta$ is 2 to 4 deg. Then there is a rapid increase to about 5 deg in the lower part of the soaked zone. This jump occurs over a distance of only a couple of kilometers across the boundary between the two zones.

Perhaps the most significant consequence of this increase in surface temperature in the superimposed ice and soaked zones is that the temperature throughout this part of a glacier will also increase by approximately the same amount. Thus if the base of a glacier were not already at the pressure melting temperature up-glacier from the soaked zone, it is likely that it would rise to the pressure melting temperature somewhere beneath the soaked or superimposed ice zones. The consequences of this for basal erosion and the entrainment of morainal material need to be examined.

REFERENCE

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A MOVIE FILM ILLUSTRATING THE NUMERICAL MODELLING OF PERIODICALLY SURGING ICE MASSES

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ABSTRACT. A general two-dimensional numerical model for a typical flow line of a glacier or ice cap has been developed which results in periodical surging for certain ranges of the input parameters. The input includes the bedrock and surface-balance profiles along the flow line, some three-dimensional parameterization depending on the cross-section shape and the flow-line patterns, the flow properties of the ice, and a numerical basal lubrication factor.